Mathl Modelling, Vol. 9, No. 1, pp. 17–28, 1987
Printed in Great Britain.

0270-0255/87 $3.00 + 0.00

A COMPARATIVE EVALUATION OF BOOKS ON MATHEMATICAL MODELLING

D. N. P. MURTHY[1] and E. Y. RODIN[2]

[1]Department of Mechanical Engineering, University of Queensland, St Lucia, Queensland 4067, Australia

[2]Department of Systems Science & Mathematics, Washington University, St Louis, MO 63130, U.S.A.

Abstract—In this paper we present a comparative evaluation of books on mathematical modelling that have appeared in the last 15 years.

1. INTRODUCTION

Over the last 15 years, a number of books have appeared on mathematical models and modelling. In this paper we carry out a comparative evaluation of these books. We start with a brief discussion on mathematical models and modelling so as to define the framework for the comparative evaluation. Following this, we carry out the evaluation.

A few comments are appropriate at this stage.

(i) The books included for evaluation have as their main aim the teaching of mathematical modelling. As such, any book whose aim is to study a particular discipline using mathematical models is not included; but, of course, some such books are borderline cases.

(ii) The authors have been fairly diligent in their search, but recognize the fact that they may have overlooked several books whose main aim is teaching mathematical modelling. The authors regret this and would appreciate such omissions brought to their attention.

(iii) The main aim of the paper is to carry out a comparative evaluation and not detailed reviews of each book. Some of the books have been reviewed in the past and these are indicated.

2. MODELS AND MODELLING

The usage of the word model is large and varied in both the "everyday sense" as well as in the "technical sense". The sense in which it is used in the books discussed in this paper is given by the following definition:

> "A model is a representation of a system (or object, or phenomenon). The model is called an adequate one if it is appropriate for the purpose (or goal) in the mind of the model builder. Otherwise it is called an inadequate model."

The representation always contains less information than the system it represents. This is important, for the representation should contain only relevant information that is appropriate for the purpose in mind. As such, a model can be viewed as a simplification or idealization of the system.

Models are of many different types and we shall not go into their taxonomy. We focus our attention on two types: (i) system characterization; (ii) mathematical models.

> "A system characterization is a descriptive model of the system. The description is done in terms of variables and relationships between variables."

The system characterization can be either verbal or in terms of a flow diagram. It can be either adequate or not depending on the purpose.

> "A mathematical model is a symbolic representation involving an abstract mathematical formulation. It is called an adequate mathematical model, if it is adequate for

the purpose in mind of the model builder. Otherwise, it is called an inadequate mathematical model."

This implies that in model building one needs to define the purpose of modelling and also the criterion for testing the adequacy of the model.

The abstract formulation, involving symbols, makes no sense outside of mathematics. The symbols have precise meanings and their manipulation is dictated by well-defined rules. The abstract formulation by itself is not a model. It is by relating the symbols of the formulation to variables and relationships of the system characterization in a satisfactory manner that the abstract formulation becomes a mathematical model. This is shown schematically in Fig. 1.

The mathematical model thus involves the following three steps:

Step 1. Carry out a system characterization (which is hopefully adequate).
Step 2. Select a mathematical formulation (which is hopefully appropriate).
Step 3. Relate the system characterization to the mathematical formulation (in a satisfactory manner).

In real life rarely can one successfully carry out either Step 1 and/or Step 2 at the very first attempt. Often one starts with a system characterization which might not be an adequate representation and/or a mathematical formulation which might not be appropriate. As a result, the end product is often an inadequate mathematical model. We use the term tentative mathematical model to indicate the relating of any system characterization (adequate or not) with any mathematical formulation (appropriate or not). Once the adequacy of a tentative mathematical model is established, it becomes an adequate mathematical model.

Thus, an iterative procedure is needed, where during each iteration either Step 1 and/or Step 2 need to be modified so as to produce a sequence of improved mathematical models and, the

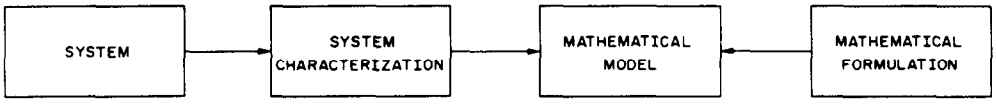

Fig. 1. Relationship between a system and its mathematical model.

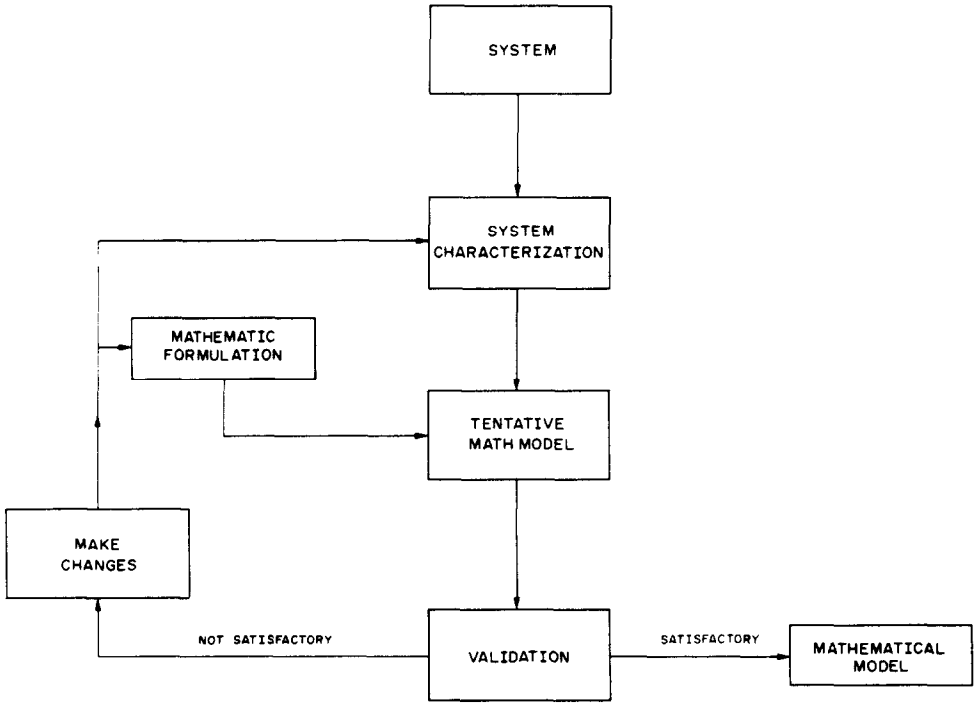

Fig. 2. The mathematical modelling process (simplified).

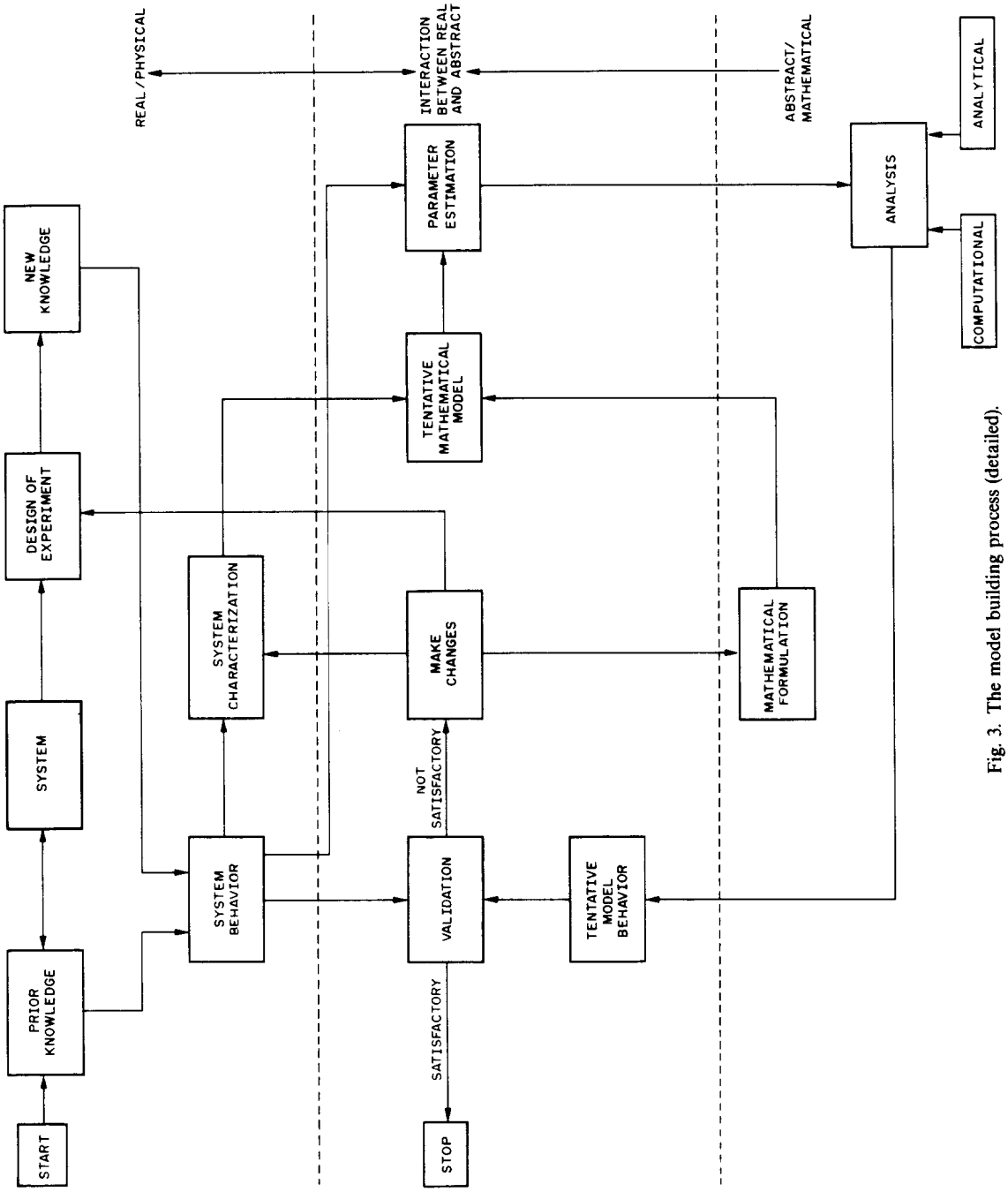

Fig. 3. The model building process (detailed).

procedure terminated once the final tentative model is validated. The iterative procedure is shown in Fig. 2. Note that one needs to define a criterion for testing adequacy. The more stringent the criterion, the more complex is the resulting mathematical model.

The diagram in Fig. 2 does not show all the features of mathematical modelling. This is shown in Fig. 3 where one can see the role of many different topics—e.g. Analysis, Parameter Estimation, Design of Experiment to name a few. Also, this diagram illustrates the demarcation between the real physical world of the system and the abstract world of the mathematical formulation and the interaction between the two.

3. FRAMEWORK FOR EVALUATION

As seen from Fig. 3, mathematical modelling requires an understanding of many different topics. Effective teaching of mathematical modelling requires that the student be exposed to all of these topics as part of the modelling subject. Broadly speaking the different topics needed for modelling are as follows:

A. Models and Model Building
B. System Characterization
C. Different Types of Mathematical Formulations
D. Analysis of Mathematical Formulations
E. Parameter Estimation
F. Design of Experiment
G. Validation.

For the purpose of comparative evaluation, we divide topics C and D into further subtopics as follows:

C1. Nondynamic Formulations (Deterministic/Probabilistic)
C2. Dynamic Deterministic Formulations
C3. Dynamic Stochastic Formulations

and

D1. Analytical Methods
D2. Computational Methods
D3. Simulation Methods.

(Note: C1–C3 and D1–D3 can be further subdivided, e.g. C2 can be subdivided into difference equations, ordinary differential equations, partial differential equations and so on. However, we stop at the subdivision indicated above.)

In addition, mathematical modelling is both an art as well as a science. It is difficult to teach the art aspect, but this can be learned by studying real-life applications or case studies. For the purpose of evaluation we include a further topic:

H. Applications/Case Studies.

This is subdivided into four subtopics as follows:

H1. Applications/Case Studies in Physical Sciences
H2. Applications/Case Studies in Biological Sciences
H3. Applications/Case Studies in Social Sciences
H4. Applications/Case Studies in Technology.

As a result, we have a total of 16 topics/subtopics and these form the basis for the comparative evaluation.

4. COMPARATIVE EVALUATION

Table 1 gives the topics/subtopics covered by each of the books included for evaluation. This is indicated by a check in the appropriate column. A few comments are appropriate at this stage:

(i) If a book concentrates on one method of analysis, we classify the book as covering that particular method of analysis and ignoring other methods.

Table 1. Comparative evaluation

Ref. No.	Topic														
	A	B	C1	C2	C3	D1	D2	D3	E	F	G	H1	H2	H3	H4
1		×		×	×	×		×				×	×	×	×
2		×		×	×	×	×	×				×	×	×	×
3	×	×	×	×		×	×	×		×	×	×	×	×	×
4	×	×	×	×	×	×						×	×	×	×
5	×	×		×		×									×
6	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
7	×	×	×	×		×		×							
8		×	×	×	×	×						×	×		
9		×	×	×	×	×						×	×		×
10	×		×			×								×	
11		×	×	×		×	×					×			×
12	×	×	×			×						×	×		×
13	×	×		×		×						×	×		×
14	×	×	×	×		×								×	
15	×	×	×	×		×						×	×	×	×
16	×	×	×	×		×	×			×			×		×
17	×	×		×		×						×		×	
18	×	×		×				×					×	×	
19	×	×	×	×		×						×	×	×	×
20	×				×	×							×		
21	×	×	×	×		×	×	×	×	×		×	×	×	×
22	×	×		×		×			×						
23		×		×			×					×			
24	×	×		×		×						×	×		×
25		×												×	
26	×	×		×								×			×
27	×	×	×							×					×
28	×	×	×	×		×	×					×	×		×
29	×	×	×	×		×		×					×	×	×
30	×	×	×	×	×	×								×	
31	×	×	×	×		×						×	×		
32	×	×	×	×		×	×					×			
33	×	×	×			×		×						×	×
34	×	×	×	×	×	×	×	×	×	×	×	×			
35	×	×	×	×		×	×						×		
36		×			×	×							×		
37	×	×	×	×	×	×		×					×	×	
38	×	×	×	×	×	×	×			×	×			×	
39	×	×		×		×		×							×
40	×		×		×			×		×					
41		×	×	×		×	×	×							×
42	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
43	×	×		×	×	×	×	×		×			×		
44	×	×	×	×	×	×							×	×	
45	×	×			×			×			×		×	×	×
46	×	×	×	×	×	×	×						×		
47		×				×						×			
48	×	×	×	×	×	×	×	×					×		
49	×	×	×			×		×						×	
50	×	×	×		×	×							×	×	
51	×	×		×		×							×		
52					×	×			×						
53	×	×	×	×	×	×	×	×	×			×	×	×	
54	×	×	×	×	×	×						×	×	×	
55		×		×		×						×			
56	×	×	×	×	×	×	×							×	
57		×		×		×	×					×	×		
58	×	×	×		×			×							×
59	×	×		×	×	×	×	×	×				×		×
60	×	×	×	×	×							×	×	×	×
61		×		×		×						×			
62	×	×			×			×					×		

(ii) If a book concentrates on some types of formulations more than other, then it is classified as examining only those types covered in greater detail.

(iii) For topics E, F and G we use the following convention: a book covers a topic if either a chapter is devoted to it or if the topic is stressed significantly throughout the book so that it is treated as being of importance in the modelling process.

(iv) For topic H (subtopics H1–H4) the basis for classification is again the relative importance given to applications/case studies from each of the four disciplines.

We briefly comment on each book.

[1] Abraham and Shaw (1982). This is the first part of a three-part series dedicated to the subject of nonlinear dynamics. Beginning with basic concepts of dynamics the book presents information on periodic behavior (classical applications, vibrations, forced vibrations, compound oscillations) visually. Each page consists of one or more four-color illustrations with brief captions and little additional text. Among the subjects included are penduli, buckling columns, percussion instruments, predators and prey, wind and bowed instruments and radio transmitters.

[2] Abraham and Shaw (1983). Part 2 of this series on nonlinear dynamics concerns chaotic behavior. The book visually describes the chaotic limit sets and attractors of theory. Poincare, Birkhoff, Lorenz and Rossler examples as well as the chaotic aspects of these examples, unpredictability, divergence, expansion, are presented in detail.

[3] Ahuja and Schachter (1983). This book presents formal models of spatial patterns. Examining the sequence of steps involved in modelling beginning with model conception analysis, fitting and ending with synthesis of data, the book illustrates the various phases of modelling and synthesis.

[4] Andrews and McLone (1976). This is a collection of modelling exercises written by different authors. Seven of the exercises are from either physical sciences or technology; two from biological sciences and five from social sciences. In addition there is a chapter each on network flow models and role of catastrophy method. The main emphasis is on model formulation.

[5] Aris (1978). The book concentrates mainly on the use of ordinary differential equation formulations (lumped models) and partial differential equation formulations (distributed modes) in modelling systems and processes in chemical engineering. The emphasis is on modelling and analytical methods of analysis. The appendices examine three case studies.

For a review of this book, see L. A. Segel, *SIAM Rev.* **21,** 571–572 (1979).

[6] Avula et al. (1984). This is a volume of papers presented at the *4th International Conference on Mathematical Modelling.* It contains seven invited lectures concerning topics such as deterministic control, multicriterion decision making, and the use of iterative graphics. Other subjects covered extensively include methodology, systems theory, parameter estimation and system identification, optimization, stochastic modelling, biomedical models, modelling in economics, transportation and mathematical modelling education.

[7] Bender (1978). This is a rather unusual book. It is divided into two parts. Part I deals mainly with models involving nondynamic formulations. The chapter entitled "Arguments from Scale" is interesting. Part II deals with models involving simple ordinary differential equation formulations. The model building and analysis are strongly interwoven and the range of applications considered is large and varied.

For a review of this book, see E. Butz, *SIAM Rev.* **21,** 402–404 (1979).

[8] Bleustein (1973). This is a collection of 20 papers delivered in a symposium devoted to the theme of "mechanics and sport". Analyses of the mechanics of human motion as well as the design of sports equipment are among the topics covered.

[9] Boyce (1981). This is a collection of seven case studies written by different authors. The topics covered are: herbicide resistance; elevator system; traffic flow; semiconductor crystal growth; railroad/information (network flow models); computer data communication; and system security verification. The case study on semiconductor crystal growth (by L. O. Wilson) is especially interesting for it describes the evolution of the modelling process very nicely.

[10] Boynton (1980). This is an introductory book that combines the logic of mathematics with the art of politics. It assumes a knowledge of algebra to study discrete time systems. Applications of the book include such special issues as policy outputs, voting distributions and behavior regulation.

[11] Bradley et al. (1981). This is a collection of seven case studies to model systems or processes from different industries and written by different authors. The topics covered are blast furnace; sound distortion in a record player; hydraulic buffers; grinding of particles; moving fresh glass; motion of automobile; and freezing and thawing of meat. Each case study is a result of group activity and often the group leader presents his modelling attempt and compares it with that of the group.

For a review of this book, see H. I. Freedman, *SIAM Rev.* **25,** 409–410 (1983).

[12] Brams et al. (1983). This book is a collection of 13 case studies written by different people.

Each case study deals with the modelling and analysis of a topic from political or related field. In some cases the model analysis is compared with the real data. Each case study ends with notes for the Instructor.

[13] Braun et al. (1983). This book deals exclusively with models involving differential equation formulations. It consists of 23 chapters divided into five parts. Part 1 is general. Part 2 deals with modelling exercises involving first-order differential equation formulations. Part 3 (4) deals with three (four) modelling exercises involving higher-order linear differential equation (partial differential equation) formulations. Part 5 deals with modelling different facets of traffic flow and Part 6 deals similarly with population interactions.

For a review of this book, see R. B. Guenther, *SIAM Rev.* **26,** 297–298 (1984).

[14] Burghes and Wood (1980). This book deals mainly with very simple mathematical formulations; their analysis and application to problems in social science. The emphasis is on deterministic nondynamic formulations, with a chapter each on difference and first- and second-order differential equations.

[15] Burghes et al. (1982). This book consists of four parts. Part 1 deals with interesting case studies. Part 2 is a collection of 20 simple modelling exercises involving mainly deterministic formulations (either nondynamic or simple ordinary differential equation formulations). In some cases real data is used for modelling and validation. Part 3 deals with the modelling process in a formal sense and the last part contains 26 topics suitable for student modelling assignments.

[16] Clark (1976). This book discusses mathematical models applicable to the optimal utilization of biological resources focusing on conservation of productive resources rather than the preservation of natural environments. A general model is put forth and then in succeeding chapters mathematical techniques of optimal control theory and dynamical systems are introduced.

[17] Close and Frederick (1978). This book provides exposure to dynamic systems. Assuming a knowledge of differential and integral calculus the authors cover mechanical, electrical, electro-mechanical, thermal and hydraulic systems. They also include five case studies drawn from a variety of systems, including a sociological system, a thermal system and a velocity control system.

[18] Coats and Parkins (1977). This book concentrates on computer modelling for social sciences. The first five chapters deal with models and modelling. The last three chapters deal with three case studies. The topics for the case studies are: human memory; bed usage in hospitals; and economics of education.

[19] Dym and Ivey (1980). This book is unusual in its presentation. It consists of two parts. Part A deals with what the authors call "mathematical ideas of interest to the model builder" and contains four chapters. The chapters entitled "Dimensional Analysis" and "Scaling" are particularly interesting and extremely important in model building. Part B deals with applications and contains 10 chapters. A chapter each is devoted to ordinary differential equation formulation models and linear programming models. Two chapters deals with oscillations and there are chapters on traffic flow and diffraction.

[20] Ewens (1968). This book investigates the changes in the genetic structure of populations brought about by selection, mutation, migration, inbreeding and random changes deriving from chance events. The author's aim is to present the information for mathematicians rather than biologists and as such a glossary of genetical terms is included.

[21] Giordano and Weir (1985). This book places emphasis on identifying problems and their underlying assumptions, determining appropriate data to be collected, constructing and selecting models, and testing and validating models. It provides numerous examples and applications in mathematical sciences, operations research, engineering, social and life sciences and management science.

[22] Goel et al. (1971). This book extensively examines the Volterra model of different biological species. It is divided into 10 sections. The first section introduces the Volterra model and discusses its implications for the two-species case. Sections 2–9 continue the discussion and finally, Section 10 gives some examples and tests the Volterra model's validity for two or three species. The book also applies statistical mechanics to many-species case within the same model.

[23] Greenspan (1973). This book concentrates on the use of difference equation formulations to model a range of problems in dynamics and fluid mechanics. The emphasis is solely on model building.

[24] Haberman (1977). This book examines modelling in three areas: mechanical vibrations; population growth; and traffic flow. In each area a series of models are developed with a lot of emphasis on the analysis and interpretation of the model. The book is unusual in the sense that system understanding, modelling and analysis are strongly interwoven.

For a review of this book, see H. I. Freedman, *SIAM Rev.* **21,** 43–44 (1979).

[25] Hawkes (1973). This is a collection of articles dealing with trends in mathematical modelling in mainly social sciences.

[26] Howard (1979). This book deals with partial differential equation formulations in modelling problems in particle dynamics, fluid mechanics, cosmology and application of models in aeronautical engineering.

[27] Hyvarinen (1970). This book deals mainly with the use of nondynamic formulations (especially regression formulations) in modelling industrial processes and contains a chapter on design of experiments and two chapters on the analysis of experimental results.

[28] James (1978). This book is a revised version of the *Proceedings of the Conference on the Use of Mathematical Modelling in Water Pollution Control.* Nineteen papers are included. Techniques of water pollution control systems are developed, as are models that apply to polluted environments, waste treatment and water resources.

[29] James and McDonald (1981). This book contains 15 case studies written by different authors. They cover a wide range of topics and the emphasis is mostly on model building. One or two case studies involve simulation. The models involve mainly nondynamic formulations and the dynamic models discussed are based on simple ordinary differential equation formulations.

[30] Kemeny and Snell (1975). This is a reprint of an old book first published in 1962. It deals with seven case studies from social sciences and one from biological science. The topics covered include: ecology; money flow; organization; scheduling; and a few others. The modelling involves both deterministic and stochastic formulations.

[31] Lancaster (1976). This book contains chapters on growth of populations, population genetics and collision of particles. In particular, the last modelling exercise discusses models under different assumptions and laws. In addition there are chapters on linear programming and on search for maximum.

[32] Lin and Segal (1974). This is a textbook introduction to applied mathematics. It is concerned with the construction, analysis and interpretation of mathematical models with applications in particular to the natural sciences.

[33] Lucas et al. (1983). This book contains a large number of modelling exercises dealing with application to economics, traffic theory, operations research and other fields. The models involve mainly nondynamic formulations. A few of the exercises involve computer simulation but the majority use analytical methods of analysis. The range of topics covered is fairly wide. There are two interesting case studies involving problems in power engineering.

For a review of this book, see M. S. Klamkin, *SIAM Rev.* **26,** 606–608 (1984).

[34] Marchuk and Kagan (1984). This book is an extensive study of ocean tides. It is divided into six chapters. In the first chapter definitions are introduced concerning tidal potential and tidal dynamics. Chapter 2 studies the equations of tidal dynamics. Chapter 3 introduces numerical methods of solution of those equations. Chapter 4 gives real examples and Chapters 5 and 6 expand the definitions to theoretical examples.

[35] Marchuk and Nisevich (1980). This book concerns the application of mathematical theory to medicine. In particular it introduces a mathematical model of viral disease and then develops methods for quantitative assessment of the seriousness of viral hepatitus. These models then are used to study the dynamics of functional recovery.

[36] Marcus-Roberts and Thompson (1982). This book is a collection of 12 case studies in biological sciences written by different authors. The case studies involve both deterministic and stochastic dynamic formulations and in some of the studies real data is used. The emphasis is on models suitable for analytical methods of analysis. Each chapter contains notes at the end for the instructor.

[37] Maki and Thompson (1973). This book takes an axiomatic approach to model building. It discusses modelling topics from biological and social sciences involving both deterministic and stochastic dynamic model formulations. The last chapter entitled "Practical Aspects of Model

Building" discusses intuitive evaluation, statistics for the model building process and simulation.

[38] Malkewich and Meyer (1973). This is a game theoretical work, concentrating on graph theoretical approaches. Graphs, digraphs, paths and connectivity are introduced. Functional models, probability and statistics and their roles are discussed. The principal application is to the theory of electrons. There are 11 chapters and appendices.

[39] McClamroch (1980). This book concentrates on models involving mainly ordinary differential equation formulations. Seven chapters are devoted to differential equation models. In each chapter several interesting case studies/applications are discussed. Both modelling and analysis are equally emphasized. Most of the case studies come from engineering and an appendix deals with CSMP simulation.

For a review of this book, see R. B. Guenther, *SIAM Rev.* **24,** 234–235 (1982).

[40] Mihram (1972). This book concentrates mainly on probabilistic nondynamic and discrete stochastic process formulations. The emphasis is on computer simulation. In addition, it contains two chapters on the design of experiment.

[41] Nicholson (1981). This book is a collection of seven case studies from various branches of engineering. The topics for the case studies are: iron and steel making; electric arc furnaces; turbo generator; mineral extraction and manufacturing. The models involve either nondynamic or simple differential equation formulations.

[42] Noble (1967). This book is based on examples of applications of undergraduate mathematics in engineering, submitted by various university mathematics and engineering departments and some industrial companies. The four sections of the book deal with applications of elementary mathematics, ordinary differential equations, field problems such as flow of fluids or heat, the diffusion of gases, stresses and strains in solids, linear algebra and probability.

[43] Okubo (1980). This book gives a mathematical treatment of the essential problem of ecology, the investigation of the interrelations between the distribution and quantity of organisms and their environment. Using stochastic mathematical models primarily the book introduces the problem and gives numerous examples.

[44] Olinick (1978). This book contains 14 chapters, with 10 devoted to modelling specific topics. In each modelling chapter often more than one model is built and analysed. The interesting aspect is the historical perspective for modelling topics. The models are mainly from biological and social sciences and involve a range of mathematical formulations.

[45] Osborne and Watts (1977). This is a collection of several articles with the emphasis on simulation. Six of them deal with case studies from mainly biological and social sciences. One chapter deals with the use of statistics in model validation.

[46] Pielou (1969). This book is based on a one-semester course on mathematical ecology. It is divided into four chapters. Chapter I introduces definitions in population dynamics. Chapter II uses population dynamics to examine spatial patterns in one-species populations. Chapter III expands this to two or more species. Finally Chapter IV concerns many-species populations.

[47] Pollard (1972). This is an introductory book on applied mathematics. First it defines falling bodies, harmonic motion, path of quickest descent and qualitative problems. Then the topics covered are the two-body problem, optimization, Hamilton's and Lagrange's equations, Laplace's equation, the wave equation and the heat equation.

[48] Ransom (1981). This book describes the uses and applications of models in development biology. A variety of model systems are discussed in the first half of the book. The second half concentrates on computer modelling.

[49] Rivett (1972). This book contains 12 chapters and a summary. It deals with the philosophical questions of model building first, followed by applications to business and economics. Investment, sequenced divisions, decision and utility theory and competitive problems are some of the subjects to which individual chapters are devoted.

[50] Roberts (1976). This book deals mainly with two types of formulations—graph theoretic and Markov chains—and their use in modelling a range of topics in social and biological sciences. This is an excellent book for the use of graph theoretic representations in system characterization. The approach is mainly axiomatic and the emphasis is on analytical methods.

For a review of this book, see R. M. Thrail, *SIAM Rev.* **21,** 424–425 (1979).

[51] Rosen (1967). This book deals with mathematical biology in an overview form. It exhibits

simply some of the basic interrelationships between diverse areas of mathematics, biology, physics and social sciences. It describes in detail mathematical techniques and applies them to the vascular system, ontogeny and phylogeny, optimality, homeostasis and input–output systems.

[52] Ross (1972). This book discusses various stochastic dynamic formulations and their use in modelling. In particular, it examines Markov chains, Poisson processes and renewal theory. The application of these to queues and reliability are discussed. The final chapter discusses parameter estimation for some of the formulations.

[53] Saaty (1973). This book consists of the notes used as a summer seminar on mathematical modelling. The notes are divided into three parts. Part I is a general discussion of mathematics and behavioral sciences with emphasis on model planning and building and the process of decision making. Part II investigates eight topics central to mathematical modelling of social sciences: allocation and optimization, conflict resolution, scheduling tasks to machines, flow and synthesis problems in networks, inventory control, queues and congestions, reliability, maintenance and renewals and search. Part III describes some structured and unstructured problems. Among these are, the travelling salesman problem, arborescences and questionnaires, epidemics and stochastic barriers and parking at an aircraft terminal.

[54] Saaty and Alexander (1981). This book is divided into three parts. Part 1 deals with methodology; Part 2 with the "framework" and Part 3 with applications. The different types of formulations considered in the book are given in Part 2. Each formulation is illustrated by a simple application involving the formulation. The chapter entitled "Equations" deals with different types of equation formulations and the chapter entitled "Stochastic Processes" looks at a range of stochastic formulations. The applications cover a wide range of topics and the emphasis is on model building and analysis.

[55] Segel (1977). This text develops and uses mathematics to analyze continuum models of fluid flow and solid deformation. The volume is divided into four parts: geometrical prerequisites for three-dimensional continuum mechanics, problems in continuum mechanics, water waves and extremum principles. The book addresses such topics as directional derivatives, change of variables in multiple integrals, line and surface integrals and the divergence theorem.

[56] Selby (1973). This volume contains six lectures given at a summer seminar on mathematical modelling, given by six different lecturers. The topics addressed are: general equilibrium political science—voting and bargaining; anthropology—kinship and clustering; sociology—algebra of dichotomies; psychology—measurements and psychophysics; and scaling and measurement—multidimensional scaling.

[57] Shepp (1983). This is the 27th volume of the *Proceedings of Symposia in Applied Mathematics.* The subject covered, computed tomography, is discussed in seven papers. Each paper suggests problems, both relevant and of "intrinsic interest" to mathematicians. The volume addresses the mathematical modelling problem of finding the appropriate level of abstraction in a mathematical model of a real-world situation.

[58] Smith (1974). This book extensively covers the subject of ecology using mathematical models. It assumes very little mathematical knowledge, although there is a great deal of mathematics present throughout the book. Topics addressed include predator–prey systems without age structure, breeding seasons and age structure, predator–prey systems with age structure, competition, migration, stability and complexity, the statistical mechanics of populations, complexity at a single trophic level, complexity with several trophic levels, coevolution and territorial behavior. Two kinds of ecological models are used: a strategic kind and a laboratory kind; both aimed at an understanding of the general principles of ecosystems.

[59] Spriet and Vansteenkiste (1982). This book examines the use of stochastic difference equation formulations and deterministic partial differential equation formulation. Parameter estimation is discussed in great detail for stochastic difference equations. The numerical solution of partial differential equations are discussed along with software packages. The application of partial differential equation to environmental problems is discussed briefly. In addition, there is a chapter on hardware trends and their impact on simulation.

[60] Weinberg (1975). This is an introductory book which assumes a knowledge of high-school algebra. The book is divided into seven chapters, each of which ends with a number of exercises. The book also includes an appendix on how to read mathematical notation. The chapters are

titled: the problem; the approach; system and illusion; interpreting observations; breaking down observations; describing behavior; and some systems questions.

[61] Zierep (1971). This book discusses dimensional analysis, similarity laws and dimensionless parameters. It uses the study of classical mechanical and thermodynamic similarity laws and the similarity laws of gas dynamics to cover subjects such as linearized subsonic and supersonic flow, transonic and hypersonic flow and a brief survey of corresponding flows with relaxation.

[62] Ziegler (1976). This book concentrates on developing a theory of modelling and simulation. The system characterization is done with the aim of stimulating the model on a computer. Various aspects of simulation are discussed in great detail. Appendix A deals with a case study—Modelling the Human Brain. This book is ideally suited for a second (or advanced) level course in modelling.

5. CONCLUSIONS

(1) As seen from Table 1, the major shortcoming is that topics E, F and G tend to receive very little attention relative to other topics. This is undesirable, for model building can be compared to a chain and the chain is only as strong as its weakest link. Most books surveyed deal only with some of the topics but any good book on modelling must treat all the topics needed for modelling with equal importance.

(2) Some of the books are better suited for lower levels of undergraduate programs as they require very little mathematical background. In contrast, others are better suited for use in senior level undergraduate program. Also, the range of applications and case studies make some better suited for a particular discipline.

(3) There is a wide variation in the style of presentation. Some cover the formulations and analysis before proceeding to case studies, while others interweave the two. It is debatable whether one is better than the other. The usefulness of a particular book as a text will depend on the method of approach used by the instructor. Many different methods of teaching have been reported in the literature.

(4) A weakness in most applications and case studies is the incomplete nature of the study. Very few studies validate the tentative mathematical model and still fewer discuss the evolution of the model with discussion on the rationale for changes made in the evolutionary process. Most books describe an application briefly and proceed merely with the mathematical analysis. This is a serious drawback and future books would need to rectify this.

REFERENCES

1. R. H. Abraham and C. D. Shaw, *Dynamics. The Geometry of Behavior, Part I: Periodic Behavior.* Aerial Press, Calif. (1982).
2. R. H. Abraham and C. D. Shaw, *Dynamics. The Geometry of Behavior, Part II: Chaotic Behavior.* Aerial Press, Calif. (1983).
3. N. Ahuja and B. Schachler, *Pattern Models.* Wiley-Interscience, New York (1983).
4. J. G. Andrews and R. R. McLone, *Mathematical Modelling.* Butterworths, London (1976).
5. R. Aris, *Mathematical Modeling Techniques.* Pitman, Mass. (1978).
6. X. J. R. Avula, R. E. Kalman, A. I. Liapis and E. Y. Rodin, *Mathematical Modelling in Science and Technology.* Pergamon Press, New York (1984).
7. E. A. Bender, *An Introduction to Mathematical Modelling.* Wiley-Interscience, New York (1978).
8. J. L. Bleustein, *Mechanics and Sport.* American Society of Mechanical Engineers, New York (1973).
9. W. E. Boyce (Ed.), *Case Studies in Mathematical Modelling.* Pitman, Mass. (1981).
10. G. R. Boynton, *Mathematical Thinking About Politics.* Longman, New York (1980).
11. R. Bradley, R. D. Gibson and M. Cross, *Case Studies in Mathematical Modelling.* Wiley-Interscience, New York (1981).
12. S. J. Brams, W. F. Lucas and P. D. Staffins (Eds), *Political and Related Models.* Springer, New York (1983).
13. M. Braun, C. S. Coleman and D. A. Drew (Eds.), *Differential Equation Models.* Springer, New York (1983).
14. D. N. Burghes and A. D. Wood, *Mathematical Models in the Social, Management and Life Sciences.* Halsted Press, (1980).
15. D. N. Burghes, Huntley and McDonald, *Applying Mathematics: A Course in Mathematical Modelling.* Halsted Press (1982).
16. C. W. Clark, *Mathematical Bioeconomics: the Optimal Management of Renewable Resources.* Wiley-Interscience, New York (1976).
17. C. M. Close and D. K. Frederick, *Modeling and Analysis of Dynamic Systems.* Houghton-Miffin, Boston, Mass. (1978).
18. R. B. Coals and A. Parkins, *Comparative Models in the Social Sciences.* Little, New York (1977).
19. C. L. Dym and E. S. Ivey, *Principles of Mathematical Modeling.* Academic Press, New York (1980).
20. W. J. Ewens, *Population Genetics.* Methuen, London (1969).

21. F. R. Giordano and M. D. Weir, *A First Course in Mathematical Modeling*. Brooks/Cole, Monterey, Calif. (1985).
22. N. S. Goel, S. C. Maitra and E. W. Montroll, *On the Volterra and other Nonlinear Models of Interacting Populations*. Academic Press, New York (1971).
23. D. Greenspan, *Discrete Numerical Methods in Physics and Engineering*. Pergamon Press, New York (1973).
24. R. Haberman, *Mathematical Models: Mechanical Vibrations, Population, Dynamics and Traffic Flow. An Introduction to Applied Mathematics*. Prentice-Hall, Englewood Cliffs, N.J. (1977).
25. N. Hawkes (Ed.), *International Seminar on Trends in Mathematical Modelling*. Springer, New York (1973).
26. J. C. Howard, *Mathematical Modelling of Diverse Phenomena*. NASA Sci. & Tech. Inf. Branch, Washington, D.C. (1979).
27. L. P. Hyvarinen, *Information Theory for Systems Engineering*. Springer, New York (1970).
28. A. James, *Mathematical Models in Water Pollution Control*. Wiley-Interscience, New York (1978).
29. D. J. G. James and J. J. McDonald, *Case Studies in Mathematical Modelling*. Wiley, New York (1981).
30. J. G. Kemeny and G. L. Snell, *Mathematical Models in the Social Sciences*. MIT Press, Cambridge, Mass. (1975).
31. P. Lancaster, *Mathematics Models of the Real World*. Prentice-Hall, Englewood Cliffs, N.J. (1976).
32. C. C. Lin and L. A. Segel, *Mathematics Applied to Deterministic Problems in the Natural Sciences*. Macmillan, New York (1974).
33. W. F. Lucas, Roberts and Thrall, *Modules in Applied Mathematics*. Springer, New York (1983).
34. G. I. Marchuk and B. A. Kagan, *Ocean Tides*. Pergamon Press, Oxford (1984).
35. G. I. Marchuk and N. I. Nisevich, *Mathematical Methods in Clinical Practice*. Pergamon Press, Oxford (1980).
36. H. Marcus-Roberts and M. Thompson (Ed.), *Life Science Models*. Springer, New York (1982).
37. D. P. Maki and M. Thompson, *Mathematical Models and Applications*. Prentice-Hall, Englewood Cliffs, N.J. (1979).
38. J. Malkewich and W. Meyer, *Graphs, Models, and Finite Mathematics*. Prentice-Hall, Englewood Cliffs, N.J. (1973).
39. N. H. McClamroach, *State Models of Dynamic Systems*. Springer, New York (1980).
40. G. A. Mihram, *Simulation: Statistical Foundations and Methodology*. Academic Press, New York (1972).
41. H. Nicholson, *Modelling of Dynamical Systems*, Vol. 2. Peregrinus, England (1981).
42. B. Noble, *Applications of Undergraduate Mathematics in Engineering*. Macmillan, New York (1967).
43. A. Okubo, *Diffusion and Ecological Problems: Mathematical Models*. Springer, New York (1980).
44. M. Olinick, *Air Introduction to Mathematical Models in Social and Life Sciences*. Addison-Wesley, Reading, Mass. (1978).
45. M. R. Osborne and R. O. Watts (Ed.), *Modelling and Simulation*. Univ. of Queensland Press, Brisbane, Qld (1977).
46. E. C. Pielou, *An Introduction to Mathematical Ecology*. Wiley-Interscience, New York (1969).
47. H. Pollard, *Applied Mathematics: An Introduction*. Addison-Wesley, Reading, Mass. (1972).
48. R. Ransom, *Computers and Embryos Models in Developmental Biology*. Wiley, New York (1981).
49. P. Rivell, *Model Building for Decision Analysis*. Wiley-Interscience, New York (1972).
50. F. S. Roberts, *Discrete Mathematical Models*. Prentice-Hall, Englewood Cliffs, N.J. (1976).
51. R. Rosen, *Optimality Principles in Biology*. Plenum Press, New York (1967).
52. S. M. Ross, *Introduction to Probability Models*. Academic Press, New York (1972).
53. T. L. Saaty, *Topics in Behavioral Mathematics*. Mathematical Association of America, (1973).
54. T. L. Saaty and J. M. Alexander, *Thinking with Models*. Pergamon Press, Oxford (1981).
55. L. A. Segel, *Mathematics Applied to Continuum Mechanics*. Macmillan, New York (1977).
56. I. A. Selby, *Notes of Lectures on Mathematics in the Behavioral Sciences*. Mathematical Association of America, (1973).
57. L. A. Shepp, *Computer Tomography*. American Mathematical Society (1983).
58. J. M. Smith, *Models in Ecology*. Cambridge Univ. Press, London (1974).
59. J. A. Spriet and G. Vansteenkiste, *Computer Aided Modelling and Simulation*. Academic Press, New York (1982).
60. G. M. Weinberg, *An Introduction to General Systems Thinking*. Wiley-Interscience, New York (1975).
61. J. Zierep, *Similarity Laws and Modeling*. Dekker, New York (1971).
62. B. P. Ziegler, *Methodology in Systems Modelling and Simulation*. Elsevier, New York (1976).