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Inflationary solution to the strong CP and μ problems

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Abstract

We show that the vacuum expectation value of the inflaton at the Peccei–Quinn axion scale can generate the supersymmetric Higgs mass μ term. This provides an inflationary simultaneous solution to the strong CP problem and the μ problem of the minimal supersymmetric Standard Model, and gives a testable prediction for the μ parameter: $\mu^2 \approx (0.25\text{–}0.5)m_0^2$, where m_0 is the soft Higgs scalar mass. Our model involves a very small Yukawa coupling of order 10^{-10}, which could originate from an extra-dimensional scenario or type I string theory.

1. Introduction

The μ problem of the minimal supersymmetric Standard Model (MSSM), the origin of the supersymmetric Higgs mass parameter $\mu H_u H_d$ where H_u, H_d are the two Higgs doublets and μ is of the same order of magnitude as the soft supersymmetric (SUSY) breaking parameters, has long been a puzzle [1]. Another puzzle is the physical nature of the scalar field which drives cosmological inflation, known as the inflaton field. It is well known that the inflaton cannot be identified with the Higgs fields of either the Standard Model or one of its SUSY extensions, and there are few physical candidates for the inflaton field in the literature [2].

The possible connection between the strong CP problem and the μ problem in supersymmetry was explored some time ago [3], and a non-renormalisable operator responsible for generating the μ term was proposed in [4]. The first simultaneous solution to the strong CP problem and μ problem based on *renormalisable* operators was proposed in [5]. In [5] the μ term is generated by the VEV of a singlet field N, in a similar way to the next-to-minimal supersymmetric Standard Model (NMSSM) [6,7]: $\lambda N H_u H_d \to \mu H_u H_d$, where $\mu = \lambda\langle N\rangle$. However, whereas in the NMSSM the vacuum expectation value (VEV) of the singlet field N takes a value of order the electroweak breaking scale, in [5] its VEV is of order the Peccei–Quinn symmetry breaking scale [8], allowing an invisible axion solution to the strong CP problem [9,10]. Since the μ parameter must be of order the TeV scale, this implies that the dimensionless Yukawa cou-

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pling λ must be extremely small, possibly of order 10^{-10} [5].

The scenario proposed in [5] also provides a model of inflation since the NMSSM operator κN^3 is replaced by the operator $\kappa\phi N^2$, where ϕ is identified as the inflaton field and N as the waterfall field of hybrid inflation [5]. Whereas the NMSSM operator κN^3 is responsible for a $\mathbb{Z}_3$ symmetry, leading to problems with cosmological domain walls when it breaks, the term $\kappa\phi N^2$ permits a global $U(1)_{\rm PQ}$ symmetry, leading to a solution to the strong CP problem [5]. It also allows hybrid inflation providing the dimensionless Yukawa couplings satisfy $\lambda \sim \kappa \sim 10^{-10}$. Such small Yukawa couplings could arise from an extra-dimensional scenario due to volume suppression [11]. Note that the presence of the term $\kappa\phi N^2$ is crucial not only to allow hybrid inflation to proceed but also to stabilise the potential in a natural way.[1]

In this Letter we discuss a model in which the μ term is provided by the same inflaton field which drives the superluminal expansion of the early universe. To be precise, we suggest a simultaneous solution to the strong CP and μ problems in the framework of hybrid inflation in which the μ term is generated by an operator $\lambda\phi H_u H_d$ where ϕ is the inflaton field. The μ term is then generated by the VEV of the inflaton field ϕ at the end of inflation: $\lambda\phi H_u H_d \to \mu H_u H_d$ where $\mu = \lambda\langle\phi\rangle$. We shall also require a term $\kappa\phi N^2$ which is crucial to maintain the stability of the potential, where N still plays the part of the waterfall field in hybrid inflation. The above variation is interesting since, unlike the original version of the model, it leads to a testable prediction of the μ parameter: $\mu^2 \approx (0.25\text{–}0.5)m_0^2$, where m_0 is the soft Higgs scalar mass.[2] The generation of the μ term by the inflaton field also implies deeper connections between SUSY Higgs phenomenology, inflation, and the strong CP problem, and from a theoretical point of view admits a type I string theory embedding [13].

We shall first outline the particle content and interactions of our model. Then, in Section 3 we discuss the potential and the minimum reached at the end of inflation. To stabilise this minimum and end inflation we must require that the ratio of the soft mass and the trilinear falls within a certain range which leads to the above prediction for the μ parameter. Then, in Section 4, we review some basic inflationary requirements. Section 5 concludes the Letter.

2. The model

To begin we define the model in terms of a superpotential and the soft potential:

$$W = \lambda\phi H_u H_d + \kappa\phi N^2, \tag{1}$$

$$V_{\rm soft} = V(0) + \lambda A_\lambda \phi H_u H_d + \kappa A_\kappa \phi N^2 + \text{h.c.} + m_0^2\big(|N|^2 + |H_u|^2 + |H_d|^2\big) - m_\phi^2|\phi|^2. \tag{2}$$

Here ϕ and N are, respectively, the inflaton and waterfall fields and are singlets of the MSSM gauge group responsible for inflation. The Higgs fields H_u, H_d have standard MSSM quantum numbers. The dimensionless couplings λ, κ are $\mathcal{O}(10^{-10})$, and we have assumed a common scalar soft mass squared for the Higgs and N fields, but allowed a different (lighter) negative, soft mass squared for the inflaton field ϕ in order to satisfy the slow roll conditions and yield an acceptable inflationary trajectory.

The generation of the μ term is similar to that of the NMSSM, but the NMSSM is plagued by domain walls [14–17] (associated with breaking a discrete symmetry) created in the early universe. Our model does not face this problem since it does not have an N^3 term and therefore replaces the discrete $\mathbb{Z}_3$ symmetry with the continuous PQ symmetry mentioned above. The PQ domain wall problem is discussed in Section 4. The charges of ϕ, N and the Higgs under the PQ symmetry must satisfy the following requirements:

$$Q_\phi + Q_{H_u} + Q_{H_d} = 0, \qquad Q_\phi + 2Q_N = 0 \tag{3}$$

and the quark fields have the usual axial PQ charges.

[1] Models with only the term $\lambda N H_u H_d$ have also subsequently been considered [12], but without the additional term $\kappa\phi N^2$ the vacuum is not necessarily stable. S.K. is grateful to R. Nevzorov for pointing this out.

[2] This soft mass is assumed to be universal for both H_u, H_d and the N field. This universality is a feature of the model's type I string construction, derived in [13].

3. The potential

In this section we construct and minimise the potential and calculate the VEVs relevant to our model. We initially search the potential in the region of zero Higgs VEV post inflation. For our model to map on to the MSSM at low energies the Higgs must be minimised at zero at high scales. Subsequently radiative electroweak symmetry breaking (EWSB) then occurs in the usual way, resulting in non-zero Higgs VEVs at low energy. We shall not discuss this radiative EWSB mechanism further in this paper, since it is well known, but instead shall confine our attention to showing that the Higgs VEVs are indeed zero at high energy. Thus the VEV of the inflaton generates an effective TeV scale μ term, leading to an effective MSSM theory valid below the PQ scale with standard EWSB.

For the first step in the derivation we write down the relevant parts of the supersymmetric scalar potential (derived from the superpotential Eq. (1)) and the soft scalar potential:

$$V_{\text{susy}} = \left|\lambda H_u H_d + \kappa N^2\right|^2 + \lambda^2|\phi H_u|^2 + \lambda^2|\phi H_d|^2 + 4\kappa^2|\phi N|^2, \tag{4}$$

$$V_{\text{soft}} = V(0) + \lambda A_\lambda \phi H_u H_d + \kappa A_\kappa \phi N^2 + \text{h.c.} + m_0^2\left(|H_u|^2 + |H_d|^2 + |N|^2\right) - m_\phi^2|\phi|^2. \tag{5}$$

The full scalar potential is given by $V = V_{\text{susy}} + V_{\text{soft}}$. Henceforth, for this section, we set $\lambda = \kappa$, $A_\lambda = A_\kappa$. This is done for simplicity here, but can be justified in terms of an explicit high scale type I string model.

Since the Higgs fields will eventually achieve TeV scale VEVs, whereas the N and ϕ fields achieve PQ scale VEVs, their contribution to the energy density will be quite negligible.[3] Of course one must check that the higgses do not also receive PQ scale VEVs, and that their zero tree-level VEVs represent a stable vacuum, which we will subsequently do. Minimising the tree level potential gives

$$\langle\phi\rangle = -\frac{A_\lambda}{4\lambda}, \tag{6}$$

$$\langle N\rangle = \pm\frac{A_\lambda}{2\sqrt{2}\lambda}\sqrt{1-\frac{4m_0^2}{A_\lambda^2}}, \tag{7}$$

$$\langle H_u\rangle = \langle H_d\rangle = 0, \tag{8}$$

where we have assumed that $m_\phi \approx 0$. We will refer to this as the "good" minimum as it is phenomenologically preferred.

Looking back at Eq. (1) we see that when ϕ moves to its VEV we obtain a supersymmetric mass term for the higgses, a μ term:

$$\mu = -\lambda\frac{A_\lambda}{4\lambda} = -\frac{A_\lambda}{4}. \tag{9}$$

Since λ is the only dimensionless coupling in Eq. (4) μ automatically appears at the electroweak scale.

The soft mass parameters are constrained by inflationary requirements, and this will lead to the prediction of the μ parameter in our approach. The requirement that inflation ends implies $A_\lambda^2 > 4m_0^2$ as a necessary condition. If $A_\lambda^2 \leqslant 4m_0^2$ then N only has a minimum at zero and never destabilises to end inflation. In our model we have this bound and an additional upper bound on the trilinears which we will now derive.

Now we need to show that the "good" solution is a minimum of the potential (in the absence of radiative corrections). It is important to check that $\langle H_{u/d}\rangle = 0$ since we do not want electroweak symmetry to be broken at the high scale. In order to check this we first need to locate the turning points to ensure that $H_u = H_d = 0$ is a valid solution. Then we must examine this point to see if it is a minimum.

Solving $\frac{\partial V}{\partial H_u} = 0$ for H_u gives us turning points for H_u and, since the potential is symmetric under interchange of H_u and H_d, the solutions to $\frac{\partial V}{\partial H_d} = 0$ and $\frac{\partial V}{\partial H_u} = 0$ must be related by exchanging H_u and H_d. As a result we can solve $\frac{\partial V}{\partial H_u} = 0$ by setting $H_u = H_d = H$. We find two non-trivial solutions namely the "good" solution in Eqs. (6)–(8), and another with $\langle H\rangle \neq 0$ which we will refer to as the "bad" solution on account of its unphysically large Higgs VEV:

$$\langle H\rangle = \pm\frac{A_\lambda}{2\lambda}\sqrt{1-\frac{4m_0^2}{A_\lambda^2}}. \tag{10}$$

The discussion of the "bad" solution will be deferred until Appendix A. We also note that there exists a trivial solution (a maximum) with all fields at zero.

Now that we have shown that $H = 0$, and by extension the "good" solution, is valid we want to determine the conditions under which this solution is a local minimum of the potential.

[3] Note that this approximation is not valid for the models in [12].

To prove this we need to show that the Hessian is positive definite. If

$$\begin{pmatrix} V_{H_u H_u} & V_{H_u H_d} & V_{H_u \phi} & V_{H_u N} \\ V_{H_d H_u} & V_{H_d H_d} & V_{H_d \phi} & V_{H_d N} \\ V_{\phi H_u} & V_{\phi H_d} & V_{\phi\phi} & V_{\phi N} \\ V_{N H_u} & V_{N H_d} & V_{N\phi} & V_{NN} \end{pmatrix} \quad (11)$$

is a positive definite matrix, then the "good" solution is a minimum. To demonstrate this is true it is sufficient to show that all the eigenvalues of Eq. (11) are positive. This requirement can be expressed in terms of the ratio between $|A_\lambda|$ and m_0 which we parametrise by $x = \frac{|A_\lambda|}{m_0}$. We find that both $x^2 > 4$ and $x^2 < 8$ must be satisfied for the point to be a minimum. Expressed as a function of the soft terms we have

$$8m_0^2 > |A_\lambda|^2 > 4m_0^2. \quad (12)$$

For $|A_\lambda|^2 > 8m_0^2$ Eq. (11) has both positive and negative eigenvalues and we would have a saddle point.

Since the μ parameter is given by Eq. (9) the constraint in Eq. (12) leads to a prediction of the μ parameter in the range:[4]

$$\mu^2 = (0.25 - 0.5)m_0^2. \quad (13)$$

4. Inflation

Any model purporting to describe inflation must satisfy some basic requirements: it must have a field that is slowly rolling for a sufficient amount of expansion, it must predict curvature perturbations in line with CMB observations and its prediction for the spectral index must be consistent with current measurements. In particular it must satisfy the slow roll conditions, $\epsilon \ll 1$ and $\eta \ll 1$, and have a spectral index compatible with $n_s = 0.99 \pm 0.04$ [18,19]. The two slow roll conditions are usually expressed as

$$\epsilon_N = \frac{1}{2} m_P^2 \left(\frac{V'}{V}\right)^2 \ll 1, \quad (14)$$

$$|\eta_N| = \left| m_P^2 \frac{V''}{V} \right| \ll 1, \quad (15)$$

where N specifies when, in terms of number of e-folds before the end of inflation, ϵ and η were evaluated. They are evaluated at the time when the scales, that are currently just re-entering, left the horizon. For our model, with its relatively small vacuum energy during inflation, $N \sim 45$. Here we are using $m_P = M_{\text{Planck}}/\sqrt{8\pi}$.

In hybrid inflation [20–24] during the inflationary epoch the inflaton field ϕ slowly rolls along some almost flat direction. A second "waterfall" field N whose mass squared is positive during inflation, and hence whose field value is held at zero during inflation, is subsequently destabilised when the inflaton reaches a critical value. After this its mass squared becomes tachyonic and it rolls out to a non-zero value, effectively ending inflation. In fact, as is the case in our model, inverted hybrid inflation [25] occurs if the soft mass squared for the inflaton is negative, and normal hybrid if the soft mass squared was positive. In both cases there is a critical point that marks the transition from positive to negative effective mass squared for N.

In the previous section we saw that there are two non-trivial minima that we labelled "good" and "bad". Which minimum is reached depends on the inflationary trajectory. If a critical point is reached at which N destabilises first then the fields will fall into the "good" minimum. On the other hand if the corresponding critical point for the Higgs is reached first then we roll out to the "bad" minimum. It is therefore important to examine the critical points for the H_u, H_d and N fields.

The critical values for the Higgs and N fields can be derived from Eq. (11) by considering the stability of the Higgs and N along a trajectory that has ϕ non-zero and all other fields set to zero. The critical values of ϕ are roots of the eigenvalue equations in the Higgs and N sectors and can be expressed in terms of the soft parameters. Clearly the ϕ sector is already unstable due to the negative soft mass squared for ϕ. In fact it has a positive gradient at this point: this is the origin of the slow roll.

The critical points at which N becomes unstable are

$$\phi_{\text{crit.}(N)} = \frac{A_\kappa}{4\kappa}\left(-1 \pm \sqrt{1 - \frac{4m_0^2}{A_\kappa^2}}\right) \quad (16)$$

and the Higgs fields destabilise at

[4] It should be pointed out at this stage that the "good" solution is not the global minimum of the potential. The ramifications of this fact and potential solutions are discussed in Appendix A.

$$\phi^-_{\mathrm{crit.}(H)} = \frac{A_\lambda}{2\lambda}\left(-1 \pm \sqrt{1 - \frac{4m_0^2}{A_\lambda^2}}\right) \tag{17}$$

and

$$\phi^+_{\mathrm{crit.}(H)} = \frac{A_\lambda}{2\lambda}\left(1 \pm \sqrt{1 - \frac{4m_0^2}{A_\lambda^2}}\right). \tag{18}$$

Within the ranges of ϕ bounded by these critical values the associated field is unstable. As a result our model requires an inverted hybrid inflationary trajectory that starts from a point with small, negative ϕ and all other fields held at zero by their positive effective masses.

As ϕ rolls away from the origin it will reach $\phi_{\mathrm{crit.}(N)}$ before $\phi^-_{\mathrm{crit.}(H)}$, assuming that m_0 is non-zero, $\lambda = \kappa$ and $A_\lambda = A_\kappa$. Therefore it the "good" minimum with $N \neq 0$ and $H_u = H_d = 0$ that is reached on this trajectory. We shall now discuss the slow roll period that occurs as ϕ moves away from the origin.

For our trajectory, with all fields except the inflaton at zero, the potential simplifies to

$$V = V(0) - \frac{1}{2}m_\phi^2\phi^2. \tag{19}$$

In this case the slow roll conditions become

$$\epsilon_N = \frac{1}{2}\frac{m_P^2 m_\phi^4 \phi_N^2}{V(0)^2} \ll 1, \tag{20}$$

$$|\eta_N| = m_P^2 \frac{|m_\phi^2|}{V(0)} \ll 1. \tag{21}$$

Since

$$\phi_N = \phi_{\mathrm{crit.}(N)} e^{N\eta} \tag{22}$$

and $\eta \ll 1$ it follows that $\phi_N \sim \phi_{\mathrm{crit.}}$. Of course we must check that the slow roll conditions are satisfied. From Eq. (21) we see that we have an upper limit on m_ϕ of 10 MeV. However, from Eqs. (20) and (22) we require that, $\eta_N < 0.25$, approximately. If this were not enforced then ϕ_N would push ϵ_N above one. This slightly lowers our upper limit on m_ϕ to 5 MeV. In our model $V(0)^{1/4} \sim 10^8$ GeV is fixed when we enforce zero vacuum energy at the minimum of the potential. This leads to a low Hubble constant during inflation of $H \approx V(0)^{1/2}/3m_P \sim 1$ MeV and a low reheat temperature after inflation.

The reheat temperature is given by

$$T_{\mathrm{RH}} \simeq 0.55 g_*^{-1/4}\sqrt{\Gamma_\phi m_P}, \tag{23}$$

where [27] the decay rate is given by

$$\Gamma_\phi \sim \frac{M_\phi^3}{64\pi f_a^2}. \tag{24}$$

M_ϕ is the mass obtained after inflation and f_a is the axion decay constant. This simplifies to

$$\Gamma_\phi \sim \frac{\lambda^2}{4\pi} M_\phi \sim 10^{-8}\ \mathrm{eV} \tag{25}$$

which leads to a reheat temperature of $T_{\mathrm{RH}} \sim$ (1–10) GeV. The low reheat temperature slightly relaxes the upper bound on the axion decay constant, allowing $f_a \sim 10^{13}$ GeV [5].

It turns out that the most stringent requirement on the masses comes from the density perturbation data. From [26] we see that

$$\delta_H = \frac{32}{75}\frac{V(0)}{m_P^4}\epsilon_N^{-1} = 1.92 \times 10^{-5}. \tag{26}$$

Satisfying this requirement with the inflaton would drive its mass down to below the eV scale. This would require a high degree of fine-tuning. If the mass of the inflaton ϕ during inflation is in the MeV range this satisfies the slow roll constraints, but precludes the possibility that the density fluctuations are provided by the inflaton itself. Thus extreme fine-tuning is alleviated [28] if we use a different field, a curvaton [29–31], to generate the curvature perturbations. There are numerous examples of this mechanism in the literature. One possibility that might be compatible with our model is the axion as curvaton. This case is explored in [32] though, at this stage, it is not clear whether this analysis is applicable to this model. Another possibility is to use the coupled curvaton mechanism [33] in which the perturbations are provided by a second light scalar field which takes a non-zero value during inflation, and whose fluctuations are subsequently converted to curvature perturbations with the help of preheating effects. Alternatively we may appeal to a type of late-decaying curvaton mechanism which is consistent with low inflation scales with a symmetry breaking phase during inflation [34].

Tied into inflation is the issue of domain walls. Since this model does not possess the $\mathbb{Z}_3$ symmetry of the NMSSM it sidesteps the domain wall problem encountered when $\mathbb{Z}_3$ breaks. However, domain walls are also created when the PQ symmetry breaks [35,36]. During inflation the inflaton has a non-zero value hence breaks PQ symmetry spontaneously. As a result the domain walls are created during inflation. As such the exponential expansion of the universe will dilute them so that, by the end of inflation, their fraction of the total energy density will be negligible.

5. Conclusions

In this Letter we have suggested that the field responsible for cosmological inflation and the field responsible for generating the μ term of the MSSM are one and the same. We have shown that the vacuum expectation value of the inflaton at the Peccei–Quinn axion scale can generate the supersymmetric Higgs mass μ term of the MSSM. This provides an inflationary simultaneous solution to the strong CP problem and the μ problem of the MSSM, and gives a testable prediction for the μ parameter: $\mu^2 \approx (0.25\text{–}0.5)m_0^2$, where m_0 is the soft Higgs scalar mass. This implies deep connections between supersymmetric Higgs phenomenology, inflation and the strong CP problem.

Our model involves very small Yukawa couplings of order 10^{-10} which could originate from an extra-dimensional scenario [11]. In [13] we will show how such small Yukawa couplings can arise from embedding the model into type I string theory. The string embedding will also post-justify the assumptions that we have made here concerning smallness and equality of the Yukawa couplings in Eqs. (1) and (2), and also the equality of the soft masses of the higgses, H_u and H_d, which we have assumed to have the same soft mass as the N field.

Finally we note that Yukawa couplings as small as 10^{-10} allow the possibility of having Dirac neutrino masses, which is testable in neutrino experiments and would open up the possibility of relating the physics of the neutrino mass scale to the physics of inflation, the strong CP problem and the μ problem discussed here.

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Appendix A. Global minima

In Section 3 we discovered that $(\langle\phi\rangle = -\frac{A_\lambda}{4\lambda}, \langle N\rangle = \pm\frac{A_\lambda}{2\sqrt{2}\lambda}\sqrt{1-4m_0^2/A_\lambda^2},\ \langle H_u\rangle = \langle H_d\rangle = 0)$ is a minimum of our potential. It was noted that this is not the global minimum. In fact this is to be found at

$$\langle H\rangle = \pm\frac{A_\lambda}{2\lambda}\sqrt{1-\frac{4m_0^2}{A_\lambda^2}}, \tag{A.1}$$

$$\langle\phi\rangle = \frac{-A_\lambda}{2\lambda}, \tag{A.2}$$

$$\langle N\rangle = 0. \tag{A.3}$$

While the existence of this “bad” solution is clearly a drawback of the model it remains physically viable if the transition probability from the local minimum to the global minimum is longer than the age of the universe [1]. We also note that, in the case of inverted hybrid inflation, the trajectory is such that the “good” minimum is reached first, as discussed in Section 4.

It is worth mentioning that the model could be altered such that the global minimum arises for $N \neq 0$ and $H_u = H_d = 0$. Specifically we could relax the assumptions that $A_\lambda = A_\kappa$ and $\kappa = \lambda$. If we examine the potentials at both minima we see that

$$V_{N\neq 0} = V(0) - \frac{A_\kappa^4}{64\kappa^2}\left(1-\frac{4m_0^2}{A_\kappa^2}\right)^2 \tag{A.4}$$

and

$$V_{H\neq 0} = V(0) - \frac{A_\lambda^4}{16\lambda^2}\left(1-\frac{4m_0^2}{A_\lambda^2}\right)^2. \tag{A.5}$$

From these equations we see that if we make $A_\kappa^2/\kappa \gg A_\lambda^2/\lambda$ then $V_{N\neq 0}$ will be promoted to the global minimum. However doing so increases the complexity of the model and loses touch with the string construction presented in [13].

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