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Gauges in the bulk II: Models with bulk scalars

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Abstract

Extending previous work in Randall–Sundrum type models, we construct low-energy effective actions for braneworlds with a bulk scalar field, with special attention to the case of BPS branes. Holding the branes at fixed coordinate position with a general ansatz for the bulk metric, and imposing the Einstein frame as a gauge condition, we obtain a scalar-tensor theory with only one scalar degree of freedom related to the proper brane separation. The formalism is applicable even when there is direct coupling of the bulk scalar and brane matter, as in the Horava–Witten model. We further show that the usual moduli space approximation actually describes a non-BPS three-brane system.

Keywords: Braneworld cosmology; Radion; Horava–Witten model

1. Introduction

Braneworld models have become the focus of intense theoretical activity in the last few years (for recent reviews see [1]). Much of the attention has been triggered by the Randall–Sundrum (RS) models [2] with a purely anti-de Sitter bulk. However, to stabilize the brane separation, and hence the hierarchy solution of the first RS model, Goldberger and Wise [3] introduced a massive bulk scalar together with brane potentials. Bulk scalar models have also been suggested to alleviate the cosmological constant problem [4] (see, however [5]), and for driving inflation [6]. Thus, phenomenological considerations lead away from the RS models and towards something closer to the five-dimensional reduction [7] of Horava–Witten M-theory [8] that inspired them.

Among the models involving a bulk scalar Φ one readily distinguishes two extreme cases. The first of these occurs when the bulk (brane) potential $U(\Phi)$ ($V(\Phi)$) is dominated by the bulk cosmological constant (brane tension)

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with small residual potentials $u(\Phi)$ ($v(\Phi)$). The problem of obtaining a four-dimensional low-energy effective action for such a situation has been addressed by Kanno and Soda [9]. Indeed, up to Kaluza–Klein corrections, their two-brane effective action readily obtains by replacing $\Phi(x, y)$ with its zero mode $\eta(x)$ and integrating over the fifth coordinate $x^5 = y$ using the first metric ansatz of Chiba [10]. This follows because at leading order in the low-energy expansion the bulk scalar plays no role in determining the bulk geometry which is identical to that of the RS model.

The opposite extreme occurs when the bulk scalar controls the bulk geometry. In the case that the potentials $U(\Phi)$ and $V(\Phi)$ derive from a superpotential $W(\Phi)$ the solution of the static vacuum geometry reduces to a set of first-order BPS-like equations [11]. The Horava–Witten model exemplifies this category. An additional feature of the Horava–Witten model is that $V(\Phi)$ is the volume modulus of the Calabi–Yau space, hence the scalar directly couples to matter, and in particular with the inclusion of nonrelativistic matter static bulk solutions do not exist [12]. In the restricted case of no Φ-matter coupling, a low-energy effective action for BPS braneworlds has been given in [13,14] using the moduli space approximation. The moduli space approximation proceeds from the static vacuum solution by replacing the Minkowski metric $\eta_{\mu\nu}$ on the brane with $g_{\mu\nu}(x)$, where x^μ are coordinates tangential to the brane, and promoting the coordinate orthogonal to the positive/negative tension brane to a field $X^{(\pm)5}(x)$; the result is a biscalar-tensor theory. Clearly one scalar corresponds to a relative displacement between the branes, however the second scalar represents a centre-of-mass displacement that is spurious on a two-brane orbifold. Indeed, perturbation theory evidences a single scalar mode for BPS branes [15]. We will say more on this point anon. While the original moduli space approximation based on moving branes in a fixed background cannot be used in the interesting case of the Horava–Witten model due to the direct Φ-matter coupling, the alternate formulation of Palma and Davis [14] can. Then one is led to a remarkable conclusion: the Horava–Witten model is cosmologically excluded due to the centre-of-mass mode [13].

In this Letter we pursue the low-energy effective action for BPS braneworlds from a different approach which extends our previous treatment [16] of RS type models. Specifically, we maintain the branes at fixed coordinate $x^5 = y$, while taking a rather general ansatz for the five-dimensional metric that includes the graviton zero mode $g_{\mu\nu}(x)$. The other metric functions, and now the scalar Φ, are restricted by imposing the $\mu - 5$ bulk Einstein equation. A residual freedom is fixed by requiring that the resulting effective action be in the Einstein frame. There is no centre-of-mass mode in this two-brane system. For an exponential superpotential the effective action can be given in closed form, and the Horava–Witten model appears as a particular case.

The remainder of this Letter is organized as follows: in Section 2, we briefly review the construction of the static vacuum solution following [11]. Then, in Section 3, we present our metric ansatz and analyze the constraints on the metric functions. Section 4 gives our construction of the effective action in the Einstein gauge with the exponential superpotential as an example. In Section 5 we discuss the Jordan gauge analogous to [10] as well as the moduli space gauge and show that the latter actually describes a different non-BPS three-brane system. Conclusions are presented in Section 6. An Appendix A gives the effective action for RS-type models.

2. Vacuum BPS branes

We begin with the action $S = S_{\text{bulk}} + S_{\text{brane}}$, with

$$S_{\text{bulk}} = \frac{1}{K_{(5)}} \int d^5x \sqrt{g_{(5)}} \left[-\frac{1}{2} R_{(5)} + \frac{1}{2} g_{(5)}^{MN} \Phi_{,M} \Phi_{,N} - U(\Phi) \right], \tag{1}$$

$$S_{\text{brane}} = -\frac{1}{K_{(5)}} \int d^4x \sqrt{-g_{(4)}} V(\Phi). \tag{2}$$

Only one brane is shown, the second being introduced below; throughout the Gibbons–Hawking surface terms are left implicit. In vacuum, the metric and bulk scalar are chosen as the x^μ independent forms

$$dS_{(5)}^2 = \mathrm{e}^{-2A(y)}\eta_{\mu\nu}\,dx^\mu\,dx^\nu - dy^2, \qquad \Phi = \Phi(y). \tag{3}$$

The nonvanishing components of the Einstein tensor are

$$G_{(5)\mu\nu} = \eta_{\mu\nu}\mathrm{e}^{-2A}\big[3A'' - 6(A')^2\big], \tag{4}$$

$$G_{(5)55} = 6(A')^2 \tag{5}$$

and those of the bulk energy–momentum tensor are

$$K_{(5)}T^{(\Phi)}_{(5)\mu\nu} = \eta_{\mu\nu}\mathrm{e}^{-2A}\left[\frac{1}{2}(\Phi')^2 + U(\Phi)\right], \tag{6}$$

$$K_{(5)}T^{(\Phi)}_{(5)\mu\nu} = \frac{1}{2}(\Phi')^2 - U(\Phi), \tag{7}$$

where prime denotes derivative with respect to y. The positive tension brane will be placed at $y = 0$ so that the Einstein equations are

$$\mathrm{e}^{-2A}\big[3A'' - 6(A')^2\big] = \mathrm{e}^{-2A}\left[\frac{1}{2}(\Phi')^2 + U(\Phi)\right] + \mathrm{e}^{-2A_0}V(\Phi_0)\delta(y), \tag{8}$$

$$6(A')^2 = \frac{1}{2}(\Phi')^2 - U(\Phi) \tag{9}$$

with $A_0 = A(0)$, $\Phi_0 = \Phi(0)$. The Φ field equation is

$$\Phi'' - 4A'\Phi' = \frac{\partial U(\Phi)}{\partial \Phi} + \frac{\partial V(\Phi)}{\partial \Phi}\delta(y). \tag{10}$$

Note it is more convenient to recast Eq. (8), by multiplying with e^{2A}, as

$$3A'' - 6(A')^2 = \frac{1}{2}(\Phi')^2 + U(\phi) + V(\Phi_0)\delta(y), \tag{11}$$

which combines with Eq. (9) to yield

$$3A'' = (\Phi')^2 + V(\Phi_0)\delta(y). \tag{12}$$

Integrating Eqs. (12) and (10) around $y = 0$ gives

$$6A'(0) = V(\Phi_0), \tag{13}$$

$$2\Phi'(0) = \frac{\partial V(\Phi_0)}{\partial \Phi_0}, \tag{14}$$

where orbifold symmetry has been assumed. Given $U(\Phi)$ in terms of a superpotential $W(\Phi)$,

$$U(\Phi) = \frac{1}{8}\left[\frac{\partial W(\Phi)}{\partial \Phi}\right]^2 - \frac{1}{6}\big[W(\Phi)\big]^2, \tag{15}$$

one readily verifies that the Einstein equations (11), (12) and scalar field equation (10) are satisfied away from the brane if

$$A' = \frac{1}{6}W(\Phi), \tag{16}$$

$$\Phi' = \frac{1}{2}\frac{\partial W(\Phi)}{\partial \Phi}. \tag{17}$$

In addition, the boundary conditions of Eqs. (13), (14) are satisfied if

$$W(\Phi_0) = V(\Phi_0), \qquad \frac{\partial W(\Phi_0)}{\partial \Phi_0} = \frac{\partial V(\Phi_0)}{\partial \Phi_0}, \tag{18}$$

i.e., $W(\Phi_0)$ is tangent to $V(\Phi_0)$. When $W(\Phi) = V(\Phi)$ the boundary conditions are automatically fulfilled by the solution of Eqs. (16), (17), and the brane is BPS. As an example, for

$$W(\Phi) = V(\Phi) = W_0 \mathrm{e}^{\alpha(\Phi_0 - \Phi)}, \tag{19}$$

$$\frac{d\Phi}{dy} = -\frac{\alpha}{2} W_0 \mathrm{e}^{\alpha(\Phi_0 - \Phi)} \tag{20}$$

integrating to

$$\Phi(y) = \Phi_0 + \frac{1}{\alpha} \ln\left(1 - \frac{\alpha^2}{2} W_0 y\right). \tag{21}$$

Then

$$\frac{dA}{dy} = \frac{1}{6} \frac{W_0}{1 - \frac{\alpha^2}{2} W_0 y}, \tag{22}$$

so that

$$A(y) = A_0 - \frac{1}{3\alpha^2} \ln\left(1 - \frac{\alpha^2}{2} W_0 y\right). \tag{23}$$

The integration constant will be chosen as $A_0 = 0$, hence

$$\mathrm{e}^{-A(y)} = \left(1 - \frac{\alpha^2}{2} W_0 y\right)^{\frac{1}{3\alpha^2}}. \tag{24}$$

Clearly the limit $\alpha \to 0$ reproduces the RS model [2], including the fine-tuning of the bulk cosmological constant and brane tension, if we set $W_0 = 6k$ where k is the AdS_5 curvature. The Horava–Witten model corresponds to $\alpha = \sqrt{2}$, and the self-tuning model [4] to $\alpha = 2/\sqrt{3}$. Note, however, using Eqs. (5), (22), $G_{(5)55}$ possesses a bulk singularity. To avoid the naked singularity it is necessary to add a second brane at the other orbifold fixed point $y = \ell < 2/\alpha^2 W_0$ with

$$V(\Phi_\ell) = -W(\Phi_\ell), \qquad \frac{\partial V(\Phi_\ell)}{\partial \Phi_\ell} = \frac{-\partial W(\Phi_\ell)}{\partial \Phi_\ell}. \tag{25}$$

It is this additional fine-tuning that undermines the self-tuning models [5].

3. Choosing a gauge

As in the RS models, the inclusion of matter entails a shift of branes from their vacuum positions—this is the basis for the moduli space approximation. Instead, by a gauge transformation, the branes can be restored to their vacuum coordinate locations [15,17] at the price of introducing an x^μ dependence in the metric itself. It is advantageous to maintain $g_{(5)\mu 5} = 0$ and separate the graviton zero mode $g_{\mu\nu}(x)$. At energies small compared to the scale of W we may neglect the Kaluza–Klein modes implicit in $g_{(5)\mu\nu}$ [18]. Thus we consider the sufficiently general ansatz

$$dS^2_{(5)} = \Psi^2(x, y) g_{\mu\nu}(x)\, dx^\mu\, dx^\nu - \varphi^2(x, y)\, dy^2. \tag{26}$$

Note that $d_5(x) = \int_0^\ell \varphi(x,y)\,dy$ measures the proper distance between the branes at fixed x^μ.

The Christoffel symbols, Ricci tensor and Ricci scalar for the metric Eq. (26) have been given in [16]. There the metric functions Ψ and φ are restricted by $R_{(5)\mu 5} = 0$ in the AdS_5 bulk. Here, due to the bulk scalar we rather have

$$G_{(5)\mu 5} = R_{(5)\mu 5} = 3\left[\left(\frac{\Psi'}{\Psi}\right)\left(\frac{\varphi_{,\mu}}{\varphi}\right) - \left(\frac{\Psi'}{\Psi}\right)_{,\mu}\right] = K_{(5)} T^{(\Phi)}_{(5)\mu 5} = \Phi_{,\mu}\Phi'. \tag{27}$$

To deal with the nonvanishing right-hand side let us take[1]

$$\Psi(x,y) = \exp\bigl(-A\bigl(F(x,y)\bigr)\bigr), \qquad \Phi(x,y) = \Phi\bigl(F(x,y)\bigr), \tag{28}$$

where $A(z)$ and $\Phi(z)$ are solutions of Eqs. (16), (17). That is to say

$$\begin{aligned} &\Phi' = \frac{1}{2}\frac{\partial W}{\partial \Phi} F', \qquad \Phi_{,\mu} = \frac{1}{2}\frac{\partial W}{\partial \Phi} F_{,\mu}, \qquad -\left(\frac{\Psi'}{\Psi}\right) = \frac{1}{6} W F', \\ &-\left(\frac{\Psi'}{\Psi}\right)_{,\mu} = \frac{1}{6} W F'_{,\mu} + \frac{1}{12}\left(\frac{\partial W}{\partial \Phi}\right)^2 F_{,\mu} F', \end{aligned} \tag{29}$$

yielding

$$W\left[F'_{,\mu} - F'\frac{\varphi_{,\mu}}{\varphi}\right] = W\varphi\left(\frac{F'}{\varphi}\right)_{,\mu} = 0. \tag{30}$$

Thus F'/φ can be at most a function of y only which is fixed to unity by the vacuum, i.e.,

$$\varphi(x,y) = F'(x,y). \tag{31}$$

Eqs. (28), (30) are consistent with the perturbative results [15]. In the terminology of [16] different choices of the one free scalar function $F(x,y)$ are 'gauges'. This is not to say that they necessarily describe the same physics, however, as we will show.

4. The effective action in the Einstein gauge

For the metric Eq. (26) the bulk action of Eq. (1) is

$$\begin{aligned} S_{\text{bulk}} = \frac{1}{K_{(5)}} \int d^4x \sqrt{-g} \int dy \biggl\{ &-\frac{R}{2}\Psi^2\varphi - 3g^{\mu\nu}(\varphi\Psi)_{,\mu}\Psi_{,\nu} + 6\frac{(\Psi\Psi')^2}{\varphi} - 4\left(\frac{\Psi^3\Psi'}{\varphi}\right)' \\ &+ \frac{1}{2}\Psi^2\varphi g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu} - \frac{1}{2}\frac{\Psi^4}{\varphi}(\Phi')^2 - \Psi^4\varphi U(\Phi)\biggr\}. \end{aligned} \tag{32}$$

Here we can omit the y integral of the total derivative term which is cancelled by the implicit Gibbons–Hawking terms. As in the RS case [16], we impose as a gauge condition that the coefficient of the four-dimensional Ricci scalar R be identical to the vacuum solution:

$$\Psi^2\varphi = \mathrm{e}^{-2A(F)} F' = \mathrm{e}^{-2A(y)}, \tag{33}$$

where we have used Eqs. (28), (31). Implicitly this determines $F(x,y)$ as

$$\int_y^{F(x,y)} dz\, \mathrm{e}^{-2A(z)} + T(x) = 0, \tag{34}$$

[1] In the case of the exponential superpotential one can find other solutions of Eq. (27) analogous to [19].

the integration function $T(x)$ being related to the radion. Observe the physical distance between the branes is

$$d_5(x) = F(x,\ell) - F(x,0). \tag{35}$$

Per definition

$$\frac{1}{K} = \frac{2}{K_{(5)}} \int_0^\ell \mathrm{e}^{-2A(y)}\, dy \tag{36}$$

so we may write

$$S_{\text{bulk}} = \int d^4x \sqrt{-g}\left\{-\frac{R}{2K} + \mathcal{L}_{\text{bulk}}\right\} \tag{37}$$

with, using Eqs. (15)–(17), (28), (31), (33), (34),

$$\begin{aligned}
\mathcal{L}_{\text{bulk}} = {} & \frac{2}{K_{(5)}} \int_0^\ell dy \biggl\{ 3\mathrm{e}^{-2A(F)} F' g^{\mu\nu} \bigl(A(F)\bigr)_{,\mu} \bigl(A(F)\bigr)_{,\nu} + 6\frac{\mathrm{e}^{-4A(F)}((F)')^2}{F'} \\
& + \frac{1}{2}\mathrm{e}^{-2A(F)} F' g^{\mu\nu} \bigl(\Phi(F)\bigr)_{,\mu} \bigl(\Phi(F)\bigr)_{,\nu} - \frac{1}{2}\frac{\mathrm{e}^{-4A(F)}}{F'}\bigl(\Phi(F)'\bigr)^2 - \mathrm{e}^{-4A(F)} F' \left[\frac{1}{8}\left(\frac{\partial W}{\partial \Phi}\right)^2 - \frac{W^2}{6}\right]\biggr\} \\
= {} & \frac{2}{K_{(5)}} \int_0^\ell dy\, F' \biggl\{ 3\mathrm{e}^{-2A(F)} g^{\mu\nu} \left(-\frac{W}{6}\mathrm{e}^{2A(F)} T_{,\mu}\right)\left(-\frac{W}{6}\mathrm{e}^{2A(F)} T_{,\nu}\right) + 6\mathrm{e}^{-4A(F)}\left(\frac{W}{6}\right)^2 \\
& + \frac{1}{2}\mathrm{e}^{-2A(F)} g^{\mu\nu} \left(-\frac{1}{2}\frac{\partial W}{\partial \Phi}\mathrm{e}^{2A(F)} T_{,\mu}\right)\left(-\frac{1}{2}\frac{\partial W}{\partial \Phi}\mathrm{e}^{2A(F)} T_{,\nu}\right) \\
& - \frac{1}{2}\mathrm{e}^{-4A(F)}\left(\frac{1}{2}\frac{\partial W}{\partial \Phi}\right)^2 - \mathrm{e}^{-4A(F)}\left[\frac{1}{8}\left(\frac{\partial W}{\partial \Phi}\right)^2 - \frac{W^2}{6}\right]\biggr\} \\
= {} & \frac{2}{K_{(5)}} \int_{F(x,0)}^{F(x,\ell)} dz \left\{ \frac{1}{2}\mathrm{e}^{2A(z)}\left[\frac{W^2}{6} + \frac{1}{4}\left(\frac{\partial W}{\partial \Phi}\right)^2\right] g^{\mu\nu} T_{,\mu} T_{,\nu} + \mathrm{e}^{-4A(z)}\left[\frac{W^2}{3} - \frac{1}{4}\left(\frac{\partial W}{\partial \Phi}\right)^2\right]\right\}.
\end{aligned} \tag{38}$$

Here $W = W(\Phi(z))$ and similarly for $\partial W/\partial\Phi$. As $2(\partial W/\partial\Phi)^2 = dW/dz$ the integral becomes a total derivative, yielding

$$\mathcal{L}_{\text{bulk}} = g^{\mu\nu} T_{,\mu} T_{,\nu} \left[\frac{W\mathrm{e}^{2A}}{2K_{(5)}}\right]_{F(x,0)}^{F(x,\ell)} - \left[\frac{W\mathrm{e}^{-4A}}{K_{(5)}}\right]_{F(x,0)}^{F(x,\ell)}. \tag{39}$$

The latter terms in Eq. (39) cancel with the brane potentials as expected since the vacuum solution does not admit a net cosmological constant. As an example, we obtain for the experimental superpotential of Eq. (19), using Eqs. (24), (36)

$$\frac{1}{K} = \frac{12(1-\beta)}{(2+3\alpha^2)K_{(5)}W_0}, \qquad \beta \equiv \left(1 - \frac{\alpha^2}{2}W_0\ell\right)^{1+\frac{2}{3\alpha^2}} \tag{40}$$

and by Eq. (34)

$$\left[1 - \frac{\alpha^2}{2}W_0 F(x,y)\right]^{1+\frac{2}{3\alpha^2}} - \left(1 - \frac{\alpha^2}{2}W_0 y\right)^{1+\frac{2}{3\alpha^2}} = \frac{(2+3\alpha^2)}{6}W_0 T(x) \equiv \phi(x). \tag{41}$$

The complete effective action in the Einstein gauge is

$$S_{\mathrm{eff}} = \int d^4x\,\sqrt{-g}\left[-\frac{R}{2K} + \frac{\omega(\phi)}{2K} g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right] + S^{(+)} + S^{(-)}, \tag{42}$$

where

$$\omega(\phi) = \frac{3}{2+3\alpha^2}\frac{1}{(1+\phi)(\beta+\phi)}. \tag{43}$$

$S^{(+)}$ describes matter on the positive tension brane at $y = 0$ which feels the metric $g^{(+)}_{\mu\nu}(x) = g_{(5)\mu\nu}(x,0) = \Psi^2(x,0)g_{\mu\nu}(x)$,

$$\Psi^2(x,0) = \left[1+\phi(x)\right]^{\frac{2}{2+3\alpha^2}} \tag{44}$$

and in the Horava–Witten model also couples to

$$W\big(\Phi(x,0)\big)/W_0 = \left[1+\phi(x)\right]^{-\frac{3\alpha^2}{2+3\alpha^2}}. \tag{45}$$

On the negative tension brane at $y = \ell$ corresponding to $S^{(-)}$ the conformal factor is

$$\Psi^2(x,\ell) = \left[\beta+\phi(x)\right]^{\frac{2}{2+3\alpha^2}} \tag{46}$$

and

$$W\big(\Phi(x,\ell)\big)/W_0 = \left[\beta+\phi(x)\right]^{-\frac{3\alpha^2}{2+3\alpha^2}}. \tag{47}$$

In the limit $\alpha \to 0$, Eqs. (42)–(44), (46) go smoothly to the expression for the RS models in [16]. Irrespective of the value of α, or whether Φ couples directly to matter, β is the key parameter controlling the strength of the scalar coupling: if the coordinate position of the second brane is sufficiently close to the bulk singularity one can satisfy the constraints on scalar-tensor theories [20], and the Horava–Witten value $\alpha = 1$ is cosmologically safe up to the issue of nonrelativistic matter [12].

5. The Jordan and moduli space gauges

Often in scalar-tensor theories one gives priority to the Jordan frame, in which the motion of fiducial test particles is geodesic, rather than the Einstein frame where the scalar and tensor fields are unmixed. One can impose the Jordan frame on the positive tension brane as a gauge condition by taking

$$F(x,y) = y\varphi(x) \tag{48}$$

as in [10,18] for the RS model.[2] Proceeding from Eq. (32) in this Jordan gauge a straightforward calculation now yields

$$S_{\mathrm{bulk}} = \int d^4x\,\sqrt{-g}\left\{-\frac{R}{K_{(5)}}\int_0^{\ell\varphi} dz\,\mathrm{e}^{-2A(z)} + \frac{g^{\mu\nu}}{2K_{(5)}}\left[W\mathrm{e}^{-2A}z_{,\mu}z_{,\nu}\right]_0^{\ell\varphi} - \left[\frac{W}{K_{(5)}}\mathrm{e}^{-4A}\right]_0^{\ell\varphi}\right\}. \tag{49}$$

[2] See, however, [17] for some cautionary remarks.

For the exponential super-potential, using Eq. (24)

$$\frac{1}{K_{(5)}}\int_0^{\ell\varphi} dz\, \mathrm{e}^{-2A(z)} = \frac{6}{K_{(5)}W_0(2+3\alpha^2)}\left[1-\left(1-\frac{\alpha^2}{2}W_0\ell\varphi\right)^{1+2/3\alpha^2}\right] \equiv \frac{1}{2K_0}[\psi], \tag{50}$$

where ψ is the Brans–Dickie scalar and K_0 a bare gravitational coupling. Including matter, the effective action is then

$$S_{\text{eff}} = \int d^4x\,\sqrt{-g}\left[-\frac{\psi R}{2K_0}+\frac{\omega(\psi)}{2K_0}g^{\mu\nu}\psi_{,\mu}\psi_{,\nu}\right]+S^{(+)}+S^{(-)}, \tag{51}$$

$$\omega(\psi) = \frac{3}{(2+3\alpha^2)(1-\psi)}. \tag{52}$$

Note $\omega(\psi)$ drives ψ to unity by the self-tuning mechanism [10]. In this gauge matter on the negative tension experiences a metric

$$g^{(-)}_{\mu\nu}(x) = (1-\psi)^{\frac{2}{2+3\alpha^2}}g_{\mu\nu}(x) \tag{53}$$

and can be coupled to

$$W\big(\Phi(\ell\varphi)\big)/W_0 = (1-\psi)^{-\frac{3\alpha^2}{2+3\alpha^2}}. \tag{54}$$

Per the gauge definition, $g^{(+)}_{\mu\nu}(x) = g_{\mu\nu}(x)$ on the positive tension brane, and moreover $W = W_0$ there which is to say the matter is implicitly decoupled from the bulk scalar. Nor, for that matter, can radiation-scalar coupling be recovered by a conformal transformation of the Jordan gauge effective action to the Einstein frame $g_{\mu\nu} \to g_{\mu\nu}/\psi$. This makes the Jordan gauge unsuitable for the Horava–Witten model.

Within a given scalar-tensor theory the Jordan and Einstein frames describe identical physics, but the Jordan and Einstein gauges are inequivalent even in the absence of direct coupling of the bulk scalar and brane matter. The coordinate length ℓ appears directly in the Einstein gauge via the coupling parameter β, whereas in the Jordan gauge it is subsumed in ψ. The two gauges only become conformally equivalent if $\alpha = \beta = 0$. In that case $\psi = (1+\phi)^{-1} = 1-\frac{\chi^2}{6}$ with χ a conformally coupled scalar [21].

Still, there is a subtlety with $\alpha = \beta = 0$: $\alpha \to 0$ followed by $\ell \to \infty$, $\beta \to 0$ in the Einstein gauge describes the second RS model. Although the coordinate distance is infinite, the AdS warp makes the physical distance finite. Displacing the brane distorts the bulk geometry as reflected in the scalar ϕ remaining in the effective action [16]. Taking $\alpha \to 0$ followed by $\ell \to \infty$ in the Jordan gauge would yield $\psi = 1$, according to Eq. (50), and no scalar which is the wrong physics.

Next, we turn to the moduli space approximation. In the original version [13] (see also [22]) the Minkowski metric of the static vacuum solution in Section 2 is promoted to $g_{\mu\nu}(x)$ and the brane positions to $X^{(\pm)5}(x)$ with $h^{(\pm)}_{\mu\nu}(x) = g_{(5)\mu\nu}(x, X^{(\pm)5}) - X^{(\pm)5}_{,\mu}X^{(\pm)5}_{,\nu}$, the induced metrics on the branes. The alternative formulation of [14] is equivalent to here imposing a moduli space gauge

$$F(x,y) = \varphi(x)y - \xi(x). \tag{55}$$

The additional field ξ represents the centre of mass displacement, or a local twist of the orbifold boundary conditions [23]. Once again a straightforward calculation proceeding from Eq. (32) yields[3]

$$S_{\text{bulk}} = \int d^4x\,\sqrt{-g}\left\{-\frac{R}{K_{(5)}}\int_{z^+}^{z^-} dz\,\mathrm{e}^{-2A}+\frac{g^{\mu\nu}}{2K_{(5)}}\big[W\mathrm{e}^{-2A}z_{,\mu}z_{,\nu}\big]^{z^-}_{z^+}-\left[\frac{W}{K_{(5)}}\mathrm{e}^{-4A}\right]^{z^-}_{z^+}\right\}. \tag{56}$$

[3] The precise form in [14] obtains by a conformal transformation $g_{\mu\nu} \to \exp(2A(z^+))g_{\mu\nu}$ to the Jordan frame on the positive tension brane.

Evidently the moduli space gauge S_{bulk}, Eq. (56), is just two copies of the Jordan gauge S_{bulk}, Eq. (49), glued together. The Einstein gauge does not allow the shift mode ξ but one could paste together two copies with scalars $\phi^{(+)}$ and $\phi^{(-)}$. The key point is that where the joint is made one must impose Israel's junction conditions, e.g., at $z=0$

$$[[A']]=0, \qquad [[\Phi']]=0, \tag{57}$$

where [[ ]] denotes the discontinuity. One recognises Eq. (57) as the junction conditions for a tensionless brane. This is not mere tautology: to discuss the centre-of-mass, as opposed to relative, motion of the positive and negative tension branes requires a third observer brane. The catch is that Eq. (57) is not BPS unless $U(\Phi)$ has a zero[4] or the superpotential is a constant. If W is constant, one has AdS_5, the RS model, and two conformally coupled scalars $\chi^{(+)}$, $\chi^{(-)}$, and through a conformal transformation only one scalar mode $(\chi^{(-)}/\chi^{(+)})$. Otherwise one is not examining the advertised two-brane BPS system, but instead a nearly BPS three-brane system similar to the Ekpyrotic model of [23].

The ramifications of the moduli space gauge becomes evident by adapting the viewpoint of a freely falling observer on the tensionless brane rather than the positive tension brane. For the exponential superpotential define

$$\left(1-\frac{\alpha^2}{2}W_0 z^+\right)^{1+\frac{2}{3\alpha^2}} = \psi\cosh^2\left(\frac{r}{2}\right), \tag{58}$$

$$\left(1-\frac{\alpha^2}{2}W_0 z^-\right)^{1+\frac{2}{3\alpha^2}} = \psi\sinh^2\left(\frac{r}{2}\right), \tag{59}$$

and ignore brane matter so

$$S_{\text{eff}} = \int d^4x\,\sqrt{-g}\left\{-\frac{R\psi}{2K_0}+\frac{g^{\mu\nu}}{2K_0}\left(\frac{3}{2+3\alpha^2}\right)\left[\frac{\psi_{,\mu}\psi_{,\nu}}{\psi}-\psi r_{,\mu}r_{,\nu}\right]\right\}. \tag{60}$$

Such observers see a Brans–Dickie theory coupled to a ghost. The instability reflects that the tensionless brane wants to sit at $z=-\infty$ where U vanishes. Like the apparent disappearance of the radion in the RS2 limit, this is the wrong physics.

6. Conclusions

BPS braneworlds are much closer to string/M-theory than the simple RS models. Our general Einstein gauge effective action, Eqs. (34), (37), (39) provide a simple treatment of the nonvacuum case without invoking the problematic third brane of the moduli space approximation. It can be used even when there is direct coupling of the bulk scalar and brane matter, unlike the Jordan gauge. The metric of Eq. (26) could be used as a starting point for calculating Kaluza–Klein corrections through the low energy expansion scheme [18]. Setting $\alpha=1$ in Eqs. (42)–(47) one has a new basis to explore inhomogeneous Horava–Witten brane cosmology and, following the methods of [21], also black holes.

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[4] That is the case examined in [23].

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Appendix A. Non-BPS branes

Following [9], suppose the bulk potential is

$$U(\Phi) = u(\Phi) - 6k^2 \tag{A.1}$$

and on the positive tension brane at $y = 0$

$$V_0(\Phi) = 6k + v_0(\Phi) \tag{A.2}$$

while on the negative tension brane at $y = \ell$

$$V_\ell(\Phi) = -6k + v_\ell(\Phi) \tag{A.3}$$

with u, v_0 and v_ℓ small. Neglecting the influence of the scalar on the bulk geometry one can replace $\Phi(x, y)$ with the zero mode $\eta(x)$ at leading order. The metric functions for the RS geometry are [16]

$$\Psi(x, y) = \left[\mathrm{e}^{-2ky} + \phi(x)\right]^{1/2}, \qquad \varphi(x, y) = \Psi^{-2}(x, y)\mathrm{e}^{-2ky}. \tag{A.4}$$

A simple calculation using Eqs. (32), (36) gives

$$\begin{aligned}\mathcal{L} = {} & -\frac{R}{2K} + \frac{3}{4K}\frac{g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}}{(1+\phi)(e^{-2k\ell}+\phi)} + \frac{g^{\mu\nu}}{2K}\eta_{,\mu}\eta_{,\nu} - \frac{1}{K}\left(\frac{1+e^{-2k\ell}}{2} + \phi\right)u(\eta) \\ & - \frac{k}{K(1-e^{-2k\ell})}\left[(1+\phi)^2 v_0(\eta) + \left(\mathrm{e}^{-2k\ell} + \phi\right)^2 v_\ell(\eta)\right].\end{aligned} \tag{A.5}$$

Note the radion ϕ remains in the limit $\ell \to \infty$.

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