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# A method of multi-objective reliability tolerance design for electronic circuits

Zhai Guofu \*, Zhou Yuege, Ye Xuerong, Hu Bo

School of Electrical Engineering and Automation, Harbin Institute of Technology, Harbin 150001, China

Received 2 September 2011; revised 21 November 2011; accepted 21 December 2011 Available online 16 January 2013

## KEYWORDS

Design of experiments; Multi-objective; Quality-cost model; Reliability design; Sensitivity analysis; Tolerance design **Abstract** Tolerance design plays an important role in reliability design for electronic circuits. The traditional method only focuses on the consistency of output response. It is not able to meet the needs of increasing development of electronic products. This paper researches the state of related fields and proposes a method of multi-objective reliability tolerance design. The characteristics of output response and operating stresses on critical components are both defined as design objectives. Critical components and their operating stresses are determined by failure mode and effect analysis (FMEA) and fault tree analysis (FTA). Sensitivity analysis is carried out to determine sensitive parameters that affect the design objectives significantly. Monte Carlo and worst-case analysis are utilized to explore the tolerance levels of sensitive parameters. Design of experiment and regression analysis are applied in this method. The optimal tolerance levels are selected in accord with a quality-cost model to improve consistency of output response and reduce failure rates of critical components synchronously. The application in light-emitting diode (LED) drivers indicates details and potential. It shows that the proposed method provides a more effective way to improve performance and reliability of electronic circuits.

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### 1. Introduction

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Electronic circuits are necessary parts of electronic systems which are widely used in aeronautic and space, industry control, and military fields. Their stability and reliability have a direct and significant effect on system reliability. Practically,

\* Corresponding author. Tel.: +86 451 86416664.

E-mail addresses: gfzhai@hit.edu.cn (G. Zhai), zhouygb05250@163. com (Y. Zhou), xuelai1981@163.com (X. Ye), huboowen@126.com (B. Hu).

Peer review under responsibility of Editorial Committee of CJA.



performance parameters of components fluctuate around their mean values due to differences of manufacturing techniques and environmental conditions (called as disturbance factors). The fluctuation causes variation and non-conformance of system performance measures. When component parameters deviate from their tolerance levels, it results in soft failure, indicating that the system is functional, but some performance measures do not conform to design specifications.<sup>1–3</sup> Especially in fields requiring high reliability, taking a helicopter's automatic driving system for example, the fluctuation of output may generate unpredicted results. Hence, research of tolerance design plays an increasingly important role in electronic circuit design for realizations of high reliability and stability.

Reliability tolerance design for electronic circuits is a topic of great interest. Italian researchers have discussed a tolerance design approach for feedback compensation networks of

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DC-DC (DC means direct current) switching regulators, identifying performance and stability constraints of crossover frequency and phase margin by means of the Monte Carlo analysis and interval arithmetic computations.<sup>4,5</sup> Spanish members have studied output power variation of resonant inverters applied to high pressure sodium (HPS) lamps due to component tolerance. They analyzed lamp aging and tolerance of circuit components, and selected the most suitable value of the resonant network. The result is validated by Monte Carlo analysis using PSpice simulation.<sup>6</sup> However, there are two problems in those approaches. Firstly, the transfer function must be known. The availability is greatly constricted by complicated large-scale circuits, which are difficult to get the system transfer function. Secondly, only stability of output response can be improved. Operating stresses on the components are not paid attention. In fact, working conditions and operating stresses are the most important factors that affect reliability of electronic systems.

With development of circuit simulation technology, Electronic Design Automation (EDA) software has powerful capabilities of modeling and computing. It can assist in performing tolerance design. Presently, tolerance analysis based on EDA simulation has been applied in many electronic systems, such as DC hybrid contactors, driving controllers, and low-pass filters.<sup>7,8</sup> However, they also just analyze factors that cause variation of output characteristics.

The traditional tolerance design method only focuses on consistency of output response. It is a single-objective method. A multi-objective tolerance design method is proposed to improve reliability of electronic circuits. In this research, output response, along with operating stresses on components such as voltage, ripple current, and power loss, are both considered as analysis objects at the same time. Meanwhile, the most important is that operating stresses are the most critical factors that affect reliability of components. Reduction of operating stresses can decrease failure rates of components. So, multiobjective tolerance analysis is not only improving robustness of output response, but also reducing operating stresses on components by tolerance design of parameters. A light-emitting diode (LED) driver is taken as a case to study, determining the optimal tolerance levels of sensitive parameters to increase the qualification rate of a batch of LED drivers, decrease the failure rate, and finally reduce the whole life-cycle cost.

#### 2. Method of multi-objective tolerance design

The multi-objective tolerance design method focuses not only on output characteristics, but also on operating characteristics of critical components. EDA simulation, sensitivity analysis, experimental design, and multiple regression analysis are adopted into the multi-objective method, which can improve design efficiency greatly. The flowchart of this multi-objective tolerance design method is shown in Fig. 1. In the figure, FMEA means failure mode and effect analysis, FTA means fault tree analysis.

#### 2.1. Identification of critical components

Critical components are the main factors that affect reliability of a system. There are three methods to determine critical



Fig. 1 Flowchart of the multi-objective tolerance design method.

components, i.e., reliability prediction, FMEA, and FTA. Reliability prediction adopts the stress analysis approach to predict failure rate of each component. It is performed under a certain stress. The components with higher failure probabilities are defined as the critical ones. FMEA analyzes all possible failure modes and their effects on the system, determining critical components in accordance with severity and probability. The components with higher risk levels are critical. FTA can identify various failure causes of components and their probabilities. The higher the critical degree is, the greater the probability of system failure is. In this way, those components with greater critical degrees are confirmed as the critical ones. The three methods are combined to identify critical components.

#### 2.2. Sensitivity analysis

The purpose of sensitivity analysis is to determine sensitivity parameters causing fluctuation of output response and operating stresses on critical components. Circuit sensitivity is the sensitivity degree of a target response to each design variables, including absolute sensitivity and relative sensitivity. The relative sensitivity is adopted generally to evaluate the degree to which the design factors affect the target response.  $f = f(x_1, x_2, ..., x_n)$  is defined as the target response in a system, in which  $x_1, x_2, ..., x_n$  are design variables. Supposing that  $x_{n}$ represents the mean value of a component's parameter,  $f_0$  represents the mean of the target response; then the relative sensitivity  $S_{x_n}^f$  can be expressed as

$$S_{x_{i}}^{f} = \frac{\mathrm{d}f/f_{0}}{\mathrm{d}x_{i}/x_{i0}} = \frac{\partial f}{\partial x_{i}} \frac{x_{i0}}{f_{0}} \quad (i = 1, 2, \dots, n)$$
(1)

Orthogonal experiment is an approach that is used in multifactor tests. It is to select several typical points which are uniform and ordered. Range analysis is applied to analyze the sensitivity of the factors in an orthogonal array.

## 2.3. Tolerance analysis

In a circuit design, the deviation between actual value and nominal value of a component cannot be ignored. Sometimes its effect may be significantly serious. It is essential to study the effect on circuit response because of parameter variation, that is, tolerance analysis. The approach most commonly utilized is the Monte Carlo analysis and the worst-case analysis. The Monte Carlo analysis is a statistical analytical method, which analyzes the response deviation by parameter sampling of components, when the parameter meets certain probability distribution. The result of this method shares the most similarity with actual condition, but consumes much more computing time. The worst-case analysis evaluates circuit performance under a set of worst cases. The result can verify whether the response is acceptable under the worst case. This method has the advantage of reducing analysis time, but the result is too conservative. It just provides the condition under the worst case that is not in accord with most of practical situations. For difference purposes, these two methods are both adopted in the research.

#### 2.4. Tolerance allocation

Tolerance allocation means that the system response tolerance is distributed to the tolerances of components. It can be expressed as

$$R_s(R_1, R_2, ..., R_i, ..., R_n) < R_s^*$$

# $g_s(R_1, R_2, ..., R_i, ..., R_n) < g_s^*$

where  $R_{s}^{*}$  is the tolerance of response,  $g_{s}^{*}$  the constraint, including cost, temperature, volume, power consumption and so on.  $R_i$  the tolerance of design variable *i*. Usually, tolerance allocation is an optimization problem. Therefore, an optimization method is adopted to solve the problem. In this research, the quality-cost model,<sup>9,10</sup> which is an optimization function for multi-objective tolerance design, is established for the optimal tolerance allocation. The tolerance levels of sensitive parameters are the design variables, and the minimum total life-cycle cost is the final purpose under the condition that the output current and electrical stresses on components fulfill design specifications. The model is consisted of quality loss and manufacturing cost to balance quality and cost increasing due to quality improving. The optimal tolerance levels are the solution of the model, which is solved by genetic algorithm<sup>11</sup> at the lowest total life-cycle cost.

#### 3. Design and simulation of a LED driver

#### 3.1. Scheme of the LED driver

The single flyback topology is selected to design a driver for LED lights. Meanwhile, in order to reduce the interference to the grid, a power factor correction circuit is utilized to improve the input current. The schematic diagram is shown in Fig. 2. In the figure, the overvoltage on the field effect transis-



Fig. 2 Scheme of the LED driver.

tor VT is reduced by a cushion circuit consisting of a capacitor  $C_c$ , a resistor  $R_c$ , and a diode  $D_c$ .

#### 3.2. Design specifications of the LED driver

The relationship between bias voltage and current of LEDs is a typical PN junction characteristic. The corresponding forwardbias current increases by orders of magnitude with only a small change in the forward-bias voltage, when the forward-bias voltage exceeds its threshold. Therefore, constant current power supply is selected to drive LEDs.<sup>12,13</sup> The load of the driver is a high power white-light LED, which optimal driving current is determined as 700 mA according to its lighting efficiency. Its light flux increases with current in a linear relation when the forward-bias current is less than 700 mA, while increasing slightly and tending to saturate when the forward-bias current is greater than 700 mA. As a result, the heat dissipation of the LED will dramatically increase, i.e., the light efficiency decreases severely. According to the LED characteristic, the design specifications of the LED driver are given:

- (1) Output current:  $(700 \pm 15)$  mA.
- (2) Operating temperature:  $-30\ 60\ ^{\circ}C$ .
- (3) Qualification rate: greater than 0.95.

### 3.3. Simulation model of the LED driver

The accuracy of analysis based on EDA simulation largely depends on the simulation model. The key component for simulation is the flyback transformer, which directly affects the accuracy of analysis results. The transformer model in PSpice library is ideal and cannot reflect power loss and leakage inductance characteristics. Eqs. (2)-(4) are dynamic equations of the transformer. They are used to establish an accurate flyback transformer simulation model. For tolerance design based on circuit simulations, sensitivity analysis and tolerance analysis are performed based on the steady values of circuit response, such as output current, electrical stresses on components. The steady values of circuit response are closely related to the transformer model. Supposing that  $R_{\rm m}$  represents the transformer core reluctance, let  $u_1$ ,  $u_2$  and  $i_1$ ,  $i_2$  stand for primary and secondary voltage and current respectively, the turn ratio is  $n_1/n_2$ , and excitation current is  $i_{mp}$ .  $R_{ac1}$ ,  $R_{\rm ac2}$  and  $L_{11}$ ,  $L_{12}$  represent the leakage resistance and inductance of primary and secondary winding respectively.

 $R_{\rm c}$  is the equivalent resistor of core loss and  $L_{\rm mp}$  is the primary magnetizing inductance. The equivalent model of the transformer is shown in Fig. 3.

$$\Phi R_{\rm m} = n_1 i_1 + n_2 i_2 \tag{2}$$

$$\begin{cases} e_1 = n_1 \frac{d\Phi}{dt} = n_1 L_{\rm mp} \frac{di_{\rm mp}}{dt} \\ e_2 = n_2 \frac{d\Phi}{dt} = n_2 L_{\rm mp} \frac{di_{\rm mp}}{dt} \end{cases}$$
(3)

$$\begin{cases} u_1 = R_{ac1}i_1 + L_{11}\frac{di_1}{dt} + e_1 \\ u_2 = R_{ac2}i_2 + L_{12}\frac{di_2}{dt} + e_2 \end{cases}$$
(4)



Fig. 3 Equivalent model of the transformer.

The PSpice model of the LED driver is shown in Fig. 4. It can simulate the actual working conditions of the circuit. The implementation of the multi-objective tolerance design with PSpice simulation also has a significant value.

#### 4. Application

The main process is stated in detail in the following sections, using the method of multi-objective tolerance design to analyze the LED driver.

#### 4.1. Identification of critical components

The multi-objective tolerance design is different from the traditional one, as it considers the output characteristic of every component that makes up a system. The reliability of the system is improved by means of reducing fluctuations of electrical stress on critical components. According to the reliability analysis, the critical components of the LED driver are metallic oxide semiconductor field effect transistor (MOSFET)  $Q_3$ , aluminum electrolytic capacitor  $C_5$ , the transformer  $T_1$ , and the power rectifier diode  $D_5$ . The component failure mechanism shows that decreasing electrical stress can effectively reduce the failure rate and enhance its reliability. This can be achieved by controlling some certain sensitive parameters and related tolerances. The main stresses that cause the critical components to fail are  $P_M$  (power consumption on  $Q_3$ ),  $P_L$  (energy dissipation on  $T_1$ ),  $V_C$  (voltage on  $C_5$ ),  $I_C$  (ripple current on  $C_5$ ), and  $V_D$  (reverse-bias voltage on  $D_5$ ). Therefore, the characteristics of these electrical stresses and output current are all set as the objectives of tolerance design. The constraints of these objectives are determined in accord with the reliability requirements, as shown in Eq. (5), in which  $x_1, x_2, \ldots, x_n$  are sensitive parameters.

$$\begin{cases}
P_{M}(x_{M_{1}}, x_{M_{2}}, \dots, x_{M_{n}}) \leq 9 W \\
P_{L}(x_{L_{1}}, x_{L_{2}}, \dots, x_{L_{n}}) \leq 5.5 W \\
V_{C}(x_{C_{1}}, x_{C_{2}}, \dots, x_{C_{n}}) \leq 150 V \\
I_{C}(x_{C_{1}}, x_{C_{2}}, \dots, x_{C_{n}}) \leq 1.65 A \\
V_{D}(x_{D_{1}}, x_{D_{2}}, \dots, x_{D_{n}}) \leq 610 V \\
I_{out}(x_{out1}, x_{out2}, \dots, x_{outn}) \leq 715 mA \\
I_{out}(x_{out1}, x_{out2}, \dots, x_{outn}) \geq 685 mA
\end{cases}$$
(5)



Fig. 4 PSpice model of the LED driver.

 Table 1
 Orthogonal design factors and levels.

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Factor	$D_6$ (V)	$U_5$ (V)	$R_1 \left( \Omega \right)$	$R_{12}~(\mathrm{k}\Omega)$	$C_1$ ( <u>nF</u> )	$R_{2}\left(\Omega\right)$
Level 1	15	2.495	100	47	10	47
Level 2	15.75	2.62	105	49.35	10.5	49.35
Factor	$L_1 (\mu H)$	$R_4~(\Omega)$	$R_3$ (k $\Omega$ )	$C_2 (\mu F)$	$C_{10}$ (µF)	C <sub>5</sub> (µF)
Level 1	6	0.33	10	0.1	1	470
Level 2	6.3	0.347	10.5	0.105	1.05	493.5
Factor	$R_{17}$ (k $\Omega$ )	$R_{18}$ (k $\Omega$ )	C <sub>15</sub> (µF)	$R_6$ (k $\Omega$ )	<i>C</i> <sub>3</sub> ( <u>nF</u> )	$R_7$ (k $\Omega$ )
Level 1	39	5.1	10	100	680	1
Level 2	40.95	5.355	10.5	105	714	1.05
Factor	$R_{10} (\Omega)$	$R_{14}~(\mathrm{k}\Omega)$	$R_{11}$ (k $\Omega$ )	$R_9$ (k $\Omega$ )	$R_{15}(\Omega)$	$R_{16}\left(\Omega\right)$
Level 1	5.1	10	15	5.1	3.9	4.7
Level 2	5.355	10.5	15.75	5.355	4.095	4.935
Factor	<i>L</i> (mH)	$R_5$ (k $\Omega$ )	$R_8$ (k $\Omega$ )			
Level 1	1.4	10	2.2			
Level 2	1.47	10.5	2.31			
-						



Fig. 5 Relative sensitivity of output current.



#### 4.2. Sensitivity analysis

To find the sensitive parameters, the PSpice simulation is operated to carry out sensitivity analysis for the factors that affect the fluctuations of those electrical stresses. Orthogonal array  $L_{32}$  (2<sup>31</sup>) is selected among the total 27 factors of the LED driver to arrange test order, as shown in Table 1. Then, all factors' relative sensitivity is calculated separately. The results are shown in Figs. 5 and 6. The sensitive parameters that affect the multi-objective stresses and output current are given in Eq. (6), where the elements in the left column such as  $U_5$ ,  $R_2$ are sensitive components correspondingly.

\_ \_ \_ \_

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ \end{bmatrix} \begin{pmatrix} V_5 \\ R_2 \\ R_5 \\ R_{11} \\ R_{14} \\ R_{15} \\ R_{16} \\ C_1 \end{bmatrix} \iff \begin{bmatrix} P_M \\ P_L \\ V_C \\ I_C \\ V_D \\ I_{out} \end{bmatrix}$$
(6)

#### 4.3. Tolerance analysis

According to the results of sensitivity analysis, there are eight parameters in total that are sensitive to the multi-objective electrical stresses and output current. They are  $U_5$ ,  $R_2$ ,  $R_5$ ,  $R_{11}$ ,  $R_{14}$ ,  $R_{15}$ ,  $R_{16}$ ,  $C_1$ . In practical applications, the tolerance levels that can be acceptable for the eight parameters are just selected using commercial values.  $U_5$  represents the output voltage of the precision regulator TL431, which has three levels: 0.5%, 1%, 2%; resistor  $R_i$  has six levels: 0.1%, 0.2%, 0.5%, 1%, 2%, 5%; and capacitor  $C_i$  has three levels: 5%, 10%, 20%. Other non-sensitive parameters are kept at the maximum tolerance region, i.e., resistor 5%, capacitor 20%.

Uniform design is adopted to arrange the process of simulation analysis, which can improve the efficiency of tolerance design. Multi-objective electrical stresses are expected to be as small as possible; therefore, only worst-case analysis is performed for these stresses.<sup>14,15</sup> Taking  $V_C$  for example, its fluctuations are desired as small as possible. When  $V_C$  is still in the range of constraint at the worst case, the result of multi-objective tolerance design fulfills the requirement.  $U_{12}$  (6<sup>5</sup>) uniform array is selected to perform worstcase analysis. The results are shown in Table 2. Utilizing multiple linear regression method to analyze the results in Table 2, the relationship between  $V_C$  and the tolerance levels of sensitive parameters is shown in Eq. (7). Using the Statistical Product and Service Solutions (SPSS) software to perform significance test for the regression equation, it shows that the significance level is close to zero, indicating the significance of the linear relationship between  $V_C$  and the tolerance levels.

$$V_C = 142.145 + 149.106t_{U_5} + 58.854t_{R_{11}} + 61.967t_{R_{14}} + 82.064t_{R_{15}} + 66.676t_{R_{16}}$$
(7)

Table 2	Uniform	n array de	esign.			
Test No.	Δ <i>U</i> <sub>5</sub> (%)	$\Delta R_{11}(\%)$	$\Delta R_{14}(\%)$	$\Delta R_{15}(\%)$	$\Delta R_{16}(\%)$	Target value(V)
1	2	2	0.5	2	5	151.501
2	1	2	5	0.5	0.2	148.372
3	2	0.1	1	5	0.2	149.997
4	0.5	0.2	0.5	0.1	0.1	143.384
5	1	5	0.2	1	0.1	147.573
6	1	0.1	0.2	0.5	5	147.586
7	0.5	1	2	5	1	149.579
8	1	0.2	5	1	2	149.043
9	2	1	0.1	0.1	1	146.610
10	0.5	0.5	0.1	2	0.5	145.192
11	2	0.5	2	0.2	0.5	147.223
12	0.5	5	1	0.2	2	148.001

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Fig. 7 Statistical bar chart of output current.

For the definite purpose of the output current  $I_{out}$ , this paper uses the Monte Carlo method to improve the output current fluctuations within the range of  $(700 \pm 15)$  mA and qualification rate higher than 95%. For example, the result of the Monte Carlo analysis with  $U_{12}$  tolerance  $\pm 2\%$ ,  $R_{11} \pm 0.1\%$ ,  $R_{14} \pm 1\%$ ,  $R_{15} \pm 2\%$ , and  $R_{16} \pm 2\%$  is shown in Fig. 7. It can be seen that the output current of the LED driver is close to normal distribution.

For distribution  $I_{out}$  obeys  $N(\mu, \sigma^2)$ , with the help of the stress-strength interference model, the qualification rate of the output current is calculated, where  $x_1$  and  $x_2$  are the constraints.

$$P(x_1 < I_{\text{out}} < x_2) = \int_{x_1}^{x_2} \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx$$
(8)

By means of Eq. (8), the qualification rate of output current in every tolerance combination can be obtained. The regression function between the qualification rate  $\alpha_{I_{out}}$  and the tolerance levels can be built using the multiple linear regression method similarly, as shown in Eq. (9). inevitably lead to increasing cost. Therefore, a quality–cost model is established to balance quality and cost, and to achieve optimization of whole life-cycle cost. The whole life-cycle cost of the LED driver includes two sections: manufacturing cost  $C_{\rm m}$ , and utilizing cost  $C_{\rm u}$  resulted from quality loss.

The manufacturing cost consists of material and component cost and assembly cost, both of which are closely related to tolerance levels of components. The relationship between the manufacturing cost  $C_{\rm m}$  and the tolerance levels of sensitive parameters is set up by curve fitting, as shown in Eq. (10), where  $C_{\rm B}$ ,  $C_{U5}$ ,  $C_R$ , and  $C_C$  are initial costs of non-sensitive components, the cost of TL431, the cost of sensitive resistor, and the cost of sensitive capacitor, respectively.

$$C_{\rm m}(\Delta t) = C_{\rm B} + C_{U_5} + \Sigma C_R + C_C \tag{10}$$

The utilizing cost  $C_u$  of the LED driver is proportional to its quality loss, which includes both internal and external loss. This LED driver is a non-repairable system, in which the external loss can be ignored while the internal loss is only related to

										$\begin{bmatrix} t_{11} \end{bmatrix}$				
$V_C$		[ 149.106	0	0	0	58.854	61.967	82.064	66.676	<i>t</i> U <sub>5</sub>		[142.145]		
$P_M$		22.764	0	0	0	7.231	6.7	11.098	8.46			8.085		
$P_L$		12.193	0	0	1.059	5.301	5.151	7.636	6.269	$t_{R_2}$		5.157		( <b>0</b> )
$I_C$	=	4.429	0.05	0.278	0	1.487	0.554	1.732	1.274	$t_{R_{11}}$	+	1.588		(9)
$V_D$		178.618	0	0	0	111.404	99.776	106.38	91.393	$t_{R_{14}}$		602.649		
$\alpha_{I_{\text{out}}}$			0	0	0	-7.657	-6.593	-5.843	-5.843	$t_{R_{15}}$		1.1285		
										$t_{R_{16}}$				

#### 4.4. Tolerance allocation

Product quality and cost are a pair of irreconcilable contradiction, which means that improving product quality will the qualification rate. The qualification rate varies with different tolerance levels of components. What's more, decrease in the qualification rate will cause increase of the utilizing cost. The relationship between the qualification rate and the utilizing cost is shown in Eq. (11).

$$C_{\rm u}(\Delta t) = K(1 - \alpha_{I_{\rm out}}) \tag{11}$$

where K is the coefficient related to internal loss, and set to 10 for the LED driver. The whole life-cycle cost of the LED driver is the sum of the manufacturing cost  $C_{\rm m}$  and the utilizing cost  $C_{\rm u}$ , as shown in Eq. (12). To find the optimal tolerance level, the genetic algorithm is adopted to solve the function under the minimum cost and the results are listed in Table 3. Meanwhile, the other non-sensitive parameters are kept within their maximum tolerance levels, that is, resistor 5% and capacitor 20%.

$$TC(\Delta t) = C_m(\Delta t) + C_u(\Delta t)$$

$$= 201.4464 + 0.000268t_{U_5}^{-1.538}$$

$$+ 0.00193t_{R_{11}}^{-0.6726} + 0.00193t_{R_{14}}^{-0.6726}$$

$$+ 0.00581t_{R_{15}}^{-0.7107} + 0.00193t_{R_{2}}^{-0.6726}$$

$$+ 0.00193t_{R_{5}}^{-0.6726} + 0.004t_{C_{1}}^{-2}$$

$$+ 10[1 - (1.123)$$

$$- 7.888t_{U_5} - 7.657t_{R_{11}} - 6.593t_{R_{14}}$$

$$- 5.843t_{R_{15}})]$$
(12)

Table 3Optimal tolerance allocation.

Sensitive parameter	$U_5$	$R_2$	$R_5$	$R_{11}$
Tolerance(%)	1	5	5	0.1
Sensitive parameter	$R_{14}$	$R_{15}$	$R_{16}$	$C_1$
Tolerance(%)	0.2	0.5	0.1	20

#### 5. Discussion and comparison

The method of multi-objective tolerance design for the LED driver mainly achieves two purposes: one is to raise the qualification rate of the output current and reduce the increase of cost; the other is to make sure that the stresses on critical components are within the specification limit and to improve the quality and stability of the LED driver. These two purposes are verified below respectively.

#### 5.1. Comparison of output characteristic

The output characteristic of the LED driver is mainly reflected in the output current. Its design specification is  $I_{out} = (700 \pm 15)$  mA. The Monte Carlo method is applied in PSpice simulation. Then, the distribution of the output current before and after tolerance design is obtained. Fig. 8 shows the comparison of output current before and after design, via multi-objective tolerance design, the robustness of the output



Fig. 8 Comparison of output current by simulation.



Fig. 9 Distribution of output current by experiment.

current is significantly improved. The deviation of output current is limited nearly between 680 mA and 720 mA. The actual output current is measured in the experiment, when LED drivers are produced according to the results after design. It is normally distributed  $I_{out}$  obeys  $N(695.91, 2.726^2)$ , as shown in Fig. 9. It demonstrates the validity of this method.

Figs. 8 and 9 also show that the simulation result is very close to the actual condition. The output current obtained from simulation fluctuates among 688–713 mA. The mean value and standard deviation are 700.16 mA and 6.511 mA, respectively. The experiment results fluctuate in the range of 689–702 mA. The mean and standard deviation are 695.91 mA and 2.726 mA, respectively. The difference between simulation and experiment results may be due to some components' nominal values deviating from their specifications.

#### 5.2. Comparison of electrical stresses

The critical factor that affects system reliability is the reliability of critical components. Fig. 10 shows the comparison of stresses on critical components before and after design. The operating stresses on critical components, such as power loss, voltage, and ripple current, are effectively decreased by the multi-objective tolerance design, thus avoiding the failure due to excessive stress. The mean value and stand deviation are both decreased. The robustness of operating stresses on



Fig. 10 Comparison of stresses on the critical components.

Table 4         Comparison of failure rate.									
No.	Component	Failure rate (1	Ratio						
		Before design	After design						
1	MOSFET	3.2638	2.2313	0.6837					
2	Electrolytic capacitor	2.8241	1.6063	0.5688					
3	Power diode	1.1640	0.8220	0.7062					
4	Transform	0.6660	0.6360	0.9550					
5	Transistor	0.3758	0.1508	0.4013					
6	Resistor	0.2544	0.2016	0.7925					

components can reduce failure rates of components and enhance the life consistency.

At last, the failure rates of critical components before and after design are predicted according to the approach of stress analysis. The comparison is listed in Table 4. It can be found that the failure rate of some components is reduced to approximately 70%. This result indicates that the reliability of the LED driver is improved greatly and the design specifications are fully satisfied.

#### 6. Conclusions

This paper proposes a novel multi-objective tolerance design method for electronic systems. It tries to improve the performance of output response and reduce operating stresses on components. The traditional method only focuses on the consistency of output response for a batch of products. It is just an approach to balance cost and performance. The reliability of products themselves is not improved radically. The essential purpose of the proposed method is to improve the working condition and operating reliability of products themselves, and to reduce the cost increase to a maximum extent. It is shown that the operating stresses on some key components and elements are the critical factors that decrease the reliability and life-cycle. From initial design perspective, system reliability can be improved by optimal design. The multi-objective tolerance design is to do such a job.

For a large and complex system, critical components can be identified by reliability prediction, FMEA, and FTA. The key operating stresses on critical components and output response are defined as objectives for multi-objective tolerance design. Sensitive parameters that cause fluctuations of design objectives are determined by sensitive analysis. According to the sensitivity rank, the tolerance levels of sensitive parameters are defined as design variables. Tolerance analysis is performed by the Monte Carlo and worst-case methods. According to the cost and quality loss, a quality-cost model is established. When the whole life-cycle cost is minimal, the tolerance levels are selected.

In the study, the whole process is assisted by PSpice simulation. The proposed method can improve system reliability by tolerance design with high efficiency. It is evaluated by a case study, showing that the operating stresses on a LED driver are reduced significantly and the performance of output current is improved effectively. It solves the problem of improving system reliability from initial parameters and tolerance design.

#### Acknowledgement

This study was supported by National Defense Basic Research Program (No. 20112060303).

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**Zhai Guofu** received his Ph.D. from Harbin Institute of Technology (HIT) in 1998. He is currently a professor in the Department of Electrical Engineering at HIT. His main research interests include electrical contacts and reliability and testing techniques of electrical apparatus.

**Zhou Yuege** received his B.S. and M.S. degrees from HIT in 2009 and 2011 respectively, and is now a Ph.D. candidate at HIT. His main research interests are reliability design and prognostic of electronic systems.

**Ye Xuerong** received his B.S. and M.S. degrees from HIT in 2005 and 2009 respectively. He is currently a lecturer in the Department of Electrical Engineering at HIT. His main research interest is reliability and testing techniques of relay.

**Hu Bo** received his B.S. degree from Dalian University of Technology in 2008 and M.S. degree from HIT in 2010. His main research interest is tolerance design of electronic circuits.