



# Gluonic phase versus LOFF phase in two-flavor quark matter

O. Kiriya<sup>a,\*</sup>, D.H. Rischke<sup>a,b</sup>, I.A. Shovkovy<sup>b,c,1</sup>

<sup>a</sup> *Institut für Theoretische Physik, J.W. Goethe-Universität, D-60438 Frankfurt am Main, Germany*

<sup>b</sup> *Frankfurt Institute for Advanced Studies, J.W. Goethe-Universität, D-60438 Frankfurt am Main, Germany*

<sup>c</sup> *Department of Physics, Western Illinois University, Macomb, IL 61455, USA*

Received 6 June 2006; received in revised form 26 September 2006; accepted 30 October 2006

Editor: J.-P. Blaizot

## Abstract

We study the gluonic phase in a two-flavor color superconductor as a function of the ratio of the gap over the chemical potential mismatch,  $\Delta/\delta\mu$ . We find that the gluonic phase resolves the chromomagnetic instability encountered in a two-flavor color superconductor for  $\Delta/\delta\mu < \sqrt{2}$ . We also calculate approximately the free energies of the gluonic phase and the single plane-wave LOFF phase and show that the former is favored over the latter for a wide range of coupling strengths.

© 2006 Elsevier B.V. Open access under [CC BY license](http://creativecommons.org/licenses/by/2.0/).

PACS: 12.38.-t; 11.30.Qc; 26.60.+c

It is widely accepted that sufficiently cold and dense quark matter is a color superconductor [1]. The most likely and, probably, the only place where color superconductivity can exist in the universe is the interior of compact stars. Thus, studies of phases of quark matter under conditions realized in the bulk of compact stars (i.e., color and electric charge neutrality, and  $\beta$ -equilibrium) have recently attracted a great deal of interest. The density regime of relevance for compact stars is up to a few times the normal nuclear density  $\rho_0 \simeq 0.16 \text{ fm}^{-3}$ . In this “moderate” density regime, the studies of QCD-motivated effective theories are most useful and have revealed a rich phase structure [2–4].

One of the most striking features of neutral and  $\beta$ -equilibrated color-superconducting phases is unconventional cross-flavor Cooper pairing of quarks with the possibility of gapless superconductivity, e.g., in the form of the gapless 2SC (g2SC) phase [5] or the gapless color-flavor-locked (gCFL) phase [6].

It was, however, quickly realized that the 2SC/g2SC phases suffer from a chromomagnetic instability, indicated by imaginary Meissner screening masses of some gluons [7]. In the 2SC phase, these instabilities occur when the ratio of the gap over the mismatch of the chemical potential,  $\Delta/\delta\mu$ , decreases below a value  $\sqrt{2}$ . Similar instabilities were found also in the gCFL phase [8].

Resolving the chromomagnetic instability and clarifying the nature of the true ground state of dense quark matter are the most pressing tasks in the study of color superconductors. It was proposed that the chromomagnetic instability in two-flavor quark matter can be removed by the formation of a single plane-wave LOFF state [9–12] (first studied by Larkin and Ovchinnikov [13], and Fulde and Ferrell [14] in the context of solid state physics, and by Alford et al. [15] for cold, dense quark matter), or a gluonic phase with vector condensation in the ground state [16]. (For a recent discussion of this issue, see also Refs. [17,18].) Alternatives include a mixed phase [19] and, in the case of three-flavor quark matter, also phases with spontaneously induced meson supercurrents [20]. While the neutral LOFF state is free from the chromomagnetic instability in the weak-coupling regime [10], this is, in fact, not the case for somewhat larger values of the coupling [21]. At the same time, the gluonic phase can resolve the instability there. So far, how-

\* Corresponding author.

E-mail addresses: [kiriya@th.physik.uni-frankfurt.de](mailto:kiriya@th.physik.uni-frankfurt.de) (O. Kiriya), [drischke@th.physik.uni-frankfurt.de](mailto:drischke@th.physik.uni-frankfurt.de) (D.H. Rischke), [shovkovy@th.physik.uni-frankfurt.de](mailto:shovkovy@th.physik.uni-frankfurt.de) (I.A. Shovkovy).

<sup>1</sup> On leave from Bogolyubov Institute for Theoretical Physics, 03143 Kiev, Ukraine.

ever, the gluonic phase has been studied only around the critical point  $\Delta/\delta\mu = \sqrt{2}$  [16].

The gluonic phase and the LOFF phases are currently viewed as the most likely candidates for resolving the chromomagnetic instability and, thus, for the true ground state of two-flavor color-superconducting quark matter. (Here we exclude the possibility of phase separation [19] which may have limitations of its own and deserves a separate in-depth study.) In order to see which of the two proposed phases is actually preferred, one first has to extend the analysis of Ref. [16] to a computation of the free energy away from the critical point  $\Delta/\delta\mu = \sqrt{2}$ , and then compare the results to the free energy of the single-plane LOFF state.<sup>2</sup> This is done in the present work. We qualitatively confirm the results of Ref. [18] and extend them by (approximately) including the neutrality condition and explicitly comparing the free energy of the gluonic phase to that of the single plane-wave LOFF phase as a function of the coupling strength.

In order to study various phases of two-flavor quark matter, we use a gauged Nambu–Jona-Lasinio (NJL) model with massless up and down quarks:

$$\mathcal{L} = \bar{\psi}(i\not{D} + \hat{\mu}\gamma^0)\psi + G_D(\bar{\psi}i\gamma_5\epsilon^b C\bar{\psi}^T)(\psi^T C i\gamma_5\epsilon^b \psi) - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu}, \quad (1)$$

where the quark field  $\psi$  carries flavor ( $i, j = 1, \dots, N_f$  with  $N_f = 2$ ) and color ( $\alpha, \beta = 1, \dots, N_c$  with  $N_c = 3$ ) indices,  $C$  is the charge conjugation matrix;  $(\epsilon)^{ik} = \epsilon^{ik}$  and  $(\epsilon^b)^{\alpha\beta} = \epsilon^{b\alpha\beta}$  are the antisymmetric tensors in flavor and color spaces, respectively. The covariant derivative and the field strength tensor are defined as

$$D_\mu = \partial_\mu - igA_\mu^a T^a, \quad (2a)$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c. \quad (2b)$$

To evaluate loop diagrams we use a three-momentum cutoff  $\Lambda$ . Hence, the model has two phenomenological model parameters, the cutoff  $\Lambda$  and the diquark coupling  $G_D$ . We use  $\Lambda = 653.3$  MeV throughout this Letter, but we consider  $G_D$  as a free parameter. Henceforth, in order to specify the diquark coupling we use  $\Delta_0$  which is the value of the 2SC gap at  $\delta\mu = 0$  (see below).

In  $\beta$ -equilibrated neutral 2SC/g2SC matter, the elements of the diagonal matrix of quark chemical potentials  $\hat{\mu}$  are given by

$$\mu_{ur} = \mu_{ug} = \bar{\mu} - \delta\mu, \quad (3a)$$

$$\mu_{dr} = \mu_{dg} = \bar{\mu} + \delta\mu, \quad (3b)$$

$$\mu_{ub} = \bar{\mu} - \delta\mu - \mu_8, \quad (3c)$$

$$\mu_{db} = \bar{\mu} + \delta\mu - \mu_8, \quad (3d)$$

<sup>2</sup> Note that while the comparison with a multi-plane wave LOFF states would be more desirable, the corresponding free energy cannot be easily estimated within a microscopic approach. For current state-of-the-art calculations using an effective theory see Ref. [22].

with

$$\bar{\mu} = \mu - \frac{\delta\mu}{3} + \frac{\mu_8}{3}, \quad \delta\mu = \frac{\mu_e}{2}. \quad (4)$$

In a gauge theory, the self-consistent solution of the Yang–Mills equations requires background gauge fields [23]. These can be viewed as electric- and color-chemical potentials which ensure electric and color-charge neutrality of the system. Note that a generalization of this holds true even in the case of inhomogeneous phases. Then, of course, the corresponding fields would not be constant in space. Instead, they would have a constant central value contribution and, on top of it, a coordinate-dependent modulation describing color-electric fields induced by the inhomogeneities.<sup>3</sup> The constant contribution would take care of the *global* neutrality, while the modulation describes the local field needed to prevent the *local* flow of currents.

On the other hand, in NJL-type models without dynamic gauge fields, one has to ensure electric and color-charge neutrality by introducing appropriate chemical potentials by hand [24]. In the case of the 2SC/g2SC phases, we only require an electron chemical potential  $\mu_e$ , and a color-chemical potential  $\mu_8$  which ensures that the color-charge density  $n_8$  is zero. In principle, in other phases like the gluonic phase one has to check that no other color-charge density is non-vanishing and necessitates the introduction of a respective color-chemical potential. Indeed, the gluonic phase introduced in Ref. [16] requires a non-vanishing temporal component of the gluon field of the third color,  $\langle A_0^3 \rangle$ . In our gauged NJL model, this is equivalent to a non-vanishing color-chemical potential  $\mu_3$  besides  $\mu_8$ . In this first exploratory study, however, we use the fact that both  $\mu_3$  and  $\mu_8$  are known to be numerically small and we simply neglect them.

In Nambu–Gor’kov space, the inverse full quark propagator  $S^{-1}(p)$  is written as

$$S^{-1}(p) = \begin{pmatrix} (S_0^+)^{-1} & \Phi^- \\ \Phi^+ & (S_0^-)^{-1} \end{pmatrix}, \quad (5)$$

with

$$(S_0^+)^{-1} = \gamma^\mu p_\mu + (\bar{\mu} - \delta\mu\tau^3)\gamma^0 + g\gamma^\mu A_\mu^a T^a, \quad (6a)$$

$$(S_0^-)^{-1} = \gamma^\mu p_\mu - (\bar{\mu} - \delta\mu\tau^3)\gamma^0 - g\gamma^\mu A_\mu^a T^a, \quad (6b)$$

and

$$\Phi^- = -i\epsilon\epsilon^b\gamma_5\Delta, \quad \Phi^+ = -i\epsilon\epsilon^b\gamma_5\Delta. \quad (7)$$

Here  $\tau^3 = \text{diag}(1, -1)$  is a matrix in flavor space. Following the usual convention, we choose the diquark condensate to point in the third (blue) direction in color space.

In the one-loop approximation, the free energy of two-flavor quark matter at  $T = 0$  is given by

$$V_g = \frac{\Delta^2}{4G_D} - \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4 i} \ln \text{Det} S^{-1}(p), \quad (8)$$

<sup>3</sup> In the special case of a mixed phase, e.g., a color-electric field is generated around the boundary layer between the two phases and prevents the generation of a color-electric current across this layer.

where “Det” stands for the determinant in Dirac, flavor, color, and Nambu–Gor’kov space. Unlike the free energy in Ref. [16], Eq. (8) does not have a quartic term in  $A_\mu^a$ . This is because we neglected the color-chemical potential  $\mu_8$  and, in addition, we take into account only one dynamic gluonic field (see below).

In the gluonic phase [16], the chromomagnetic instability at  $\Delta/\delta\mu < \sqrt{2}$  triggers a non-vanishing vacuum expectation value of the spatial component of

$$K_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} A_\mu^4 - iA_\mu^5 \\ A_\mu^6 - iA_\mu^7 \end{pmatrix}. \quad (9)$$

(For a simpler version of such a phenomenon, see also Ref. [25].) Using the  $SO(3)_{\text{rot}}$  rotational symmetry and the  $SU(2)_c$  color symmetry, one can choose  $B \equiv g\langle A_z^6 \rangle \neq 0$  without loss of generality. Consequently, the non-zero vacuum expectation value of  $B$  breaks  $SO(3)_{\text{rot}}$ , leaving only  $SO(2)_{\text{rot}}$  [16]. Furthermore, a non-vanishing  $B$  together with  $\Delta$  and  $\mu_e$  breaks the original symmetry of QCD down to

$$U(1)_{\tilde{Q}} \otimes U(1)_{\tau_L^3} \otimes U(1)_{\tau_R^3} \otimes SO(2)_{\text{rot}}, \quad (10)$$

where  $U(1)_{\tau_{L/R}^3}$  is a subgroup of the  $SU(2)_{L/R}$  chiral symmetry and the charge  $\tilde{Q}$  is given by

$$\tilde{Q} = Q_f \otimes \mathbb{1}_c - \mathbb{1}_f \otimes T^3 - \frac{1}{\sqrt{3}} \mathbb{1}_f \otimes T^8, \quad (11)$$

with  $Q_f = \text{diag}(\frac{2}{3}, -\frac{1}{3})$  being the flavor matrix of the electric charges of quarks.

The reduced symmetry of the ground state with  $B \neq 0$  allows for additional condensates,  $C = g\langle A_z^1 \rangle \neq 0$  and  $D = g\langle A_0^3 \rangle \neq 0$ . In fact, as discussed in Ref. [16], such condensates are required by the equation of motion. As discussed above, the gluonic field  $D$  is nothing but a color chemical potential  $\mu_3$ . The field  $C$ , on the other hand, induces electric superconductivity in the ground state and, therefore, is physically more interesting. However, including all three gluonic fields makes the analysis quite involved. In this work, we retain only the  $B$  field that is directly connected to the Meissner masses of gluons 4–7 and, thus, is the most relevant field for the chromomagnetic instability.

It is straightforward to show that the mass of the  $B$  field at  $B = 0$  (i.e., in the 2SC/g2SC phases) coincides with the Meissner screening masses of gluons of adjoint color 4–7 calculated in the hard-dense-loop (HDL) approximation [7],

$$M_B^2 = \left. \frac{\partial^2 V_g}{\partial B^2} \right|_{B=0} = \frac{\bar{\mu}^2}{6\pi^2} \left[ 1 - \frac{2\delta\mu^2}{\Delta^2} + 2 \frac{\delta\mu\sqrt{\delta\mu^2 - \Delta^2}}{\Delta^2} \theta(\delta\mu - \Delta) \right]. \quad (12)$$

In order to derive this expression we neglected terms of order  $O(\bar{\mu}^2/\Lambda^2)$  and  $O(\Delta^2/\mu^2)$ .

In order to study the effect of the condensate field  $B$  on the free energy of the 2SC/g2SC phases, we calculate the difference of the thermodynamic potentials in a dense medium and in vacuum at the same value of  $B$ ,

$$\Omega_g \equiv V_g(\Delta, B, \delta\mu, \mu) - V_g(0, B, 0, 0). \quad (13)$$

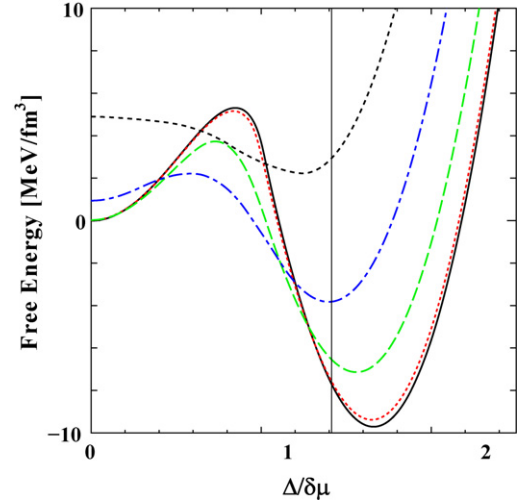


Fig. 1. The free energy (13) measured with respect to the normal phase at  $B = 0$ , as a function of  $\Delta/\delta\mu$  for  $B = 0$  MeV (solid), 50 MeV (dotted), 150 MeV (dashed), 250 MeV (dot-dashed), 350 MeV (short-dashed). The thin vertical line denotes the critical point of the chromomagnetic instability  $\Delta/\delta\mu = \sqrt{2}$ . We used  $\bar{\mu} = 500$  MeV and  $\delta\mu = 80$  MeV, and the value of the coupling constant has been chosen so that  $\Delta_0 = 132$  MeV. The contribution of electrons is not included.

In a gauge theory, this subtraction in the one-loop free energy also takes care of the renormalization of the gauge coupling constant. As a result, the cutoff dependence of the free energy can be completely removed in this approximation.

Let us look at the free energy  $\Omega_g$  in detail. Fig. 1 shows the free energy  $\Omega_g$  (measured with respect to the normal phase at  $B = 0$ ) as a function of  $\Delta/\delta\mu$ . The results are plotted for  $\bar{\mu} = 500$  MeV and  $\delta\mu = 80$  MeV, with the diquark coupling chosen so that  $\Delta_0 = 132$  MeV. Here we do not restrict  $\Delta/\delta\mu$  to its physical value, determined by the stationary point of  $\Omega_g(\Delta)$ , but treat it as a free parameter.

Several important features of the free energy as a function of the  $B$  field are evident from Fig. 1. When  $\Delta/\delta\mu > \sqrt{2}$ , one can see that the free energy monotonically increases with  $B$ . (Strictly speaking, since our model reproduces the HDL result only up to terms of order  $O(\bar{\mu}^2/\Lambda^2)$  and  $O(\Delta^2/\mu^2)$ , the actual critical point is somewhat lower than  $\sqrt{2}$ .) In other words,  $\Omega_g(B)$  has a global minimum at  $B = 0$ , and the 2SC phase is stable against gluon condensation in this regime. This is also clear from a different representation of the results, shown in Fig. 2.

When  $\Delta/\delta\mu < \sqrt{2}$ , on the other hand, we observe the onset of the chromomagnetic instability. For small  $B$ , the free energy first decreases with increasing  $B$  and then grows at larger  $B$ . This can be seen clearly in Fig. 2. The behavior of  $\Omega_g$  agrees well with Eq. (12) at small  $B$ . In this regime, the 2SC/g2SC phase is no longer the ground state. It is unstable with respect to the formation of a non-zero  $B$  condensate, i.e., the so-called gluonic phase. The corresponding ground state is determined by the minimum of  $\Omega_g(B)$ . The Meissner masses squared, which are given by the curvature of the free energy at the minimum, are non-negative in this state. (It is interesting to note that, although the free energy in the normal phase,  $\Delta/\delta\mu = 0$ , cf.

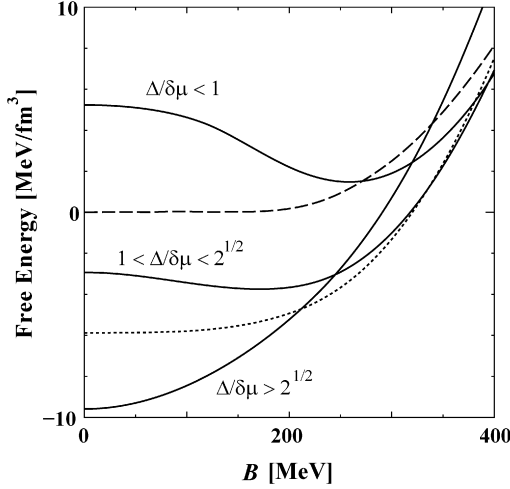


Fig. 2. The free energy (13) measured with respect to the normal phase at  $B = 0$ , as a function of  $B$  in three regimes with different values of  $\Delta/\delta\mu$ . The solid lines correspond to the cases of  $\Delta/\delta\mu = 0.8, 1.2, 1.6$  (top to bottom). The dashed and dotted line denote the free energy at  $\Delta/\delta\mu = 0$  and at the critical point  $\Delta/\delta\mu \simeq \sqrt{2}$ , respectively. Other parameters are the same as in Fig. 1.

dashed line in Fig. 2, increases with  $B$ , this increase is extremely slow over quite a large range of  $B$  values. The reason is that the quadratic term in the Taylor expansion of the free energy is vanishing, see Eq. (12). Let us note that the results of Fig. 2 are in qualitative agreement with those shown in Fig. 2 of Ref. [18].

From the results for the free energy it is clear that the gluonic phase resolves the chromomagnetic instability of the 2SC/g2SC phases. A neutral LOFF state is another candidate for the solution to the instability: it has been shown that such a state is free from the chromomagnetic instability, however, only in the weak-coupling regime [10] (for strong coupling, this is not the case [21]). In order to determine the energetically most favored state, it is necessary to compare the free energies of the 2SC/g2SC phases, the neutral LOFF state and the gluonic phase.

To this end we use the following approximation derived in Ref. [21] for the 2SC/g2SC phases and the neutral LOFF state:

$$\Omega = \Omega_{2SC} + \Omega_{g2SC/LOFF}, \quad (14)$$

where the 2SC and g2SC/LOFF parts of the free energy are given by

$$\Omega_{2SC} = -\frac{\mu_e^4}{12\pi^2} - \frac{\mu_{ub}^4}{12\pi^2} - \frac{\mu_{db}^4}{12\pi^2} - \frac{\bar{\mu}^4}{3\pi^2} + \frac{\Delta^2}{4G_D} - \frac{\bar{\mu}^2 \Delta^2}{\pi^2} \ln \frac{4(\Lambda^2 - \bar{\mu}^2)}{\Delta^2} - \frac{\Delta^2}{\pi^2} (\Lambda^2 - 2\bar{\mu}^2), \quad (15a)$$

$$\Omega_{g2SC/LOFF} = \frac{2\bar{\mu}^2 q^2}{\pi^2} + \frac{\bar{\mu}^2}{\pi^2} \left\{ \frac{(q + \delta\mu)^3}{q} \left[ \frac{1}{2} (1 - x_1^2) \ln \frac{1 + x_1}{1 - x_1} - x_1 + \frac{2}{3} x_1^3 \right] + (q \rightarrow -q) \right\}, \quad (15b)$$

with the dimensionless parameter  $x_1$  being

$$x_1 = \theta \left( 1 - \frac{\Delta^2}{(\delta\mu + q)^2} \right) \sqrt{1 - \frac{\Delta^2}{(\delta\mu + q)^2}} \quad (16)$$

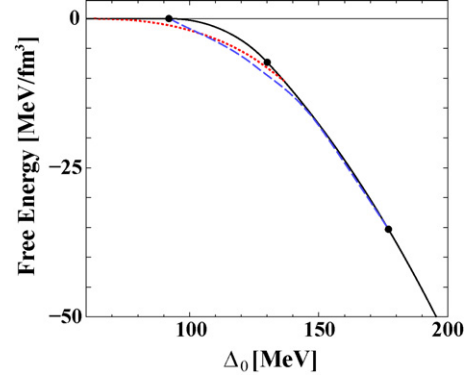


Fig. 3. The free energy of the neutral 2SC/g2SC phase (solid line), the neutral LOFF state (dotted line), and the gluonic phase (dashed line) as a function of  $\Delta_0$ . The three dots on the bold solid line ( $\Delta_0 = 92, 130, 177$  MeV from left to right) denote the edge of the g2SC window with the normal phase, the phase transition point between 2SC and g2SC phases ( $\Delta = \delta\mu$ ), and the critical point of the chromomagnetic instability ( $\Delta = \sqrt{2}\delta\mu$ ). The quark chemical potential is taken to be  $\mu = 400$  MeV.

and

$$q = |\vec{q}|, \quad \vec{q} = \frac{g}{2\sqrt{3}} \langle \vec{A}^8 \rangle. \quad (17)$$

Note that the wave vector of the diquark condensate  $\vec{q}$  is equivalent to a gauge field condensate  $\langle \vec{A}^8 \rangle$  in the case of single plane-wave LOFF pairing. In Eq. (15), the 2SC/g2SC part of the free energy is obtained by taking the  $q \rightarrow 0$  limit. Also a non-zero color chemical potential  $\mu_8$  has been neglected there.

The free energy of a given phase can be computed by solving the gap equations, e.g.,  $\partial\Omega/\partial\Delta = 0$  and  $\partial\Omega/\partial q = 0$ , and the neutrality condition  $\partial\Omega/\partial\delta\mu = 0$ . To simplify the calculations in the gluonic phase, we evaluate the free energy approximately as follows: (i) we obtain  $\Delta^*$  and  $\delta\mu^*$  in the 2SC/g2SC phase by solving the coupled set of equations  $\partial\Omega/\partial\Delta = 0$  and  $\partial\Omega/\partial\delta\mu = 0$ , (ii) by using these solutions, we calculate  $\Omega_g(B, \Delta^*, \delta\mu^*)$  which is an approximate value for the free energy in the gluonic phase. For the densities of interests,  $B$  is at most of the order of 100 MeV, whereas  $q$  is of the order of tens of MeV. We performed a preliminary test of the quality of our approximation by varying  $B$  from 0 to 300 MeV and computing the value of  $\delta\mu$  necessary to ensure electric neutrality. We found that this value changes at most by 10%.

We illustrate the comparison of the free energies of all three phases in Fig. 3 (cf. Fig. 2 in Ref. [21]). We take  $\mu = 400$  MeV and choose the normal phase as a reference point for the free energy. In Ref. [21], it has been demonstrated that the neutral LOFF state is more stable than the 2SC/g2SC phases in the whole LOFF window  $63 \text{ MeV} < \Delta_0 < 137 \text{ MeV}$ , which includes the entire g2SC window  $92 \text{ MeV} < \Delta_0 < 130 \text{ MeV}$ . However, whereas the Meissner masses squared of gluons 4–7 in the weakly coupled neutral LOFF state are positive [9,10], they remain negative in the intermediate and the strongly coupled regimes (at all values of  $\Delta_0$  above 81 MeV) [21]. In contrast, the gluonic phase removes the instability and is energetically favored over the 2SC/g2SC phases in the whole window in which the instability takes place (at all values of  $\Delta/\delta\mu$  below  $\sqrt{2}$ ). The gluonic phase and the neutral LOFF state coexist

in the region  $92 \text{ MeV} < \Delta_0 < 137 \text{ MeV}$ , but, as our results indicate, the gluonic phase is more stable than the neutral LOFF state in a wide region  $\Delta_0 > 103 \text{ MeV}$ , which is close to the edge of the g2SC window with the normal phase. We argue therefore that the instability in the 2SC/g2SC phase is resolved by the formation of the gluonic phase.

In summary, we explored the gluonic phase away from the critical point  $\Delta/\delta\mu = \sqrt{2}$ . We demonstrated that the energetically favored state of a neutral two-flavor color superconductor is not the 2SC/g2SC phase in the intermediate- and strong-coupling regimes but the gluonic phase in which the dynamic gluonic field  $B = g\langle A_z^6 \rangle$  acquires a vacuum expectation value. In particular, the whole g2SC phase is replaced by the gluonic phase which is chromomagnetically stable.

We also compared the free energies of the 2SC/g2SC phase, the neutral LOFF state, and the gluonic phase. We found that the gluonic phase is energetically favored in the intermediate- and strong-coupling regimes. The encouraging results of this analysis should be further improved in the future by (i) taking into account the most general ansatz for the gauge-field configuration in the gluonic phase, (ii) by calculating the free energy in a self-consistent manner. We already performed a preliminary investigation including the effect of the gluon field  $D$ , responsible for enforcing Gauss' law, and found that, for  $\Delta_0 \lesssim 130 \text{ MeV}$ , the additional cost in the free energy is of order  $0.01 \text{ MeV}/\text{fm}^3$ , and thus negligible. For  $\Delta_0 \gtrsim 130 \text{ MeV}$ , however, the effect of the  $D$  field could be an order of magnitude larger.

It is appropriate to mention that the instability related to the 8th gluon was not studied in the present work. In this sense, the neutral LOFF state is appealing, because the Meissner mass of the 8th gluon is automatically zero in this state. It should also be mentioned that in the strongly coupled LOFF state [9,11] the longitudinal Meissner mass squared of the 8th gluon is negative. Although this instability was not addressed in this work, it is unlikely, however, that the LOFF state is energetically favored in the strong-coupling regime.

### Note added

It has recently been demonstrated that, in the three-flavor case, realistic crystal structures are more robust than a single plane-wave LOFF state [22]. In the two-flavor case, Bowers and Rajagopal [26] already indicated that a LOFF state with multiple plane waves would have a lower free energy than that with a single plane wave. The result shown in Fig. 3 would be altered by the inclusion of crystal structures with more plane waves. This is therefore an important project that needs to be addressed in future work.

### Acknowledgements

I.A.S. acknowledges discussions with V.A. Miransky. This work was supported in part by the Virtual Institute of the Helmholtz Association under grant No. VH-VI-041, by the Gesellschaft für Schwerionenforschung (GSI), and by the Deutsche Forschungsgemeinschaft (DFG).

### References

- [1] K. Rajagopal, F. Wilczek, in: M. Shifman (Ed.), *At the Frontier of Particle Physics/Handbook of QCD*, World Scientific, Singapore, 2001; M.G. Alford, *Annu. Rev. Nucl. Part. Sci.* 51 (2001) 131; D.K. Hong, *Acta Phys. Pol. B* 32 (2001) 1253; S. Reddy, *Acta Phys. Pol. B* 33 (2002) 4101; D.H. Rischke, *Prog. Part. Nucl. Phys.* 52 (2004) 197; M. Buballa, *Phys. Rep.* 407 (2005) 205; M. Huang, *Int. J. Mod. Phys. E* 14 (2005) 675; I.A. Shovkovy, *Found. Phys.* 35 (2005) 1309.
- [2] S.B. Rüster, I.A. Shovkovy, D.H. Rischke, *Nucl. Phys. A* 743 (2004) 127; I.A. Shovkovy, S.B. Rüster, D.H. Rischke, *J. Phys. G* 31 (2005) S849; K. Fukushima, C. Kouvaris, K. Rajagopal, *Phys. Rev. D* 71 (2005) 034002.
- [3] S.B. Rüster, V. Werth, M. Buballa, I.A. Shovkovy, D.H. Rischke, *Phys. Rev. D* 72 (2005) 034004; S.B. Rüster, V. Werth, M. Buballa, I.A. Shovkovy, D.H. Rischke, *Phys. Rev. D* 73 (2006) 034025; D. Blaschke, S. Fredriksson, H. Grigorian, A.M. Öztaş, F. Sandin, *Phys. Rev. D* 72 (2005) 065020.
- [4] K. Iida, T. Matsuura, M. Tachibana, T. Hatsuda, *Phys. Rev. Lett.* 93 (2004) 132001; H. Abuki, M. Kitazawa, T. Kunihiro, *Phys. Lett. B* 615 (2005) 102; H. Abuki, T. Kunihiro, *Nucl. Phys. A* 768 (2006) 118.
- [5] I. Shovkovy, M. Huang, *Phys. Lett. B* 564 (2003) 205; M. Huang, I. Shovkovy, *Nucl. Phys. A* 729 (2003) 835.
- [6] M. Alford, C. Kouvaris, K. Rajagopal, *Phys. Rev. Lett.* 92 (2004) 222001; M. Alford, C. Kouvaris, K. Rajagopal, *Phys. Rev. D* 71 (2005) 054009.
- [7] M. Huang, I.A. Shovkovy, *Phys. Rev. D* 70 (2004) 051501(R); M. Huang, I.A. Shovkovy, *Phys. Rev. D* 70 (2004) 094030.
- [8] R. Casalbuoni, R. Gatto, M. Mannarelli, G. Nardulli, M. Ruggieri, *Phys. Lett. B* 605 (2005) 362; R. Casalbuoni, R. Gatto, M. Mannarelli, G. Nardulli, M. Ruggieri, *Phys. Lett. B* 615 (2005) 297, Erratum; M. Alford, Q.H. Wang, *J. Phys. G* 31 (2005) 719; K. Fukushima, *Phys. Rev. D* 70 (2005) 074002.
- [9] I. Giannakis, H.C. Ren, *Phys. Lett. B* 611 (2005) 137; I. Giannakis, H.C. Ren, *Nucl. Phys. B* 723 (2005) 255.
- [10] I. Giannakis, D.f. Hou, H.C. Ren, *Phys. Lett. B* 631 (2005) 16.
- [11] M. Huang, *Phys. Rev. D* 73 (2006) 045007.
- [12] D.K. Hong, hep-ph/0506097.
- [13] A.I. Larkin, Y.N. Ovchinnikov, *Zh. Eksp. Teor. Fiz.* 47 (1964) 1136, *Sov. Phys. JETP* 20 (1965) 762.
- [14] P. Fulde, R.A. Ferrell, *Phys. Rev.* 135 (1964) A550.
- [15] M. Alford, J.A. Bowers, K. Rajagopal, *Phys. Rev. D* 63 (2001) 074016.
- [16] E.V. Gorbar, M. Hashimoto, V.A. Miransky, *Phys. Lett. B* 632 (2006) 305.
- [17] K. Iida, K. Fukushima, hep-ph/0603179.
- [18] K. Fukushima, *Phys. Rev. D* 73 (2006) 094016.
- [19] S. Reddy, G. Rupak, *Phys. Rev. C* 71 (2005) 025201; F. Neumann, M. Buballa, M. Oertel, *Nucl. Phys. A* 714 (2003) 481; I. Shovkovy, M. Hanauske, M. Huang, *Phys. Rev. D* 67 (2003) 103004.
- [20] A. Kryjevski, hep-ph/0508180; T. Schäfer, *Phys. Rev. Lett.* 96 (2006) 012305.
- [21] E.V. Gorbar, M. Hashimoto, V.A. Miransky, *Phys. Rev. Lett.* 96 (2006) 022005.
- [22] K. Rajagopal, R. Sharma, hep-ph/0605316.
- [23] A. Gerhold, A. Rebhan, *Phys. Rev. D* 68 (2003) 011502(R); D.D. Dietrich, D.H. Rischke, *Prog. Part. Nucl. Phys.* 53 (2004) 305.
- [24] M. Buballa, I.A. Shovkovy, *Phys. Rev. D* 72 (2005) 097501.
- [25] V.P. Gusynin, V.A. Miransky, I.A. Shovkovy, *Phys. Lett. B* 581 (2004) 82; V.P. Gusynin, V.A. Miransky, I.A. Shovkovy, *Mod. Phys. Lett. A* 19 (2004) 1341.
- [26] J.A. Bowers, K. Rajagopal, *Phys. Rev. D* 66 (2002) 065002.