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Global existence for the system of the macroscopic balance equations of charge transport in semiconductors ☆

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Abstract

Global existence of a solution to the nonlinear balance equations of charge transport in semiconductors based on the maximum entropy principle [Contin. Mech. Thermodyn. 11 (1999) 307–325; Contin. Mech. Thermodyn. 12 (2000) 31–51] is proven for a typical 1D problem under certain restrictions on the doping profile and the initial data.

0. Introduction

0.1. In the hierarchy of macroscopic models of charge transport in semiconductors beyond *the drift-diffusion equations* [21,32,39] and *the energy-transport models* [1,17,30],

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one finds *the hydrodynamical models* which are obtained from an infinite set of moment equations of the *Boltzmann transport equation* by a suitable truncation procedure.

As is well known, a closure assumption is required in order to come from the moment equations system to a closed system of evolution equations. In [6,35], the balance equations for density, momentum, energy, and energy flux of electrons have been considered for silicon and, by using the maximum entropy principle (MEP) for the closure, explicit constitutive relations for the stress tensor, the flux of energy flux tensor, and production terms have been obtained both in the parabolic and the Kane dispersion relation approximations for the energy conduction bands (for a complete review see [5,7]). The model fits into *extended thermodynamics* [22,34] and Levermore's theory of moments [25] and does not contain any fitting parameters. General mathematical properties of the constitutive equations have been studied in [36] where it has been shown that the evolution equations form a hyperbolic system in the physically relevant region of the fields space. Applications of the model have been presented in [36] for 1D problems and in [37] for a 2D simulation of a silicon MESFET.

0.2. In the present paper we consider a typical 1D problem, representing *the* $n^+ - n - n^+$ *ballistic diode*, that has been intensively investigated by employing different numerical methods and models [4,8,15,18,19,36,38]. Roughly speaking, physically, the situation is given by a semiconductor divided into three parts: two regions of high doping (the n^+ regions) with a region of low doping (the n region) in between.

The dynamics of charge carriers depends on the applied potential (*the bias voltage*). When the applied voltage is negligible the system is expected to tend to *the global thermodynamical equilibrium* where the charge is at rest with the same temperature of the crystal.

In papers [13,14] it has been proved that for the model under consideration the equilibrium solution is asymptotically stable in the parabolic band case under certain restriction on the doping profile. However these papers contain only a brief discussion of the question on global existence for the 1D problem mentioned above. In the present paper we discuss this question in detail. Note also that similar problems for various hydrodynamical models are considered in [2,3,26–29,31].

The plan of the paper is the following. The basic equations are presented in Section 1 where they are reformulated in a more suitable way for the subsequent analysis given in Section 2. Global existence is discussed in Section 2.

1. Preliminaries. Basic equations and formulations of the problem

1.1. In papers [6,35–37], a *system of moment equations* well-reasoned from the physical point of view was proposed and used to describe the charge transport process in concrete semiconductor devices. These equations have the form of *conservation laws*. The system was obtained from the Boltzmann transport equation using a suitable truncation procedure (see [6,35]). Note that the variety of truncation procedures in mathematical modelling of charge transport causes the existence of a great number of mathematical models.

Following [13,14], in a dimensionless form and for the 1D case the quasilinear system of the above equations mentioned above reads (the reduction to the dimensionless form is described in detail in [13,14])

$$\begin{cases} R_t + J_x = 0, \\ J_t + \left(\frac{2}{3}RE\right)_x = RQ + c_{11}J + c_{12}I, \\ (RE)_t + I_x = JQ + cP, \\ I_t + \left(\frac{10}{9}RE^2\right)_x = \frac{5}{3}REQ + c_{21}J + c_{22}I, \end{cases} \tag{1.1}$$

$$\varepsilon\varphi_{xx} = R - \rho. \tag{1.2}$$

Here R is the electron density, $J = Ru$, $I = Rq$, u is the electron velocity, q is the energy flux, E is the electron energy, $P = R(\frac{2}{3}E - 1)$, $Q = \varphi_x$, φ is the electric potential, $\rho = \rho(x)$ is the doping density. The coefficients $c_{11}, \dots, c_{22}$, c of the system (1.1) are smooth functions of energy E. The precise (but rather cumbersome) expressions for these functions in the parabolic band case are reported in [13,14]. The constant $\varepsilon > 0$ appearing in the Poisson equation (1.2) is a dimensionless dielectric constant. It is also described in [13,14] for various values of physical parameters.

For system (1.1) we take the following boundary conditions at $x = 0, 1$ (for $t > 0$) corresponding to the ballistic diode problem that is well known in physics of semiconductors (see [15,18,19] and Fig. 1):

$$\begin{cases} R(t,0) = R(t,1) = 1, \\ E(t,0) = E(t,1) = \frac{3}{2}. \end{cases} \tag{1.3}$$

We pose also the initial data for $t = 0$, $0 < x < 1$:

$$\begin{cases} R(0,x) = R_0(x), \\ J(0,x) = J_0(x), \\ E(0,x) = E_0(x), \\ I(0,x) = I_0(x), \end{cases} \tag{1.4}$$

where $R_0(x), E_0(x) > 0$. For the Poisson equation (1.2) we take the following boundary conditions at $x = 0, 1$ ($t > 0$):

$$\varphi(t,0) = 0, \qquad \varphi(t,1) = b, \tag{1.5}$$

where $b > 0$ represents the bias voltage across the diode. System (1.1), (1.2) is considered in the domain $t > 0$, $0 < x < 1$.

1.2. Considering Eq. (1.2) as an ordinary differential equation (with parameter t) for the unknown function $\varphi(t,x)$ with the boundary conditions (1.5), one obtains (see [15])

$$\varphi = \varphi(t,x) = bx + \beta \int_0^1 G(x,s)\big(R(t,s) - \rho(s)\big)\,ds, \quad \beta = \frac{1}{\varepsilon}, \tag{1.6}$$

where $G(x,s)$ is the Green function:

$$G(x,s) = \begin{cases} s(x-1), & \text{if } 0 < s \leqslant x, \\ x(s-1), & \text{if } x < s < 1. \end{cases}$$

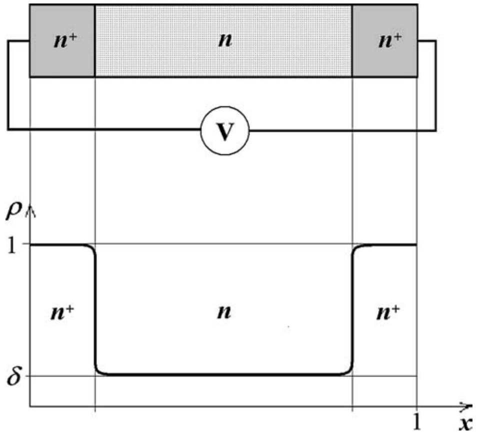

Fig. 1. Schematic representation of an $n^+ - n - n^+$ ballistic diode.

Differentiating (1.6) with respect to x, one has the following expression for $Q = \varphi_x$:

$$Q = Q(t,x) = b + \beta\int_0^x \bigl(R(t,s) - \rho(s)\bigr)\,ds - \beta\int_0^1 (1-s)\bigl(R(t,s) - \rho(s)\bigr)\,ds. \tag{1.7}$$

Note that the initial data $R_0(x)$ and $\varphi(0,x) = \varphi_0(x)$ are related by (1.6).

As for the doping density function $\rho(x)$, we will assume in the sequel that it has the typical profile (see, for example, [15]) depicted in Fig. 1.

1.3. System (1.1) is written in the divergent form. Consider also the nondivergent form of this system:

$$\tilde{\boldsymbol{V}}_t + \mathcal{B}\tilde{\boldsymbol{V}}_x = \boldsymbol{F}(Q, \tilde{\boldsymbol{V}}). \tag{1.8}$$

Here

$$\tilde{\boldsymbol{V}} = \begin{pmatrix} R \\ J \\ RE \\ I \end{pmatrix}, \qquad \mathcal{B} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{10}{9}E^2 & 0 & \frac{20}{9}E & 0 \end{pmatrix},$$

$$\boldsymbol{F} = \begin{pmatrix} 0 \\ RQ + c_{11}J + c_{12}I \\ JQ + cP \\ \frac{5}{3}REQ + c_{21}J + c_{22}I \end{pmatrix},$$

to find Q one uses expression (1.7). Calculating the eigenvalues λ of the matrix $\mathcal{B}$, one has

$$\begin{cases} \lambda_{1,2} = \pm\bigl(\frac{10+2\sqrt{10}}{9}E\bigr)^{1/2}, \\ \lambda_{3,4} = \pm\bigl(\frac{10-2\sqrt{10}}{9}E\bigr)^{1/2}. \end{cases} \tag{1.9}$$

If a natural physical condition

$$E > 0$$

holds, then system (1.8) is t-hyperbolic (strictly hyperbolic) (see [20]). Note that, as it follows from (1.9), the number of boundary conditions (see conditions (1.3)) for each of the boundaries $x = 0, 1$ corresponds exactly to the number of outgoing characteristics for these boundaries.

Write system (1.1) as

$$\tilde{\boldsymbol{V}}_t + \boldsymbol{W}_x = \boldsymbol{F}(Q, \tilde{\boldsymbol{V}}),$$

where

$$\boldsymbol{W} = \begin{pmatrix} J \\ \frac{2}{3}RE \\ I \\ \frac{10}{9}RE^2 \end{pmatrix}.$$

Making the change of unknowns

$$\boldsymbol{V} = \begin{pmatrix} R \\ J \\ P \\ \Theta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & \frac{2}{3} & 0 \\ 0 & -\frac{5}{2} & 0 & 1 \end{pmatrix} \tilde{\boldsymbol{V}},$$

we obtain one more form of the original system (1.1):

$$\boldsymbol{V}_t + \boldsymbol{\Lambda}_x = \mathcal{F}(Q, \boldsymbol{V}). \tag{1.10}$$

Here

$$\boldsymbol{\Lambda} = \begin{pmatrix} J \\ R + P \\ \frac{2}{3}(J + \Theta) \\ \frac{5}{2}P(1 + \sigma) \end{pmatrix}, \quad \mathcal{F} = \begin{pmatrix} 0 \\ RQ + \mu_{11}J + \mu_{12}\Theta \\ \frac{2}{3}(JQ + cP) \\ \frac{5}{2}(PQ + \mu_{21}J + \mu_{22}\Theta) \end{pmatrix}, \quad \sigma = \frac{P}{R},$$

$$\mu_{11} = c_{11} + \frac{5}{2}c_{12}, \qquad \mu_{12} = c_{12},$$

$$\mu_{21} = \frac{2}{5}c_{21} - c_{11} + \frac{5}{2}\mu_{22}, \qquad \mu_{22} = \frac{2}{5}c_{22} - c_{12}.$$

The boundary conditions (1.3) for this system read as

$$\begin{cases} R(t, 0) = R(t, 1) = 1, \\ P(t, 0) = P(t, 1) = 0. \end{cases} \tag{1.11}$$

To construct initial data for (1.10) one uses conditions (1.4):

$$\begin{cases} P(0, x) = P_0(x) = R_0(x)\bigl(\frac{2}{3}E_0(x) - 1\bigr), \\ \Theta(0, x) = \Theta_0(x) = I_0(x) - \frac{5}{2}J_0(x), \\ R(0, x) = R_0(x), \\ J(0, x) = J_0(x). \end{cases} \tag{1.12}$$

The component Q in (1.10) is determined by (1.7).

1.4. We give an equivalent formulation of the initial boundary value problem (1.10)–(1.12). Write the Poisson equation (1.2) in the form

$$\varepsilon Q_x = R - \rho. \tag{1.13}$$

With regard to the first equation in system (1.10), differentiating (1.13) with respect to t yields

$$(\varepsilon Q_t + J)_x = 0,$$

or

$$\varepsilon Q_t = J_*(t) - J,$$

where $J_*(t)$ is yet an unknown function. Since

$$\int_0^1 Q(t,s)\,ds = b, \tag{1.14}$$

then

$$\varepsilon \int_0^1 Q_t(t,s)\,ds = \varepsilon \frac{d}{dt}\left(\int_0^1 Q(t,s)\,ds\right) = 0 = J_*(t) - \int_0^1 J(t,s)\,ds.$$

Thus,

$$J_*(t) = \int_0^1 J(t,s)\,ds.$$

Therefore, we have the following relation for Q:

$$\varepsilon Q_t = l[J] = \int_0^1 J(t,s)\,ds - J(t,x). \tag{1.15}$$

The initial value $Q(0,x) = Q_0(x)$ for the function $Q(t,x)$ is found from expression (1.7).

Thus, instead of the initial boundary value problem (1.10)–(1.12) we can consider problem (1.10), (1.15), (1.11), (1.12) coupled with the initial data $Q_0(x)$ for $Q(t,x)$ and the two obvious relations

$$\begin{cases} \varepsilon Q_0'(x) = R_0(x) - \rho(x), \\ \int_0^1 Q_0(s)\,ds = b. \end{cases} \tag{1.16}$$

It is clear that, by virtue of (1.16), Eq. (1.13) and relation (1.14) are satisfied for all $t > 0$. At last, the potential $\varphi = \varphi(t,x)$ is determined from the relation

$$\varphi = \varphi(t,x) = \int_0^x Q(t,s)\,ds \tag{1.17}$$

when the solution to the newly formulated problem is found.

1.5. It follows from elementary physical considerations that problem (1.10)–(1.12) for $b = 0$ has the stationary *solution of global thermodynamical equilibrium*

$$\begin{cases} J(t,x) \equiv \hat{J} = 0, \\ P(t,x) \equiv \hat{P} = 0, \\ \Theta(t,x) \equiv \hat{\Theta} = 0, \\ R(t,x) = \hat{R}(x) = e^{\hat{\varphi}(x)}, \\ \varphi(t,x) = \hat{\varphi}(x), \end{cases} \tag{1.18}$$

where $\hat{\varphi}(x)$ satisfies the Poisson equation

$$\varepsilon\hat{\varphi}'' = e^{\hat{\varphi}} - \rho \tag{1.19}$$

with the boundary conditions

$$\hat{\varphi}(0) = \hat{\varphi}(1) = 0. \tag{1.20}$$

Note that ε is sufficiently small for real semiconductor devices (see [13–15]).

The boundary value problem (1.19), (1.20) is studied in detail in [12]. The following theorem was proved there.

Theorem 1.1. *Suppose the doping density* $\rho = \rho(x) \in C^2[0,1]$, $0 < \delta \leqslant \rho(x) \leqslant 1$, *and the function* $(\rho(x) - 1)$ *is finite* (*see Fig.* 1). *Then there is a number* $\varepsilon_0 > 0$ *such that for all* ε, $0 < \varepsilon \leqslant \varepsilon_0$, *there exists a unique solution to the boundary value problem* (1.19), (1.20) *in the form*

$$\hat{\varphi}(x) = \ln\rho(x) + O(\varepsilon^{1/2}). \tag{1.21}$$

Here $C^2[0,1]$ is the space of smooth functions with the norm

$$\|\rho\|_{C^2[0,1]} = \max_{x\in[0,1]}\left|\rho(x)\right| + \max_{x\in[0,1]}\left|\rho'(x)\right| + \max_{x\in[0,1]}\left|\rho''(x)\right|.$$

Note that the number ε_0 depends on δ. Unfortunately, the dielectric constant ε satisfy the inequality $0 < \varepsilon \leqslant \varepsilon_0$ if δ is close to 1. One of the reasons of this fact is coarseness of the number ε_0 estimation in the theorem.

The proof of this theorem is constructive in the sense that it gives an exact form of the second addend in (1.21). At last, note that if

$$\rho(x) \equiv 1,$$

then problem (1.19), (1.20) has the unique solution

$$\hat{\varphi}(x) \equiv 1.$$

1.6. Basing again on the physical considerations, one expects that *in the case* $b = 0$ *the solution to the initial boundary value problem* (1.10)–(1.12) *tends to the global equilibrium state* (1.18) *as* $t \to \infty$ *for any initial data. The main result of this paper is the demonstration of this fact under some restrictions on the doping density and the initial data.* To do this we prove first the global existence theorem. The key point of this proof is the construction of a global a priori estimate. Besides, the proof is based on the local (short-time) existence theorem for strictly hyperbolic systems in two independent variables with rather arbitrary boundary conditions (see paper [24]).

1.7. In the next section the technique of construction of global a priori estimate is described (see also papers [13,14]). To do this we need another formulation of the initial boundary value problem (1.10)–(1.12) in the case $b = 0$. We introduce a new independent variable $H = H(t, x)$ such that

$$\begin{cases} J = H_t, \\ R = -H_x. \end{cases} \tag{1.22}$$

Then the first equation in system (1.10) holds automatically. Accounting for (1.22), Eq. (1.15) can be rewritten as

$$\big(\varepsilon Q - l[H]\big)_t = 0$$

that is

$$\varepsilon Q(t, s) = l[H] + A_0(x).$$

To determine the arbitrary function $A_0(x)$ we use relation (1.13),

$$A_0'(x) = -\rho(x),$$

or

$$A_0(x) = C - \int_0^x \rho(s)\, ds.$$

Here C is yet an arbitrary constant. But, by taking into account (1.14), one has

$$C = \int_0^1 (1 - s)\rho(s)\, ds$$

and

$$A_0(x) = -\int_0^x \rho(s)\, ds + \int_0^1 (1 - s)\rho(s)\, ds.$$

Consequently,

$$Q(t, x) = \beta\big(l[H] + A_0(x)\big).$$

It is convenient to introduce one more auxiliary function $U = U(t, x)$ by the rule (see also (1.18))

$$H(t, x) = U(t, x) - \int_0^x \hat{R}(s)\, ds. \tag{1.23}$$

By means of (1.7), (1.22) we have the expression for Q:

$$Q(t, x) = \beta l[U] + \hat{Q}(x), \tag{1.24}$$

where

$$\hat{Q}(x) = \hat{\varphi}'(x) = \beta \int_0^x \big(\hat{R}(s) - \rho(s)\big)\, ds - \beta \int_0^1 (1-s)\big(\hat{R}(s) - \rho(s)\big)\, ds.$$

In new terms relations (1.22) take the form

$$\begin{cases} J = U_t, \\ \hat{R} - R = U_x = \mathcal{L}. \end{cases} \tag{1.25}$$

Using (1.24), (1.25), one rewrites the second equation in (1.10) as

$$J_t - \mathcal{L}_x + P_x - \mu_{11} J - \mu_{12}\Theta + Q\mathcal{L} - \lambda = 0, \tag{1.26}$$

where $\lambda = \hat{R}\beta l[U]$. Differentiating (1.26) with respect to x yields

$$\begin{aligned} &\mathcal{L}_{tt} - \mathcal{L}_{xx} + P_{xx} + \tau_1 \mathcal{L}_t + \tau_2 P_t + \chi_1 \mathcal{L} + \chi_2 P \\ &\quad + \hat{Q}(\mathcal{L}_x - \mu_{12} J - \lambda) + \mathcal{F}_0 = 0. \end{aligned} \tag{1.27}$$

Here

$$\begin{aligned} &\tau_1 = \mu_{12} - \mu_{11}, \qquad \tau_2 = \frac{3}{2}\mu_{12}, \\ &\chi_1 = \hat{R}\beta + \hat{\varphi}'', \qquad \chi_2 = -c\mu_{12}, \\ &\mathcal{F}_0 = \frac{\lambda}{\hat{R}}\mathcal{L}_x - \beta\mathcal{L}^2 - \mu_{12}\frac{\lambda}{\hat{R}} J - f_1 f_2, \\ &f_1 = \mu_{11}' J + \mu_{12}'\Theta, \qquad f_2 = \frac{1}{R}\big(P_x + \sigma(\mathcal{L}_x - \hat{R}')\big), \\ &\mu_{11}' = \frac{d}{d\sigma}\mu_{11} = \frac{3}{2}\frac{d}{dE}\mu_{11}, \qquad \mu_{12}' = \frac{d}{d\sigma}\mu_{12} = \frac{3}{2}\frac{d}{dE}\mu_{12}. \end{aligned}$$

Differentiating cross-wise the two last equations in (1.10), we eliminate Θ from the left-hand parts and come to the relation

$$\begin{aligned} &\tilde{a} P_{tt} - \tilde{b} P_{xx} + \mathcal{L}_{xx} + \tau_3 \mathcal{L}_t + \tau_4 P_t + \chi_3 \mathcal{L} + \chi_4 P \\ &\quad + \frac{\hat{Q}}{\hat{a}}\left(-2\mathcal{L}_x + \frac{7}{2} P_x + \hat{a}\hat{d} J - \mu_{12}\Theta\right) + G_0 = 0, \end{aligned} \tag{1.28}$$

where

$$\begin{aligned} &G_0 = \frac{1}{\hat{a}}\left(-5Rf_2^2 + \frac{5}{2}\hat{R}''\sigma^2 - \mathcal{F}_0 - g_0\right), \\ &g_0 = \frac{\lambda}{\hat{R}}(\lambda - Q\mathcal{L} - \hat{Q}\mathcal{L} + \mu_{11} J + \mu_{22}\Theta - P_x + \mathcal{L}_x) + \beta l[U] J + c'\sigma(P_t + \sigma\mathcal{L}_t) \\ &\qquad - \frac{5}{2}\left(\frac{\lambda}{\hat{R}} P_x - \beta P\mathcal{L} + \mu_{22}\frac{\lambda}{\hat{R}} J + (\mu_{21}' J + \mu_{22}'\Theta) f_2\right), \\ &\tau_3 = \frac{1}{\hat{a}}\left(\frac{5}{2}(\mu_{21} - \mu_{22}) - \tau_1\right), \qquad \tau_4 = -\frac{1}{\hat{a}}\left(c + \frac{15}{4}\mu_{22} + \tau_2\right), \end{aligned}$$

$$\chi_3 = -\frac{\chi_1}{\hat a} + \frac{\hat Q^2}{\hat a}, \qquad \chi_4 = \frac{c}{\hat a}\left(\frac{5}{2}\mu_{22} + \mu_{12}\right) + \frac{5\hat\varphi''}{2\hat a},$$
$$\hat d = \frac{1}{\hat a}\left(\frac{5}{2}\mu_{22} + \tau_1\right), \qquad c' = \frac{d}{d\sigma}c = \frac{3}{2}\frac{d}{dE}c,$$
$$\mu'_{21} = \frac{d}{d\sigma}\mu_{21} = \frac{3}{2}\frac{d}{dE}\mu_{21}, \qquad \mu'_{22} = \frac{d}{d\sigma}\mu_{22} = \frac{3}{2}\frac{d}{dE}\mu_{22},$$
$$\tilde a = \frac{3}{2\hat a}, \quad \hat a = 1 - \frac{5}{2}\sigma^2, \quad \tilde b = \frac{7/2 + 5\sigma}{\hat a}.$$

Note that we use the third equation in (1.10) to exclude Θ_x while deriving (1.27), (1.28).

Combining (1.27), (1.28), we obtain the system

$$A\boldsymbol{L}_{tt} - B\boldsymbol{L}_{xx} + T\boldsymbol{L}_t + X\boldsymbol{L} + \hat Q(Y\boldsymbol{L}_x + Z\boldsymbol{\mathcal{N}} - \lambda\boldsymbol{\mathcal{M}}) + \boldsymbol{\Lambda}_0 = 0. \tag{1.29}$$

Here

$$\boldsymbol{L} = \begin{pmatrix} \mathcal{L} \\ P \end{pmatrix}, \quad \boldsymbol{\mathcal{N}} = \begin{pmatrix} J \\ \Theta \end{pmatrix}, \quad \boldsymbol{\mathcal{M}} = \begin{pmatrix} \frac{\tilde a}{\tilde b} \\ 1 \end{pmatrix},$$
$$A = \begin{pmatrix} \frac{\tilde b}{\tilde a} & 1 \\ 1 & \tilde a \end{pmatrix}, \qquad B = \begin{pmatrix} \frac{\tilde b - 1}{\tilde a} & 0 \\ 0 & \tilde b - 1 \end{pmatrix},$$
$$T = \begin{pmatrix} T_1 & T_2 \\ T_3 & T_4 \end{pmatrix} = \begin{pmatrix} T_1 & T_0 \\ T_0 & T_4 \end{pmatrix} + \begin{pmatrix} 0 & \tilde T_0 \\ -\tilde T_0 & 0 \end{pmatrix} = ST + CT,$$
$$X = \begin{pmatrix} X_1 & X_2 \\ X_3 & X_4 \end{pmatrix} = \begin{pmatrix} X_1 & X_0 \\ X_0 & X_4 \end{pmatrix} + \begin{pmatrix} 0 & \tilde X_0 \\ -\tilde X_0 & 0 \end{pmatrix} = SX + CX,$$
$$Y = \begin{pmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{pmatrix}, \quad Z = \begin{pmatrix} Z_1 & Z_2 \\ Z_3 & Z_4 \end{pmatrix}, \quad \boldsymbol{\Lambda}_0 = \begin{pmatrix} \frac{\tilde b\mathcal{F}_0 + G_0}{\tilde a} \\ \mathcal{F}_0 + G_0 \end{pmatrix},$$
$$T_1 = \frac{\tilde b\tau_1 + \tau_3}{\tilde a}, \quad T_2 = \frac{\tilde b\tau_2 + \tau_4}{\tilde a}, \quad T_3 = \tau_1 + \tau_3, \quad T_4 = \tau_2 + \tau_4,$$
$$T_0 = \frac{T_2 + T_3}{2}, \qquad \tilde T_0 = \frac{T_2 - T_3}{2},$$
$$X_1 = \frac{\tilde b\chi_1 + \chi_3}{\tilde a}, \quad X_2 = \frac{\tilde b\chi_2 + \chi_4}{\tilde a}, \quad X_3 = \chi_1 + \chi_3, \quad X_4 = \chi_2 + \chi_4,$$
$$X_0 = \frac{X_2 + X_3}{2}, \qquad \tilde X_0 = \frac{X_2 - X_3}{2},$$
$$Y_1 = \frac{\tilde b\hat a - 2}{\tilde a\hat a}, \quad Y_2 = \frac{7}{2\tilde a\hat a}, \quad Y_3 = \frac{\hat a - 2}{\hat a}, \quad Y_4 = \frac{7}{2\hat a},$$
$$Z_1 = \frac{\hat d - \tilde b\mu_{12}}{\tilde a}, \quad Z_2 = -\frac{\mu_{12}}{\hat a\tilde a}, \quad Z_3 = \hat d - \mu_{12}, \quad Z_4 = -\frac{\mu_{12}}{\tilde a}.$$

Finally, from (1.26) and the last equation in (1.10) we obtain the following equation:

$$\boldsymbol{\mathcal{N}}_t = \begin{pmatrix} 1 & -1 \\ 0 & -\frac{5}{2} \end{pmatrix}\boldsymbol{L}_x + \hat Q\,\mathrm{diag}\left(-1, \frac{5}{2}\right)\boldsymbol{L}$$
$$+ \begin{pmatrix} \mu_{11} & \mu_{12} \\ \frac{5}{2}\mu_{21} & \frac{5}{2}\mu_{22} \end{pmatrix}\boldsymbol{\mathcal{N}} + \lambda\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -\frac{\lambda}{\hat R}\mathcal{L} \\ \frac{5}{2}\left(\frac{P\lambda}{\hat R} - (R\sigma^2)_x\right) \end{pmatrix}. \tag{1.30}$$

The boundary conditions (1.11) in terms of $\boldsymbol{L}$ read as

$$\boldsymbol{L}(t,0)=\boldsymbol{L}(t,1)=0. \tag{1.31}$$

1.8. At the end of this section we observe also the following. Since we discuss the existence of a global and sufficiently smooth solution (classical solution) to the initial boundary value problem (1.10)–(1.12), the initial data are supposed to satisfy *compatibility conditions*. In other words, following conditions should hold for the initial data and their derivatives at the points $(0,0)$ and $(0,1)$ on the (t,x)-plane:

$$\begin{cases} P_0(0)=P_0(1)=0,\\ R_0(0)=R_0(1)=1,\\ J_0'(0)=J_0'(1)=0,\\ \Theta_0'(0)=J_0(0)Q_0(0),\\ \Theta_0'(1)=J_0(1)Q_0(1), \end{cases} \tag{1.32}$$

where $Q_0(0)$, $Q_0(1)$ are determined by formula (1.7).

1.9. We made quite a few efforts to construct system (1.29) since a trivial extension (simply by differentiating) of original system (1.10) is not enough to obtain a priori estimate (see the next section). The purpose of change of variables (1.23) is clear. The homogeneous boundary conditions (see (1.31)) are obtained by means of this change.

2. Construction of a global a priori estimate. Global existence theorem

2.1. As was mentioned in the previous section, the global existence theorem for the initial boundary value problem (1.10)–(1.12) follows from the global a priori estimate and the local existence theorem for this problem (see also papers [10,11,16]). Below briefly describe the technique of obtaining such estimate (see [14] for details).

Differentiating (1.29) with respect to t and taking into account (1.30), we obtain

$$\begin{aligned} &A\boldsymbol{D}_{tt}-B\boldsymbol{D}_{xx}+T\boldsymbol{D}_t+X\boldsymbol{D}\\ &\quad+\hat{Q}\big(Y\boldsymbol{D}_x+\hat{Z}\boldsymbol{L}_x+\hat{Q}\tilde{Z}\boldsymbol{L}+\Omega\boldsymbol{\mathcal{N}}-\hat{R}\beta l[J]\boldsymbol{\mathcal{M}}+\lambda\hat{\boldsymbol{\mathcal{M}}}\big)+\hat{\boldsymbol{\mathcal{K}}}=0. \end{aligned} \tag{2.1}$$

Here

$$\begin{aligned} &\boldsymbol{D}=\boldsymbol{L}_t,\\ &\hat{Z}=Z\begin{pmatrix}1 & -1\\ 0 & -\frac{5}{2}\end{pmatrix}=\begin{pmatrix}Z_1 & -Z_1-\frac{5}{2}Z_2\\ Z_3 & -Z_3-\frac{5}{2}Z_4\end{pmatrix},\\ &\tilde{Z}=Z\,\mathrm{diag}\Big(-1,\frac{5}{2}\Big)=\begin{pmatrix}-Z_1 & \frac{5}{2}Z_2\\ -Z_3 & \frac{5}{2}Z_4\end{pmatrix},\\ &\Omega=Z\begin{pmatrix}\mu_{11} & \mu_{12}\\ \frac{5}{2}\mu_{21} & \frac{5}{2}\mu_{22}\end{pmatrix}=\begin{pmatrix}Z_1\mu_{11}+Z_2\frac{5}{2}\mu_{21} & Z_1\mu_{12}+Z_2\frac{5}{2}\mu_{22}\\ Z_3\mu_{11}+Z_4\frac{5}{2}\mu_{21} & Z_3\mu_{12}+Z_4\frac{5}{2}\mu_{22}\end{pmatrix},\\ &\hat{\boldsymbol{\mathcal{M}}}=Z\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}Z_1\\Z_3\end{pmatrix}, \end{aligned}$$

$$\hat{\mathcal{K}} = \mathcal{K} + \hat{Q}Z\begin{pmatrix} -\frac{\lambda}{\hat{R}}\mathcal{L} \\ \frac{5}{2}\left(\frac{P\lambda}{\hat{R}} - (R\sigma^2)_x\right)\end{pmatrix},$$

$$\mathcal{K} = A_t\boldsymbol{D}_t - B_t\boldsymbol{L}_{xx} + T_t\boldsymbol{D} + X_t\boldsymbol{L} + \hat{Q}(Y_t\boldsymbol{L}_x + Z_t\boldsymbol{\mathcal{N}} - \lambda\boldsymbol{\mathcal{M}}_t) + (\boldsymbol{\Lambda}_0)_t.$$

Using system (1.29), we estimate $\boldsymbol{L}_{xx}$:

$$\boldsymbol{L}_{xx} = B^{-1}\big(A\boldsymbol{L}_{tt} + T\boldsymbol{D} + X\boldsymbol{L} + \hat{Q}(Y\boldsymbol{L}_x + Z\boldsymbol{\mathcal{N}} - \lambda\boldsymbol{\mathcal{M}}) + \boldsymbol{\Lambda}_0\big). \tag{1.29$'$}$$

We suppose that a smooth solution to problem (1.10)–(1.12) lies in a small neighborhood of the global thermodynamical equilibrium state (1.18). Then the inequalities

$$\tilde{a} > 0, \qquad \tilde{b} > 0,$$
$$\tilde{b} - 1 = \frac{5}{2\hat{a}}(1+\sigma)^2 > 0$$

are satisfied. Therefore, the matrices $A, B > 0$. So, there exists the matrix B^{-1} appearing in (1.29$'$).

2.2. Below we will need the following obvious relations (we recall that A, B are symmetric matrices)

$$2(\boldsymbol{D}_t, A\boldsymbol{D}_{tt}) = (\boldsymbol{D}_t, A\boldsymbol{D}_t)_t - (\boldsymbol{D}_t, A_t\boldsymbol{D}_t),$$
$$2(\boldsymbol{D}_t, B\boldsymbol{D}_{xx}) = 2(\boldsymbol{D}_t, B\boldsymbol{D}_x)_x - (\boldsymbol{D}_x, B\boldsymbol{D}_x)_t - 2(\boldsymbol{D}_t, B_x\boldsymbol{D}_x) + (\boldsymbol{D}_x, B_t\boldsymbol{D}_x),$$
$$(\boldsymbol{D}, A\boldsymbol{D}_{tt}) = (\boldsymbol{D}, A\boldsymbol{D}_t)_t - (\boldsymbol{D}_t, A\boldsymbol{D}_t) - (\boldsymbol{D}, A_t\boldsymbol{D}_t),$$
$$(\boldsymbol{D}, B\boldsymbol{D}_{xx}) = (\boldsymbol{D}, B\boldsymbol{D}_x)_x - (\boldsymbol{D}_x, B\boldsymbol{D}_x) - (\boldsymbol{D}, B_x\boldsymbol{D}_x)$$

and so on.

Multiplying (2.1) by $2\boldsymbol{D}_t$, one obtains

$$\begin{aligned}&\big\{(\boldsymbol{D}_t, A\boldsymbol{D}_t) + (\boldsymbol{D}_x, B\boldsymbol{D}_x) + (\boldsymbol{D}, SX\boldsymbol{D})\big\}_t - 2(\boldsymbol{D}_t, B\boldsymbol{D}_x)_x\\ &\quad + 2\big\{(\boldsymbol{D}_t, ST\boldsymbol{D}_t) + (\boldsymbol{D}_t, CX\boldsymbol{D}) + \hat{Q}^2(\boldsymbol{D}_t, \tilde{Z}\boldsymbol{L})\\ &\quad + \hat{Q}\big((\boldsymbol{D}_t, Y\boldsymbol{D}_x) + (\boldsymbol{D}_t, \hat{Z}\boldsymbol{L}_x) + (\boldsymbol{D}_t, \Omega\boldsymbol{\mathcal{N}}) - \hat{R}\beta l[J](\boldsymbol{D}_t, \boldsymbol{\mathcal{M}}) + \lambda(\boldsymbol{D}_t, \hat{\boldsymbol{\mathcal{M}}})\big)\big\}\\ &\quad + 2(\boldsymbol{D}_t, \hat{\mathcal{K}}) - (\boldsymbol{D}_t, A_t\boldsymbol{D}_t) - \big(\boldsymbol{D}, (SX)_t\boldsymbol{D}\big)\\ &\quad + 2(\boldsymbol{D}_t, B_x\boldsymbol{D}_x) - (\boldsymbol{D}_x, B_t\boldsymbol{D}_x) = 0.\end{aligned} \tag{2.2}$$

Multiplying the same system by $2\boldsymbol{D}$, we come to the expression

$$\begin{aligned}&\big\{2(\boldsymbol{D}, A\boldsymbol{D}_t) + (\boldsymbol{D}, ST\boldsymbol{D}) + \hat{Q}^2(\boldsymbol{L}, S\tilde{Z}\boldsymbol{L})\big\}_t - 2(\boldsymbol{D}, B\boldsymbol{D}_x)_x\\ &\quad + 2\big\{-(\boldsymbol{D}_t, A\boldsymbol{D}_t) + (\boldsymbol{D}_x, B\boldsymbol{D}_x) + (\boldsymbol{D}, SX\boldsymbol{D}) + (\boldsymbol{D}, CT\boldsymbol{D}_t)\\ &\quad + \hat{Q}\big((\boldsymbol{D}, Y\boldsymbol{D}_x) + (\boldsymbol{D}, \hat{Z}\boldsymbol{L}_x) + \hat{Q}(\boldsymbol{D}, C\tilde{Z}\boldsymbol{L}) + (\boldsymbol{D}, \Omega\boldsymbol{\mathcal{N}})\\ &\quad - \hat{R}\beta l[J](\boldsymbol{D}, \boldsymbol{\mathcal{M}}) + \lambda(\boldsymbol{D}, \hat{\boldsymbol{\mathcal{M}}})\big)\big\} + 2(\boldsymbol{D}, \hat{\mathcal{K}}) - \big(\boldsymbol{D}, (ST)_t\boldsymbol{D}\big)\\ &\quad - 2(\boldsymbol{D}, A_t\boldsymbol{D}_t) + 2(\boldsymbol{D}, B_x\boldsymbol{D}_x) - \hat{Q}^2\big(\boldsymbol{L}, (S\tilde{Z})_t\boldsymbol{L}\big) = 0.\end{aligned} \tag{2.3}$$

Analogously, multiplying (1.29) first by $2\boldsymbol{D}$ and then by $2\boldsymbol{L}$, one gets

$$\begin{aligned}
&\big\{(\boldsymbol{D}, A\boldsymbol{D}) + (\boldsymbol{L}_x, B\boldsymbol{L}_x) + (\boldsymbol{L}, SX\boldsymbol{L})\big\}_t - 2(\boldsymbol{D}, B\boldsymbol{L}_x)_x \\
&\quad + 2\big\{(\boldsymbol{D}, ST\boldsymbol{D}) + (\boldsymbol{D}, CX\boldsymbol{L}) + \hat{Q}\big((\boldsymbol{D}, Y\boldsymbol{L}_x) + (\boldsymbol{D}, Z\boldsymbol{\mathcal{N}}) - \lambda(\boldsymbol{D}, \boldsymbol{\mathcal{M}})\big)\big\} \\
&\quad + 2(\boldsymbol{D}, \boldsymbol{\Lambda}_0) - (\boldsymbol{D}, A_t\boldsymbol{D}) - \big(\boldsymbol{L}, (SX)_t\boldsymbol{L}\big) + 2(\boldsymbol{D}, B_x\boldsymbol{L}_x) - (\boldsymbol{L}_x, B_t\boldsymbol{L}_x) = 0, \qquad (2.4)
\end{aligned}$$

$$\begin{aligned}
&\big\{2(\boldsymbol{L}, A\boldsymbol{D}) + (\boldsymbol{L}, ST\boldsymbol{L})\big\}_t - 2(\boldsymbol{L}, B\boldsymbol{L}_x)_x \\
&\quad + 2\big\{-(\boldsymbol{D}, A\boldsymbol{D}) + (\boldsymbol{L}_x, B\boldsymbol{L}_x) + (\boldsymbol{L}, SX\boldsymbol{L}) + (\boldsymbol{L}, CT\boldsymbol{D}) \\
&\quad + \hat{Q}\big((\boldsymbol{L}, Y\boldsymbol{L}_x) + (\boldsymbol{L}, Z\boldsymbol{\mathcal{N}}) - \lambda(\boldsymbol{L}, \boldsymbol{\mathcal{M}})\big)\big\} \\
&\quad + 2(\boldsymbol{L}, \boldsymbol{\Lambda}_0) - \big(\boldsymbol{L}, (ST)_t\boldsymbol{L}\big) - 2(\boldsymbol{L}, A_t\boldsymbol{D}) + 2(\boldsymbol{L}, B_x\boldsymbol{L}_x) = 0. \qquad (2.5)
\end{aligned}$$

At last, multiplying (1.26) by $2J$, the third equation in (1.10) by $2P$, and the fourth equation in (1.10) by 2Θ, and summing up the obtained expressions yields

$$\begin{aligned}
&\left\{J^2 + \mathcal{L}^2 + \frac{2}{5}\Theta^2 + \frac{3}{2}P^2\right\}_t - 2(J\mathcal{L} - JP - P\Theta)_x \\
&\quad + 2\big\{-\mu_{11}J^2 - \tilde{\mu}_{12}J\Theta - \mu_{22}\Theta^2 - cP^2 - \lambda J + \hat{Q}(J\mathcal{L} - JP - P\Theta)\big\} \\
&\quad + 2\frac{\lambda}{\hat{R}}(J\mathcal{L} - JP - P\Theta) + 2\Theta(\sigma P_x + Pf_2) = 0, \qquad (2.6)
\end{aligned}$$

where $\tilde{\mu}_{12} = \mu_{12} + \mu_{21}$.

2.3. Now, taking into account the boundary conditions (1.31), we integrate (2.2)–(2.6) with respect to x from 0 to 1, multiply them by positive arbitrary constants α_1, α_2, α_3, α_4, α_5, sum up the result, and finally come to

$$\frac{d}{dt}\Phi^{(0)} + \Phi^{(1)} = \Pi. \qquad (2.7)$$

Here

$$\begin{aligned}
\Phi^{(0)} &= \Phi^{(0)}(t) \\
&= \int_0^1 \left\{(\boldsymbol{L}_p, \mathcal{A}_0\boldsymbol{L}_p) + \alpha_1(\boldsymbol{D}_x, B\boldsymbol{D}_x) + \alpha_3(\boldsymbol{L}_x, B\boldsymbol{L}_x) + \alpha_5\left(J^2 + \frac{2}{5}\Theta^2\right)\right\} dx,
\end{aligned}$$

$$\begin{aligned}
&\boldsymbol{L}_p = \begin{pmatrix} \boldsymbol{L}_{tt} \\ \boldsymbol{L}_t \\ \boldsymbol{L} \end{pmatrix}, \qquad \mathcal{A}_0 = \begin{pmatrix} \alpha_1 A & \alpha_2 A & 0 \\ \alpha_2 A & \mathcal{R} & \alpha_4 A \\ 0 & \alpha_4 A & \mathcal{R}_1 \end{pmatrix}, \\
&\mathcal{R} = \alpha_1 SX + \alpha_3 A + \alpha_2 ST, \\
&\mathcal{R}_1 = \alpha_2\hat{Q}^2 S\tilde{Z} + \alpha_3 SX + \alpha_4 ST + \alpha_5 A_1, \qquad A_1 = \operatorname{diag}\left(1, \frac{3}{2}\right),
\end{aligned}$$

$$\Phi^{(1)} = \Phi^{(1)}(t) = 2\int_0^1 \big\{(\boldsymbol{L}_p, \mathcal{A}_1\boldsymbol{L}_p) + \alpha_2(\boldsymbol{D}_x, B\boldsymbol{D}_x) + \alpha_4(\boldsymbol{L}_x, B\boldsymbol{L}_x) + \alpha_5(-\mu_{11}J^2 - \tilde{\mu}_{12}J\Theta - \mu_{22}\Theta^2 - \lambda J)\big\}\,dx,$$

$$\mathcal{A}_1 = \begin{pmatrix} \alpha_1 ST - \alpha_2 A & \alpha_1 CX & 0 \\ \alpha_2 CT & \alpha_2 SX + \alpha_3 ST - \alpha_4 A & \alpha_3 CX \\ 0 & \alpha_4 CT & \alpha_4 SX + \alpha_5 \mathrm{diag}(0, -c) \end{pmatrix}.$$

The expression for $\Phi^{(1)}$ is given here only for the case $\rho \equiv 1$ (i.e., $\hat{\varphi} \equiv 0$, see Section 1) because of its unhandiness in the general case. The term Π is not given here by the same reason but it can be easily written down (see formulas (2.2)–(2.6)).

We will suppose in the sequel that the initial boundary value problem (1.10)–(1.12) has a smooth (classical) local solution on some interval $[0, t_*]$, where t_* is a sufficiently small positive number. We introduce the constant

$$M_* = \max\Big\{\max_{t\in[0,t_*]} \|\boldsymbol{L}(t)\|_{C[0,1]}, \max_{t\in[0,t_*]} \|\boldsymbol{L}_x(t)\|_{C[0,1]}, \max_{t\in[0,t_*]} \|\boldsymbol{L}_t(t)\|_{C[0,1]}\Big\}.$$

We assume that the constant M_* is sufficiently small and local solution slightly differs from the equilibrium state (1.18) (at least this concerns the functions $R(t,x)$ and $P(t,x)$). Likewise, suppose that the doping density $\rho(x)$ is slightly differs from 1. Therefore, it is enough to show that the integrand appearing in the expression for $\Phi^{(0)}$ is a positive definite quadratic form only at the equilibrium state. Indeed, since the constants M_* and $\max_{x\in[0,1]}|\rho(x)-1| = 1-\delta$ are small, this property remains valid in a neighborhood of the equilibrium state for doping densities slightly differing from 1. The positive definiteness of the quadratic forms being the integrands for the integrals $\Phi^{(0)}$ and $\Phi^{(2)}$ (see below) is shown in [14].

We rewrite (2.7) as

$$\frac{d}{dt}\Phi^{(0)} + \Phi^{(2)} \leqslant |\Pi|, \tag{2.8}$$

where

$$\Phi^{(2)} = \Phi^{(2)}(t) = 2\int_0^1 \bigg\{(\boldsymbol{L}_p, \mathcal{A}_1\boldsymbol{L}_p) + \alpha_2(\boldsymbol{D}_x, B\boldsymbol{D}_x) + \alpha_4(\boldsymbol{L}_x, B\boldsymbol{L}_x) + \alpha_5\bigg(-\mu_{11}J^2 - \tilde{\mu}_{12}J\Theta - \mu_{22}\Theta^2 - \frac{\beta\hat{\varepsilon}}{2}J^2 - \frac{\beta}{4\hat{\varepsilon}}\mathcal{L}_x^2\bigg)\bigg\}\,dx,$$

$\hat{\varepsilon} > 0$ is a constant.

Relation (2.8) is obtained as follows. First, we estimate the integral $\Phi^{(1)}$ (recall that we suppose $\rho(x) \equiv 1$):

$$\Phi^{(1)} \geqslant 2\int_0^1 \big\{(\boldsymbol{L}_p, \mathcal{A}_1\boldsymbol{L}_p) + \alpha_2(\boldsymbol{D}_x, B\boldsymbol{D}_x) + \alpha_4(\boldsymbol{L}_x, B\boldsymbol{L}_x) + \alpha_5(-\mu_{11}J^2 - \tilde{\mu}_{12}J\Theta - \mu_{22}\Theta^2)\big\}\,dx - 2\alpha_5\bigg|\int_0^1 \hat{R}\beta l[U]J\,dx\bigg|.$$

It follows from

$$l[U]=\int_0^1 U(t,s)\,ds-U(t,x)=\int_0^1\left(\int_x^s U_z(t,z)\,dz\right)ds=\int_0^1\left(\int_x^s \mathcal{L}(t,z)\,dz\right)ds$$

that

$$\bigl|l[U]\bigr|\leqslant\int_0^1\bigl|\mathcal{L}(t,s)\bigr|\,ds\leqslant\left(\int_0^1\mathcal{L}^2(t,s)\,ds\right)^{1/2}.$$

So, we have

$$\left|2\int_0^1\hat{R}\beta l[U]J\,dx\right|\leqslant 2\beta\int_0^1\bigl|l[U]\bigr||J|\,dx\leqslant\beta\left(\hat{\varepsilon}\int_0^1 J^2\,dx+\frac{1}{2\hat{\varepsilon}}\int_0^1\mathcal{L}_x^2\,dx\right). \tag{2.9}$$

The Cauchy inequality with $\hat{\varepsilon}>0$ as well as the Poincare inequality (see [33])

$$\int_0^1\mathcal{L}^2\,ds\leqslant\frac{1}{2}\int_0^1\mathcal{L}_x^2\,dx$$

were used while deriving (2.9). From (2.9) we deduce the inequality

$$\Phi^{(1)}\geqslant\Phi^{(2)}$$

that yields (2.8).

Since the quadratic forms being the integrands for the integrals $\Phi^{(0)}$ and $\Phi^{(2)}$ are positive definite, we have

$$\Phi^{(2)}\geqslant M_1\Phi^{(0)}, \tag{2.10}$$

where $M_1>0$ is some constant (which is finally determined through the constant M_*).

Using well-known Sobolev embedding theorems (see [23,33,40]), we estimate the right-hand side in (2.8) as follows:

$$|\Pi|\leqslant M_2\bigl(\Phi^{(0)}\bigr)^{3/2}. \tag{2.11}$$

Here $M_2>0$ is a constant finally determined through M_*. The embedding theorems mentioned above are formulated as follows:

$$\bigl\|\boldsymbol{L}(t)\bigr\|_{C^1[0,1]}\leqslant M_b\bigl\|\boldsymbol{L}(t)\bigr\|_{W_2^2(0,1)}\leqslant M_bM_3\bigl(\Phi^{(0)}(t)\bigr)^{1/2},$$
$$\bigl\|\boldsymbol{\mathcal{N}}(t)\bigr\|_{C^1[0,1]}\leqslant M_b\bigl\|\boldsymbol{\mathcal{N}}(t)\bigr\|_{W_2^2(0,1)}\leqslant M_bM_4\bigl(\Phi^{(0)}(t)\bigr)^{1/2}.$$

Here M_b is the embedding constant (see [40]), $M_3, M_4>0$ are constants determined by M_*, $W_2^2(0,1)$ is the Sobolev space (see [23]).

2.4. Using now the known technique based on Sobolev embedding theorems, from (2.8), (2.10), (2.11) we derive the inequality

$$\frac{d}{dt}\Phi^{(0)} + M_1\Phi^{(0)} \leqslant M_2(\Phi^{(0)})^{3/2}. \tag{2.12}$$

The function

$$f(\Phi^{(0)}) = -M_1\Phi^{(0)} + M_2(\Phi^{(0)})^{3/2}$$

is negative if

$$0 < \Phi^{(0)} < \left(\frac{M_1}{M_2}\right)^2.$$

So, (2.12) yields the following a priori estimate (that is yet local)

$$\Phi^{(0)}(t) \leqslant e^{-\nu t}\Phi^{(0)}(0), \quad 0 < t \leqslant t_*, \tag{2.13}$$

where ν is a constant, $0 < \nu < M_1$. This proves the following local existence theorem for the initial boundary value problem (1.10)–(1.12).

Theorem 2.1. *Suppose the initial data* $\boldsymbol{V}(0,x) = \boldsymbol{V}_0(x)$ *belong to* $W_2^2(0,1)$. *Suppose also the compatibility conditions* (1.32) *are satisfied. Then there is a number* $t_* > 0$ *such that in the domain* $0 < t \leqslant t_*$, $0 < x < 1$ *there exists a unique solution* $\boldsymbol{V}(t,x)$ *of problem* (1.10)–(1.12) *belonging to* $C^1([0,t_*]\times[0,1])$. *Moreover, in view of the a priori estimate* (2.13),

$$\boldsymbol{V}(t,x) \in W_2^2(0,1), \quad 0 < t \leqslant t_*.$$

We note once again that one can use the results of paper [24] (see also paper [9]) to prove the existence of a classical solution to problem (1.10)–(1.12) locally in time.

2.5. Let us now discuss how the a priori estimate (2.13) can be used to extend the local solution of problem (1.10)–(1.12) to the whole time interval. Actually, if the initial data $\boldsymbol{V}_0(x)$ satisfy

$$0 < \Phi^{(0)}(0) < \left(\frac{M_1}{M_2}\right)^2, \tag{2.14}$$

then it follows from (2.13) that

$$\Phi^{(0)}(t_*) \leqslant \Phi^{(0)}(0).$$

Since

$$\|\boldsymbol{L}(t_*)\|_{C^1[0,1]} \leqslant M_b M_3\big(\Phi^{(0)}(t_*)\big)^{1/2},$$
$$\|\boldsymbol{L}_t(t_*)\|_{C[0,1]} \leqslant M_5\big(\Phi^{(0)}(t_*)\big)^{1/2},$$

where $M_5 > 0$ is a constant determined through M_*, then we can select the initial data (i.e., $\Phi^{(0)}(0)$) such that $\boldsymbol{V}(t_*)$ lies in the same small neighborhood of the equilibrium state (1.18) (characterized by the same constant M_*). Hence, we can take $\boldsymbol{V}(t_*)$ as initial data to construct a classical solution to problem (1.10)–(1.12) for $t_* < t \leqslant 2t_*$ so that estimate (2.13) is fulfilled:

$$\Phi^{(0)}(t) \leqslant e^{-\nu(t-t_*)}\Phi^{(0)}(t_*) \leqslant e^{-\nu t}\Phi^{(0)}(0).$$

Thus, estimate (2.13) is global. This implies the global existence theorem for a classical solution to the initial boundary value problem (1.10)–(1.12).

Theorem 2.2. *Suppose the conditions of Theorem* 2.1 *are fulfilled. Suppose also the initial data satisfy* (2.14). *Then for any t, $0 < t \leqslant t_1 < \infty$ (t_1 is arbitrary), there exists a unique smooth solution to problem* (1.10)–(1.12):

$$\begin{aligned}
&\boldsymbol{\mathcal{N}}(t,x) \in W_2^2(0,1),\\
&\boldsymbol{L}(t,x) \in W_2^2(0,1) \cap \mathring{W}_2^1(0,1),\\
&R(t,x) = \hat{R}(x) - \mathcal{L}(t,x) \in W_2^2(0,1),\\
&\varphi(t,x) \in W_2^4(0,1) \cap \mathring{W}_2^1(0,1);
\end{aligned}$$

and the estimate

$$\Phi^{(0)}(t) \leqslant e^{-\nu t}\Phi^{(0)}(0)$$

holds for this solution.

2.6. Let us prove that

$$\varphi(t,x) \in W_2^4(0,1) \cap \mathring{W}_2^1(0,1).$$

Indeed, in view of (1.24), (1.17), one obtains

$$\varphi(t,x) = \beta\int_0^x l[U]\,d\zeta + \hat{\varphi}(x) = \beta\int_0^x\int_0^1\int_\zeta^s \mathcal{L}(t,z)\,dz\,ds\,d\zeta + \hat{\varphi}(x). \tag{2.15}$$

Consequently,

$$\begin{aligned}
&Q = \varphi_x(t,x) = \beta\int_0^1\int_x^s \mathcal{L}(t,z)\,dz\,ds + \hat{Q},\\
&\varphi_{xx}(t,x) = -\beta\mathcal{L} + \hat{\varphi}'',\\
&\varphi_{xxx}(t,x) = -\beta\mathcal{L}_x + \hat{\varphi}''',\\
&\varphi_{xxxx}(t,x) = -\beta\mathcal{L}_{xx} + \hat{\varphi}^{(\mathrm{IV})}.
\end{aligned}$$

This yields the above property of φ.

2.7. Since, by virtue of Theorem 2.2,

$$\boldsymbol{L}(t,x), \boldsymbol{\mathcal{N}}(t,x) \in W_2^2(0,1), \quad t \in (0,\infty),$$

then $\Phi^{(0)}(t) \to 0$ as $t \to \infty$, i.e., $\boldsymbol{L}(t,x), \boldsymbol{\mathcal{N}}(t,x) \to 0$ as $t \to \infty$ in the $W_2^2(0,1)$-norm. At the same time, $R(t,x) = (\hat{R}(x) - \mathcal{L}(t,x)) \to \hat{R}$ as $t \to \infty$ in $W_2^2(0,1)$. In view of (2.15), $\varphi(t,x) \to \hat{\varphi}(x)$ as $t \to \infty$ in the $W_2^4(0,1)$-norm. So, we come to the following theorem.

Theorem 2.3. *If the initial data for problem* (1.10)–(1.12) *slightly differ from the equilibrium state* (1.18) *and the doping density* $\rho(x)$ *slightly differs from* 1, *then this equilibrium state is asymptotically stable* (*by Lyapunov*).

3. Conclusions

The construction of a global a priori estimate is proposed in this paper as a basis to prove the global existence theorem (see also [14]). This estimate is such that the condition of smallness of the constant $\max_{x\in[0,1]} |\rho(x) - 1| = 1 - \delta$ can be apparently eliminated. However the smallness of the initial data

$$\boldsymbol{L}(0,x) = \boldsymbol{L}_0(x), \qquad \boldsymbol{\mathcal{N}}(0,x) = \boldsymbol{\mathcal{N}}_0(x)$$

seems to be essential for the technique described in the paper. Obviously, to avoid this assumption it needs to weaken the requirements on the solution to (1.10)–(1.12), i.e., to consider weak solutions to this problem.

References

[1] N.B. Abdallah, P. Degond, On a hierarchy of macroscopic models for semiconductors, J. Math. Phys. 37 (1996) 3308–3333.
[2] G. Alì, A.M. Anile, Moment equations for charged particles: global existence results, Preprint, 2003.
[3] G. Alì, D. Bini, S. Rionero, Global existence and relation limit for smooth solutions to the Euler–Poisson model for semiconductors, SIAM J. Math. Anal. 32 (2002) 572–587.
[4] A.M. Anile, M. Junk, V. Romano, G. Russo, Cross-validation of numerical schemes for extended hydrodynamical models of semiconductors, Math. Models Methods Appl. Sci. 10 (2000) 833–861.
[5] A.M. Anile, G. Mascali, V. Romano, Lecture Notes in Mathematics, Springer, 2003.
[6] A.M. Anile, V. Romano, Non parabolic band transport in semiconductors: closure of the moment equations, Contin. Mech. Thermodyn. 11 (1999) 307–325.
[7] A.M. Anile, V. Romano, Hydrodynamical modeling of charge carrier transport in semiconductors, MECCANICA 35 (2000) 249–296.
[8] A.M. Anile, V. Romano, G. Russo, Extended hydrodynamical model of carrier transport in semiconductors, SIAM J. Appl. Math. 61 (2000) 74–101.
[9] A.M. Blokhin, Well posedness of a mixed problem for the nonstationary model of a flow around an infinite cone, Numer. Methods Mech. Contin. Medium 10 (1979) 10–25.
[10] A.M. Blokhin, A.D. Birkin, Global resolving of a problem on supersonic flow around a cone, Math. Modelling 8 (1996) 89–104.
[11] A.M. Blokhin, A.D. Birkin, Global resolving of a problem about piston, Siberian J. Industrial Math. 2 (1999) 13–24.
[12] A.M. Blokhin, A.S. Bushmanova, E.V. Mishchenko, On solution of a nonlinear boundary value problem for a singularly perturbed differential equation, Comput. Tech. 6 (1999) 27–57.
[13] A.M. Blokhin, R.S. Bushmanov, V. Romano, Asymptotic stability of the equilibrium state for the hydrodynamical model of charge transport in semiconductors based on the maximum entropy principle, Internat. J. Engrg. Sci. 42 (2004) 915–934.
[14] A.M. Blokhin, R.S. Bushmanov, V. Romano, Nonlinear asymptotic stability of the equilibrium state for the MEP model of charge transport in semiconductors, Math. Methods Appl. Sci., submitted for publication.
[15] A.M. Blokhin, A.A. Iordanidy, Numerical investigation of a gas dynamical model for charge transport in semiconductors, COMPEL 18 (1999) 6–37.

[16] A.M. Blokhin, Yu.L. Trakhinin, Symmetrization of Radiation hydrodynamics equations and global resolving of Cauchy problem, Siberian Math. J. 37 (1996) 1101–1109.
[17] D. Chen, E.C. Kan, U. Ravaioli, C.-W. Shu, R. Dutton, An improved energy-transport model including nonparabolicity and non-Maxwellian distribution effects, IEEE Electron Device Lett. 13 (1992) 26–28.
[18] C.L. Gardner, Numerical simulation of a steady-state electron shock wave in a submicrometer semiconductor device, IEEE Trans. Electron Devices 38 (1991) 392–398.
[19] C.L. Gardner, J.W. Jerome, D.J. Rose, Numerical methods for the hydrodynamic device model: subsonic flow, IEEE Trans. Computeraided Design 8 (1989) 501–507.
[20] S.K. Godunov, Equations of Mathematical Physics, Nauka, Moscow, 1979 (in Russian).
[21] W. Hänsch, The Drift-Diffusion Equation and Its Applications in MOSFET Modeling, Springer-Verlag, Wien, 1991.
[22] D. Jou, J. Casas-Vazquez, G. Lebon, Extended Irreversible Thermodynamics, Springer-Verlag, Berlin, 1993.
[23] O.A. Ladyzenskaja, Mathematical Aspects of Dynamics of Viscous Noncompressible Liquid, Nauka, Moscow, 1970 (in Russian).
[24] D.-t. Lee, W.-t. Yu, Some existence theorems for quasi-linear hyperbolic systems of partial differential equations in two dependent variables, Scientia Sinica 13 (1964) 529–562.
[25] C.D. Levermore, Moment closure hierarchies for kinetic theories, J. Stat. Phys. 83 (1996) 331–407.
[26] H. Li, P. Markowich, M. Mei, Asymptotic behaviour of solutions of the hydrodynamical model of semiconductors, Proc. Roy. Soc. Edinburgh Sect. A 132 (2002) 359–378.
[27] H. Ling, W. Shu, The asymptotic behaviour of global smooth solutions to the macroscopic models of semiconductors, Chinese Ann. Math. Ser. B 22 (2001) 195–210.
[28] H. Ling, W. Shu, Asymptotic behaviour of global smooth solutions to the full 1D hydrodynamical model for semiconductors, Math. Models Methods Appl. Sci. 12 (2002) 777–796.
[29] T. Luo, R. Natalini, Z.P. Xin, Large-time behaviour of the solutions to a hydrodynamical model for semiconductors, SIAM J. Appl. Math. 59 (1998) 810–830.
[30] E. Lyumkis, B. Polsky, A. Shir, P. Visocky, Transient semiconductor device simulation including energy balance equation, COMPEL 11 (1992) 311–325.
[31] P.A. Marcati, R. Natalini, Weak solutions to hydrodynamical model for semiconductors and relaxation to the drift-diffusion equation, Arch. Rational Mech. Anal. 129 (1995) 129–145.
[32] P. Markowich, C.A. Ringhofer, C. Schmeiser, Semiconductor Equations, Springer-Verlag, Wien, 1990.
[33] S. Mizohata, The Theory of Partial Differential Equations, Cambridge Univ. Press, Cambridge, 1973.
[34] I. Müller, T. Ruggeri, Rational Extended Thermodynamics, Springer-Verlag, Berlin, 1998.
[35] V. Romano, Non-parabolic band transport in semiconductors: closure of the production terms in the moment equations, Contin. Mech. Thermodyn. 12 (2000) 31–51.
[36] V. Romano, Non-parabolic band hydrodynamical model of silicon semiconductors and simulation of electron devices, Math. Methods Appl. Sci. 24 (2001) 439–471.
[37] V. Romano, 2D simulation of a silicon MESFET with a non-parabolic hydrodynamical model based on the maximum entropy principle, J. Comput. Phys. 176 (2002) 70–92.
[38] V. Romano, G. Russo, Numerical solution for hydrodynamical models of semiconductors, Math. Models Methods Appl. Sci. 10 (2000) 1099–1120.
[39] S. Selberherr, Analysis and Simulation of Semiconductor Devices, Springer-Verlag, Wien, 1984.
[40] S.L. Sobolev, Some Application of Functional Analysis in Mathematical Physics, American Mathematical Society, 1991.