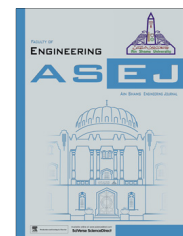




Ain Shams University
Ain Shams Engineering Journal

www.elsevier.com/locate/asej
www.sciencedirect.com



MECHANICAL ENGINEERING

Numerical investigation of inward solidification inside spherical capsule by using temperature transforming method

Mustafa Asker^{a,*}, Hadi Ganjehsarabi^b, M. Turhan Coban^c

^a Department of Mechanical Engineering, Adnan Menderes University, Central Campus, Aytepe Mevkii, 09010 Aydın, Turkey

^b Department of Mechanical Engineering, Erzincan University, 24100 Erzincan, Turkey

^c Department of Mechanical Engineering, Ege University, 35100 Bornova, Izmir, Turkey

Received 9 March 2015; revised 18 January 2016; accepted 25 February 2016

KEYWORDS

Entropy generation;
Phase change material;
Solidification;
Spherical capsules

Abstract In this study, a numerical analysis of the inward solidification of phase change material inside spherical capsule is carried out. The spherical capsule that is subjected to convection from the outside surface initially is not at its melting temperature. The control volume approach and temperature transforming method are applied to solve the dimensionless energy equation. The model solution results are validated through a comparison with published experimental data for similar case and it shows a considerably good agreement. The analysis results show that the larger diameter spherical capsules have significantly greater solidification time compared to those with smaller diameters of spherical capsules. In addition, the entropy generation inside spherical capsule is analyzed. It is found that the entropy generation increases with increasing sphere capsule diameter, attains a maximum and then decreases.

© 2016 Faculty of Engineering, Ain Shams University. Production and hosting by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

1. Introduction

During the last decade, thermal energy storage (TES) systems and applications have developed and been extended to industrial use in many countries.

* Corresponding author. Tel.: +90 256 213 7503/3658; fax: +90 256 213 66 86.

E-mail address: mustafa.asker@adu.edu.tr (M. Asker).

Peer review under responsibility of Ain Shams University.



Production and hosting by Elsevier

Latent heat thermal energy storage system is considered as one of the suitable methods of storing thermal energy because of their advantages in providing high storage density and has the capacity to store energy with small temperature difference [1]. Phase change material (PCM) encapsulation can be enhanced in various ways using cylindrical geometries or spherical capsules. The spherical capsules offer advantages over cylindrical capsules, including high energy storage capacity per unit of volume and easy packing into the storage tank.

Several detailed reviews of the phase-change materials and their various applications in thermal energy storage systems have been published by many researchers [2–6].

The investigation of the thermal behavior of PCMs in spherical capsules plays a key role in designing latent heat

<http://dx.doi.org/10.1016/j.asej.2016.02.009>

2090-4479 © 2016 Faculty of Engineering, Ain Shams University. Production and hosting by Elsevier B.V.

This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Please cite this article in press as: Asker M et al., Numerical investigation of inward solidification inside spherical capsule by using temperature transforming method, Ain Shams Eng J (2016), <http://dx.doi.org/10.1016/j.asej.2016.02.009>

Nomenclature

Bi	Biot number, $Bi = \frac{hr_0}{k_l}$
c	specific heat, J/kg K
C	dimensionless specific heat, $C = \frac{(\rho c)}{\rho_L c_L}$
C_{sl}	dimensionless specific heat, $C_{SL} = \frac{(\rho_S c_L)}{\rho_L c_L}$
D	diameter of the spherical capsule, m
Fo	Fourier number, $Fo = \frac{\alpha t}{r_0^2}$
g	gravitational acceleration, m/s ²
h	convective heat transfer coefficient, W/m ² K
H	enthalpy
k	thermal conductivity, W/m K
K	dimensionless thermal conductivity, $K = \frac{k}{k_L}$
L	latent heat of fusion, J/kg
Nu	Nusselt number, $Nu = \frac{hD}{k_{cf}}$
Pr	Prandtl number
r	radial position, m
r_o	radius of the spherical capsule, m
R	dimensionless radial position, $R = \frac{r}{r_o}$
Ra	Rayleigh number, $Ra_D = \frac{g\beta(T_s - T_{cf})D^3}{\alpha\nu}$
S	source term, $S = \frac{s^*}{\rho_L c_L (T_m - T_{cf})}$
\dot{S}_{gen}'''	entropy generation per unit volume, W/m ³ K
Ste	Stefan number, $Ste = \frac{c_L (T_m - T_{cf})}{L}$
T	temperature, K
T_i	initial temperature, K
T_m	phase change temperature, K
T_s	surface temperature of the spherical container, K
T_{cf}	coolant fluid temperature, K

Greek symbols

α	thermal diffusivity, m ² /s
β	coefficient of thermal expansion, 1/K
Δr	radial distance between grid points, m
ΔR	dimensionless radial distance between grid points
Δt	time step, s
μ	dynamic viscosity, kg/ms
θ	dimensionless temperature, $\theta = \frac{T - T_m}{T_m - T_{cf}}$
$\delta\theta_m$	dimensionless phase-change temperature range
ρ	density, kg/m ³

Subscripts

cf	coolant fluid
e	control volume face between P and E
E	neighbor grid point on the east side
k	time level
L	liquid phase
P	central grid point
s	solid phase, Interface
w	control volume face between P and W
W	neighbor grid point on the west side
wf	working fluid

Superscript

o	previous time step
-----	--------------------

storage systems [7]. Ismail and Moraes [8] investigated numerically and experimentally the effects of varying certain operating parameters on the time for complete solidification of the various phase-change materials encapsulated in spherical and cylindrical shells. Ismail and Henriquez [9] developed a simplified transient one-dimensional model to simulate a storage system consisted of spherical capsules filled with PCM placed inside a cylindrical tank fitted with a working fluid circulation system to charge and discharge the storage tank.

Adine and Qarnia [10] presented a numerical study of a latent heat storage unit consisting of a shell-and-tube. The shell space is filled with two phase change materials (PCMs), P116 and n-octadecane, with different melting temperatures. They also investigated the effects of various design parameters such as heat transfer fluid inlet temperature, mass flow rate and lengths of the PCMs sections on the thermal performances of the latent heat storage unit.

Lamberg and Siren [11] developed a simplified analytical model which predicts the solid-liquid interface location and temperature distribution of the fin in the solidification process with a constant end-wall temperature in the finned two dimensional PCM storage.

Tao [12] conducted a numerical method for solidification of a saturated liquid in cylinders and spheres. He assumed that the thermal conductivity, heat capacity of solid phase and convective heat transfer coefficient are constant.

Yang and Zhao [13] developed a feasible technique that combines the numerical calculation and the transformation of dimensionless variables in heat diffusion equations in order to analyze the solidification of PCM within a single encapsulated particle. Their results show that the smaller microcapsule can realize the rapid solidification under the constant temperature boundary condition. Pedroso and Domoto [14] studied the total solidification time in spherical coordinate by using perturbation method. They assumed that thermophysical properties of phase change material and the surface of the sphere are constant.

Riley et al. [15] studied analytically the inward solidification of a sphere and a circular cylinder for which is initially at melting temperature. They assumed that the thermal properties are constant. Their analysis was made for small Stefan number.

Hill and Kucera [16] presented a semi analytical procedure for the problem of freezing a saturated liquid inside spherical container by taking into consideration the effect of radiation at the container surface.

Prud'homme et al. [17] applied strained coordinate method to analyze the inward solidification of slabs, cylinders and spheres for three different types of boundary conditions. They assumed constant thermal properties for the liquid which is initially at the fusion temperature.

Ismail and Henriquez [18] presented numerical solution based upon finite difference approximations for the

solidification of phase change material in a spherical container under constant external wall temperature or convection boundary condition. They carried out a parametric study of the effects of the diameter of sphere, wall thickness, wall material, external temperature and the initial phase change material temperature on the solidified mass fraction and the time for the complete solidification. In another work, Ismail et al. [19] studied numerically the process of solidification of water inside spherical capsule. They solved their model by using finite difference approach and moving grid scheme and compared their results with an earlier experimental work results. Furthermore, they performed a parametric study of the effects of the material of the shell, initial temperature of the phase change material and the external temperature of the spherical capsule on the solidified mass fraction and the time for the complete solidification.

Eames and Adref [20] investigated experimentally freezing and melting processes for water contained in spherical elements of the type used in thermal (ice) storage systems. Bilir and Ilken [21] reported the result of a numerical study on the inward solidification of a liquid phase change material encapsulated in cylindrical and spherical container with a third kind of boundary condition. They used enthalpy method with control volume approach. They derived correlations that express the dimensionless total solidification time of the PCM in terms of Stefan Number, Biot Number and Superheat Parameter.

Chan and Tan [22] studied the solidification of n-hexadecane inside a spherical enclosure. They showed that the solidification phase front propagates uniformly inward toward the center of the sphere.

Veerappan et al. [23] developed an analytical model based on quasi steady approximations for solidification and melting of PCMs in a spherical container where the sphere is initially at its melting temperature. The external surface of the spherical shell is assumed to be at isothermal boundary conditions. However, the resistance of the outer shell is neglected. They investigated the effect of design parameters such as size of the spherical capsule, external fluid temperature and the initial PCM temperature on the solidified mass fraction and the time for complete solidification.

In order to solve phase change problems, various numerical methods such as the enthalpy method [24,25], the equivalent heat capacity method [26] and the temperature-transforming model [27] have been developed. The temperature-transforming model eliminates the time-step and grid-size restrictions. It is also insensitive to the phase-change temperature range [28]. The temperature transforming model has been used to solve many different problems related to latent heat thermal energy storage systems [29–31].

The primary objective of this work was to analyze the inward solidification inside spherical capsule which is initially not at its melting temperature under the effect of convection at the outside surface. The specific objectives are as follows:

- To investigate the solidification of phase change material inside spherical capsule.
- To solve the problem numerically by using control volume approach and temperature transforming method.
- To examine the effect of sphere diameter and coolant fluid temperature on the time elapsed for a total solidification.

- To determine the effect of various design parameters such as sphere diameter and coolant fluid temperature on entropy generation.

2. Mathematical model

The present analysis will consider one spherical capsule with radius r_o . The spherical capsule is filled with phase change material initially at T_i , which is higher than its melting temperature T_m . Water is used as a phase change material. The thermophysical properties of water are given in Table 1. The outside surface of the spherical capsule is subjected to convection heat transfer. The coolant temperature is kept at T_{cf} . The following assumptions are made:

- The thermophysical properties of PCM are independent of temperature but they differ for the solid and liquid phases of the PCM.
- The PCM is homogenous and isotropic.
- The effect of natural convection inside the PCM liquid is neglected.

It should be noted that the subcooling phenomenon, encountered especially in water crystallization, is neglected in the proposed model. In addition, the volume expansion due to solidification of water is also ignored. The heat conduction in the PCM is formulated by a temperature transforming method using a fixed grid numerical model [27]. This model assumes that solidification process occurs over a range of phase change temperatures from $(T_m - \delta T_m)$ to $(T_m + \delta T_m)$, but it can also be used to simulate the solidification process occurring at a single temperature by taking a small range of phase change temperature, $2\delta T_m$. This method has the advantage of eliminating the time step and grid size limitations that are normally encountered in other fixed grid methods [29].

Based upon the aforementioned assumptions, the governing energy equation in spherical coordinate can be written as follows:

$$\frac{\partial H}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(kr^2 \frac{\partial T}{\partial r} \right) \quad (1)$$

The relation between the enthalpy and temperature can be expressed in linear form as follows:

$$H = cT + s^* \quad (2)$$

where c , s^* are the dimensional specific heat and source term, respectively. Eq. (1) can be nondimensionalized by using the following dimensionless parameters:

Table 1 Thermophysical properties of water.

Properties	Water (liquid)	Water (solid)
Density, kg/m ³	999.8	916.8
Specific heat, kJ/kg K	4210	2040
Thermal conductivity, W/m K	0.567	1.88
Melting temperature, °C	0	
Latent heat of fusion, kJ/kg	333,500	

$$R = \frac{r}{r_o}, \quad C = \frac{\rho c}{\rho_L c_L}, \quad \theta = \frac{T - T_m}{T_m - T_{cf}}, \quad \delta\theta_m = \frac{\delta T}{T_m - T_{cf}},$$

$$K = \frac{k}{k_L}, \quad Fo = \frac{\alpha_L t}{r_o^2}$$

$$Bi = \frac{hr_o}{k_L}, \quad S = \frac{s^*}{\rho_L c_L (T_m - T_{cf})}, \quad Ste = \frac{c_L (T_m - T_{cf})}{L} \quad (3)$$

Here s^* is the dimensional source term. By using the aforesaid dimensionless parameters, the dimensionless forms of energy equation for spherical capsule can be written as follows:

$$\frac{\partial(C\theta)}{\partial Fo} = \frac{1}{R^2} \frac{\partial}{\partial R} \left[KR^2 \frac{\partial \theta}{\partial R} \right] - \frac{\partial S}{\partial Fo} \quad (4)$$

where

$$C(\theta) = \begin{cases} C_{sl} & \theta < -\delta\theta_m & \text{Solid} \\ \frac{1}{2}(1 + C_{sl}) + \frac{1}{2Ste\delta\theta_m} & -\delta\theta_m \leq \theta \leq \delta\theta_m & \text{Mushy} \\ 1 & \theta > \delta\theta_m & \text{Liquid} \end{cases} \quad (5)$$

$$S(\theta) = \begin{cases} C_{sl}\delta\theta_m & \theta < -\delta\theta_m & \text{Solid} \\ \left[\frac{1}{2}\delta\theta_m(1 + C_{sl}) + \frac{1}{2Ste} \right] & -\delta\theta_m \leq \theta \leq \delta\theta_m & \text{Mushy} \\ C_{sl}\delta\theta_m + \frac{1}{Ste} & \theta > \delta\theta_m & \text{Liquid} \end{cases} \quad (6)$$

$$K(\theta) = \begin{cases} K_{sl} & \theta < -\delta\theta_m & \text{Solid} \\ K_{sl} + \frac{(1-K_{sl})(\theta + \delta\theta_m)}{2\delta\theta_m} & -\delta\theta_m \leq \theta \leq \delta\theta_m & \text{Mushy} \\ 1 & \theta > \delta\theta_m & \text{Liquid} \end{cases} \quad (7)$$

A mushy region is formed where solid coexists with liquid in equilibrium. According to temperature transforming method, mushy phase will occur when the temperature at node (i) satisfies the condition of $-\delta\theta_m \leq \theta \leq \delta\theta_m$. The values of $C(\theta)$, $S(\theta)$ and $K(\theta)$ at the mushy phase can be calculated by using Eqs. (5)–(7), respectively.

The initial condition for Eq. (4) is

$$Fo = 0 \rightarrow \theta = \theta_i \quad (8)$$

While the boundary conditions for Eq. (4) are

$$R = 0 \rightarrow \frac{\partial \theta}{\partial R} = 0 \quad (9)$$

$$R = 1 \rightarrow K \frac{\partial \theta}{\partial R} = -Bi(\theta + 1) \quad (10)$$

In the current study the model of Ismail et al. [19] is used in which the spherical capsule is immersed in a pool of ethanol. The thermophysical properties of ethanol are obtained from Reid et al. [32]. The convection heat transfer coefficient can be calculated by using Churchill correlations as follows [33]:

$$\overline{Nu}_D = \frac{\bar{h}D}{k_{cf}} = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + (0.469/Pr)^{9/16} \right]^{4/9}} \quad (11)$$

Here Rayleigh number can be calculated as follows:

$$Ra_D = \frac{g\beta(T_s - T_{cf})D^3}{\alpha\nu} \quad (12)$$

2.1. Solution method

The problem is solved by using the control volume approach described by Versteeg and Malalasekera [34], and the dimensionless energy Eq. (4) is integrated over the control volume and time as follows:

$$\int_V \int_{Fo} \frac{\partial(C\theta)}{\partial Fo} dV dFo = \int_V \int_{Fo} \frac{1}{R^2} \frac{\partial}{\partial R} \left[KR^2 \frac{\partial \theta}{\partial R} \right] dV dFo - \int_V \int_{Fo} \frac{\partial S}{\partial Fo} dV dFo \quad (13)$$

The dimensionless energy Eq. (4) is integrated across the control volume in r direction and time. Performing the integration and rearranging it, the following finite volume equations can be obtained as follows:

$$\frac{C_P\theta_P - C_P^0\theta_P^0}{\Delta Fo} \frac{(R_e^3 - R_w^3)}{3} = \left[K_e R_e^2 \frac{\theta_E - \theta_P}{\Delta R_e} - K_w R_w^2 \frac{\theta_P - \theta_W}{\Delta R_w} \right] - \frac{S_P - S_P^0}{\Delta Fo} \frac{(R_e^3 - R_w^3)}{3} \quad (14)$$

where θ_P^0 is the dimensionless temperature at the previous time step. C_P^0 and S_P^0 are the dimensionless specific heat and source term at previous time step, respectively. R_e and R_w are the dimensionless radii for the control volume interfaces at e and w , respectively. K_e and K_w are thermal conductivities for the control volume interfaces at e and w , respectively.

The dimensionless thermal conductivity at the control surface is calculated by using harmonic mean which is described by Patankar [35]. Fig. 1 illustrates the discretization of the control volume for nodal points. The discretized equation can be written as follows:

$$a_P\theta_P = a_E\theta_E + a_W\theta_W + b \quad (15)$$

where

$$a_E = \frac{K_e R_e^2}{\Delta R_e}, \quad a_W = \frac{K_w R_w^2}{\Delta R_w} \quad (16)$$

$$b = \left[\frac{C_P^0\theta_P^0}{\Delta Fo} - \frac{S_P - S_P^0}{\Delta Fo} \right] \frac{(R_e^3 - R_w^3)}{3} \quad (17)$$

$$a_P = a_E + a_W + \frac{C_P\theta_P}{\Delta Fo} \frac{(R_e^3 - R_w^3)}{3} \quad (18)$$

The algebraic equations have been solved by using Thomas algorithm. Since the dimensionless energy equation is nonlinear heat conduction equation, iteration is needed at each time step which is declared as follows:

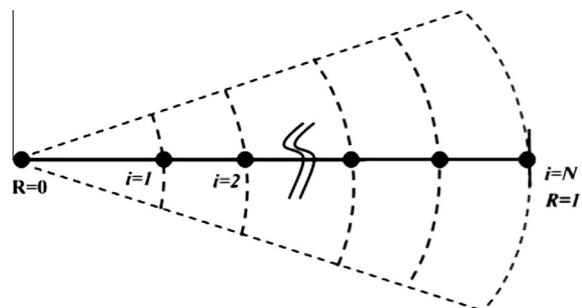


Figure 1 Nodal point for the control volume.

$$|\theta_i^{k+1} - \theta_i^k| \leq 10^{-9} \quad (19)$$

During the calculation process, the energy balance is checked to verify the numerical results, and thus at each time step, the change in the stored energy of the PCM must be equal to the total heat rejected from the sphere as follows:

$$\int_0^{Fo} 4\pi R_p^2 Bi(\theta_p + 1) dFo = \sum_{i=1}^N \int_{R_w}^{R_c} 4\pi(H^* - H_0^*) R^2 dR \quad (20)$$

where N represents the number of nodes.

The left-hand side of Eq. (20) represents the heat rejected from the PCM while the right hand-side represents the energy stored in the PCM. The numerical deviation between two sides of Eq. (20) is taken less than 0.00000001%.

3. Entropy generation

After the temperature distribution is obtained it's possible to calculate the local entropy generation. The entropy generation rate per unit volume may be expressed based on the second law of thermodynamics in the following manner [36]:

$$\dot{S}_{gen}^m dr = \frac{q + \left(\frac{\partial q}{\partial r}\right) \partial r}{T + \left(\frac{\partial T}{\partial r}\right) \partial r} - \frac{q}{T} + \rho \frac{\partial s}{\partial t} dr \quad (21)$$

The first two terms in the right side of Eq. (21) represent the entropy transfer associated with heat transfer, and the last term represents the rate of entropy accumulation in the element. Dividing Eq. (21) by dr , the local entropy generation becomes the following:

$$\dot{S}_{gen}^m = \frac{1}{T} \frac{\partial q}{\partial r} - \frac{q}{T^2} \frac{\partial T}{\partial r} + \rho \frac{\partial s}{\partial t} \quad (22)$$

if $\partial s = \partial u/T$, where u is internal energy, inside the element dr

$$\rho \frac{\partial s}{\partial t} = \frac{\rho}{T} \frac{\partial u}{\partial t} \quad (23)$$

From the first law of thermodynamics, written for one point in the convective medium, $(\partial u/\partial t) = -\partial q/\partial x$, so we can write down Eq. (21) as

$$\rho \frac{\partial s}{\partial t} = -\frac{1}{T} \frac{\partial q}{\partial r} \quad (24)$$

Combining Eq. (21) with Eq. (22) we obtain entropy generation as

$$\dot{S}_{gen}^m = -\frac{q}{T^2} \frac{\partial T}{\partial r} \quad (25)$$

Applying Fourier's law, the local entropy generation rate per unit volume for one dimensional spherical coordinate becomes [36]:

$$\dot{S}_{gen}^m = \frac{k}{T^2} \left(\frac{\partial T}{\partial r}\right)^2 \quad (26)$$

Since the viscous effect is neglected then the total entropy generation per unit volume can be determined by integrating Eq. (26) over the volume. It should be noted here that Eq. (26) is derived by applying the second law of thermodynamics on small element. In addition, the element size is small enough so that the thermodynamic state of the fluid inside the element is considered independent of position. However, the thermodynamic state of the element may be changed with time [36].

4. Results and discussion

A numerical investigation has been performed to study the effects of design parameters on solidification and entropy generation of phase change material inside a spherical capsule. The finite volume method is used in order to solve the governing equations.

In this study, the solidification process starts from the outside surface which is in contact with the coolant fluid, and proceeds inward toward the center of the sphere due to the heat transfer mechanism. The analysis presented here is the case of pure conduction and convection effect in the liquid phase is neglected. It's because of the fact that the initial temperature of the spherical capsule is considered to be near the phase change temperature, and as a result, the natural convection effect inside the liquid phase can be neglected. The solidification process is completed when all the liquid is solidified and the system has reached its thermal equilibrium with outside coolant temperature. Throughout the solution, the number of grids is selected to be $N = 500$, the time step is $\Delta t = 0.1$ and the phase change temperature range is 0.01.

In order to verify and validate the numerical model used in this work, the result are compared with the existing published numerical and experimental data obtained by [19]. The present numerical results for temperature variation of phase change material at the center of sphere and the numerical and experimental data from [19] are depicted in Fig. 2. In this figure, the initial temperature, the coolant fluid temperature and the diameter of spherical capsule are taken as 25.8 °C, -7.5 °C and 0.128 m, respectively. Fig. 2 shows a good agreement between the present numerical model results and the published experimental data which gives more confidence in the method of tackling this problem. Bilir and İlken [21] presented figure for the surface temperature of a sphere with time which can be verified against that solution. The numerical model was implemented at fixed values of $D = 0.06$ m, $T_{cf} = -15$ °C, $T_i = 25$ °C, and $h = 218.7$ W/m² K and the results are illustrated in Fig. 3. Excellent agreement between the obtained result and published data is verified. In addition, no oscillations in the surface temperature are occurred and this result also shows the accuracy of the numerical model.

In order to examine the stability of temperature transforming method, the changes in interface position and the temperature at the center of spherical capsule as time is varied are calculated for several phase change temperature range values $\delta T = 0.001$, $\delta T = 0.01$, $\delta T = 0.05$ and $\delta T = 0.1$. It is found that the method is insensitive to the phase change temperature range. As a result, a small phase change temperature range is used to simulate the solidification front.

The variation of surface temperature (T_s) and temperature at the center of the spherical capsule (T_{center}) is illustrated in Fig. 4. This figure shows that the central part of the capsule keeps existing in its liquid phase for approximately 0.4 h while the solid phase at the center starts initiating in approximately 0.72 h. The mushy phase occurs between these two periods. The reason of sudden decrease in the temperature at the center of the spherical capsule at approximately 0.72 h is due to removing sensible heat from the solidified mass while phase change is occurred at the transformation from mushy phase to solid phase. After approximately 1.1 h the system reaches its thermal equilibrium and the solidification process is completed. In Fig. 4, the

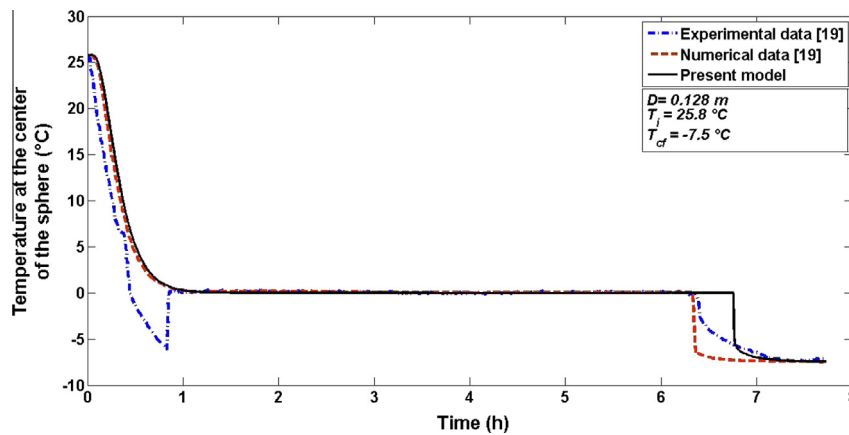


Figure 2 Comparison of present model results with the experimental and numerical data taken from [19].

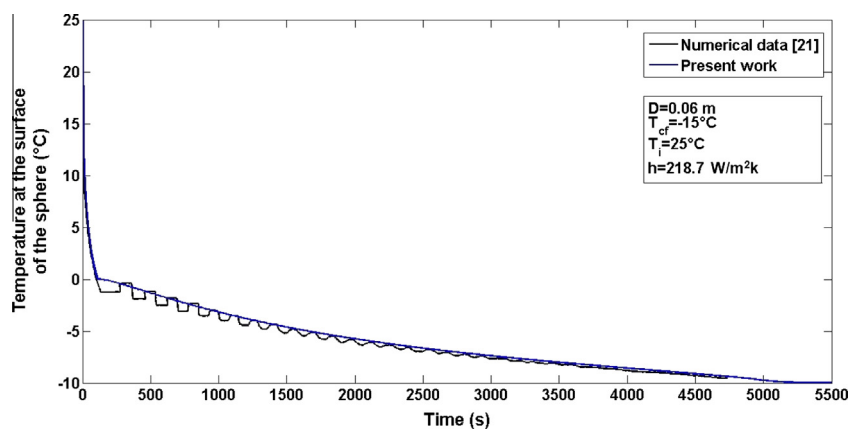


Figure 3 Comparison of the surface temperature of a sphere with Ref. [21].

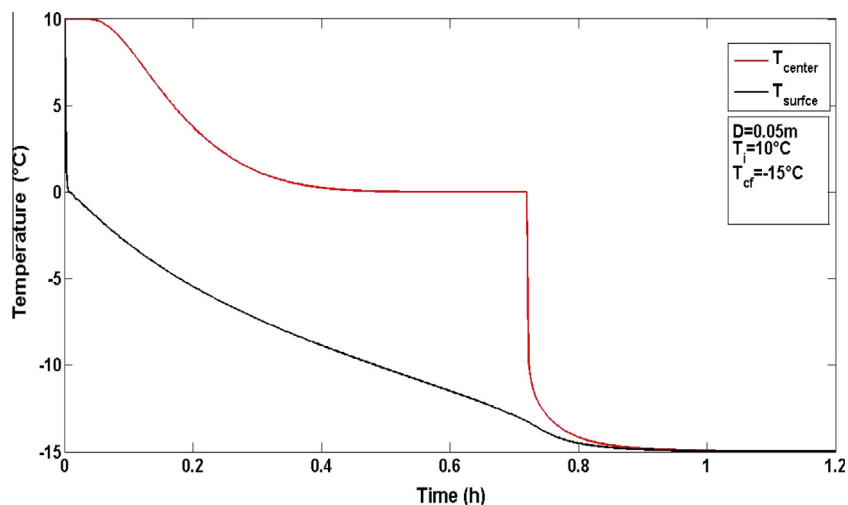


Figure 4 Transient distributions of surface temperature and center temperature of sphere capsule.

dimensionless phase change temperature at the beginning of solidification would be $\delta\theta_m = 6.666 \times 10^{-4}$. The outer surface of the phase change material is at liquid phase which is the case of $\theta > 6.667 \times 10^{-4}$, and the value of dimensionless specific heat, thermal conductivity and source term at that point would

be $C = 1$, $K = 1$ and $S = 5.2814$, respectively. The temperature distribution is calculated from Eq. (14). The temperature continuously decreases while the value of C , K and S is kept constant at liquid region. The mushy phase is occurred at the node in the range of $-6.667 \times 10^{-4} \leq \theta \leq 6.667 \times 10^{-4}$. In this case the

value of dimensionless specific heat and source term at that point will be changed as $C = 3961.53$ and $S = 2.641$, respectively. However, the thermal conductivity varies linearly with temperature. In the range of $\theta < -6.667 \times 10^{-4}$, the liquid phase is occurred and in this case $C = 0.4443$, $K = 3.3156$ and $S = 2.962 \times 10^{-4}$, respectively. The process continues until the center of the sphere is solidified completely and the temperature of the center of sphere is equal to coolant fluid temperature.

Fig. 5 shows the transient variation of the solid-liquid interface position for different sphere diameters $D = 0.02$ m, $D = 0.05$ m, $D = 0.1$ m, $D = 0.15$ m and $D = 0.2$ m. The initial temperature is 0.5 °C and the coolant fluid temperature is -15 °C. It can be seen that at $D = 0.02$ m the time elapsed for complete solidification of the liquid is found to be 0.166 h (10 min) while its 8.6 h at $D = 0.2$ m. It can also be observed that larger diameters of spherical capsule have significantly greater solidification time compared to those of smaller diameters of spherical capsules. Since the ice forms the first layer at the last grid node (N) and as the ice thickness increases toward the center of the sphere the thermal resistance will also increase; therefore, the distance of heat transfer to take place

will be larger as the sphere diameter varies increasing from $D = 0.02$ m to $D = 0.2$ m.

For the same sphere diameters and temperature conditions Fig. 6 shows the behavior of temperature profile at the center of the sphere with respect to time as the diameter of the sphere capsule increases from 0.02 m to 0.2 m. It should be noted that for all the cases considered, fixed values of $T_i = 0.5$ °C and $T_{cf} = -15$ °C with variable sphere capsule diameters $D = 0.02$ – 0.2 m are used. The temperature at the center of spherical capsule at liquid state decreases until it eventually becomes constant as it passes to the mushy phase, suddenly drops to the solid phase and then further decreases slightly. It can be observed that spherical capsules with smaller diameters require significantly lower solidification time compared to those with larger diameters because PCMs spheres with smaller diameters have higher solidification rates.

The effect of the coolant fluid temperature on the interface position is depicted in Fig. 7. The coolant fluid temperature is varied from -5 °C to -25 °C in steps of 5 °C, while initial temperature and the diameter of spherical capsules are taken as 0.5 °C and 0.1 m, respectively. At $T_{cf} = -5$ °C the time for total solidification is 7.6 h while at $T_{cf} = -25$ °C the time for

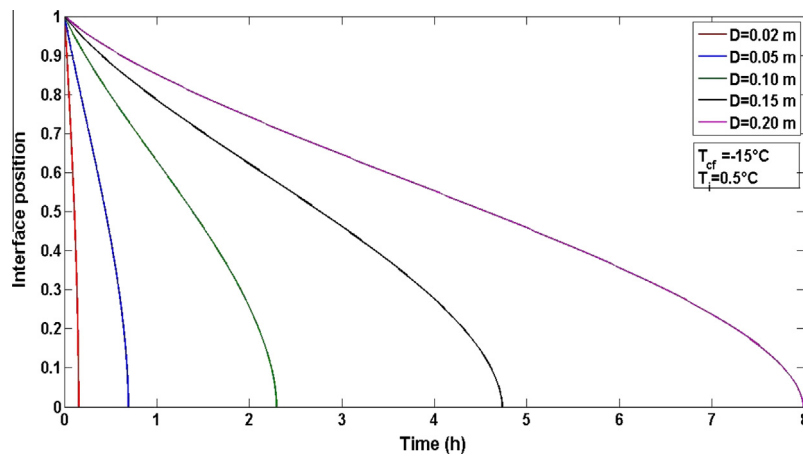


Figure 5 Effects of the diameter of sphere capsule on interface position.

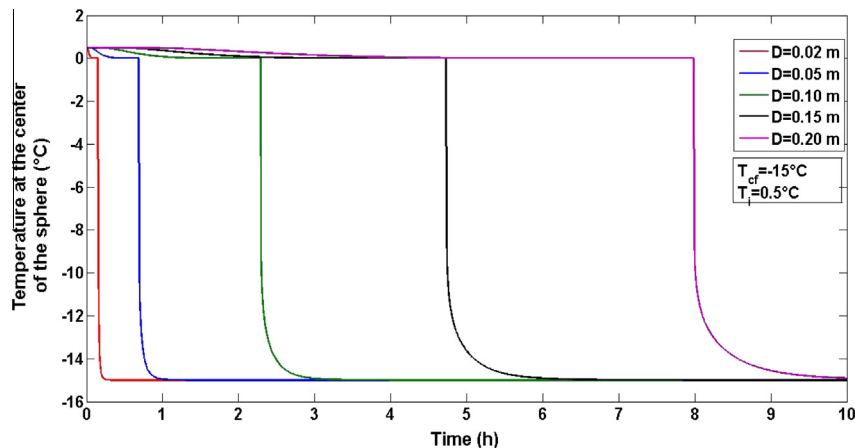


Figure 6 Variation of the temperature at the center of the sphere with respect to the time and the diameter of spherical capsule.

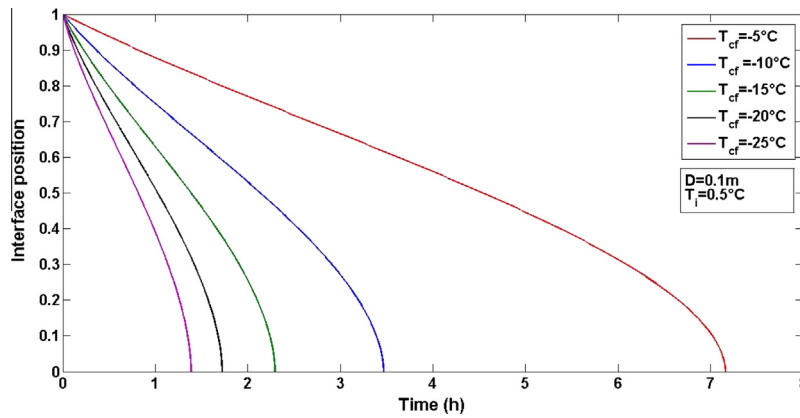


Figure 7 Variation of interface position with respect to the solidification time and the coolant fluid temperature.

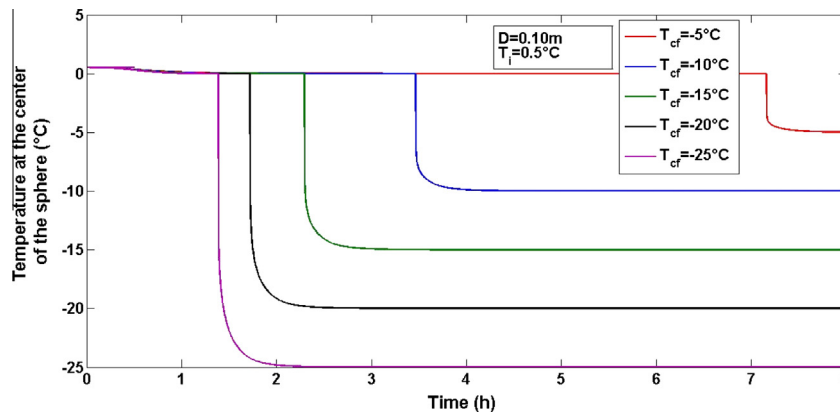


Figure 8 Variation of the temperature at the center of the sphere with respect to the time and the coolant fluid temperature.

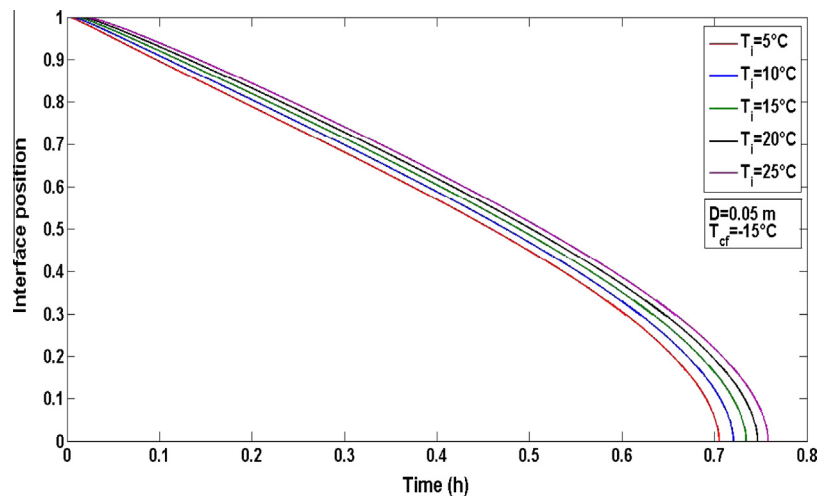


Figure 9 Variation of interface position with respect to the solidification time and the initial temperature.

complete solidification is 1.5 h. It can be seen in this figure that reducing coolant fluid temperature from -5°C to -25°C significantly decreases the time for complete phase change due to the higher convective heat transfer rate. The temperature variation of phase change material at the center of spherical capsule at different coolant fluid temperatures is illustrated in

Fig. 8. As shown in Fig. 8, for a given value of the time, the temperature at the center of sphere capsule decreases and eventually becomes constant, drops suddenly and then finally decreases slightly before it becomes stabilized. It is concluded that the lower is the coolant fluid temperature, the shorter is the solidification time in sphere capsule. Fig. 9 presents the effect of the

initial temperature on the interface position. The initial temperature was varied from 5 °C to 25 °C in steps of 5 °C, while keeping the other parameters constant. As can be seen from Fig. 9, decreasing initial temperature from 25 °C to 5 °C has very small effect on the time for complete phase change.

Fig. 10 illustrates the temperature distribution at the center of the sphere capsule for various values of initial temperatures T_i ($T_i = 5, 10, 15, 20, 25$ °C). In this figure, the coolant fluid temperature and the diameter of spherical capsule are fixed and taken as -15 °C and 0.05 m, respectively. Beside, the diameter of spherical capsule is considered to be in small value in order to satisfy the assumption of ignoring natural convection effect inside the liquid phase. For particular value of the initial temperature, the magnitude of temperature is higher for higher value of the initial temperature. For a given value of the time, at $T_i = 10$ °C, the temperature at the center of sphere capsule is constant for values of the time between 0.0 and 0.08 h (4.8 min) and then decreases and eventually becomes constant as it passes to mushy phase for values of the time between 0.4 h (24 min) and 0.72 h (43.2 min), drops suddenly and then finally decreases slightly to reach the value of -15 °C at values of the time greater than 0.9 h (54 min).

Fig. 11 shows the variation of entropy generation with respect to solidification time for different values of sphere

diameter. In this figure, the diameter of the spherical capsule has significant effect on the entropy generation. It can be observed that an increase in the diameter of spherical capsule results in an increase in entropy generation. For particular value of the time, the entropy generation is higher for higher value of the diameter of spherical capsule.

According to Eq. (25), entropy generation is directly proportional to heat transfer rate and temperature gradient. At the beginning of the solidification process, the heat transfer rate increases rapidly until it reaches to interphase position. Therefore, the entropy generation increases sharply at the beginning up to a maximum point. After the peak point, the solidification rate and heat transfer rate decrease smoothly because of the increasing thermal resistance. As a result, the entropy generation drops off to its lowest value where the solidification time is completed. The effect of the coolant fluid temperature on the entropy generation is depicted in Fig. 12. In this figure, initial temperature and the diameter of spherical capsule are taken as 0.5 °C and 0.1 m, respectively. For particular value of the time, the magnitude of entropy generation is higher for lower value of the coolant fluid temperature. For a given value of the coolant fluid temperature, the entropy generation increases with solidification time, attains a maximum and then decreases.

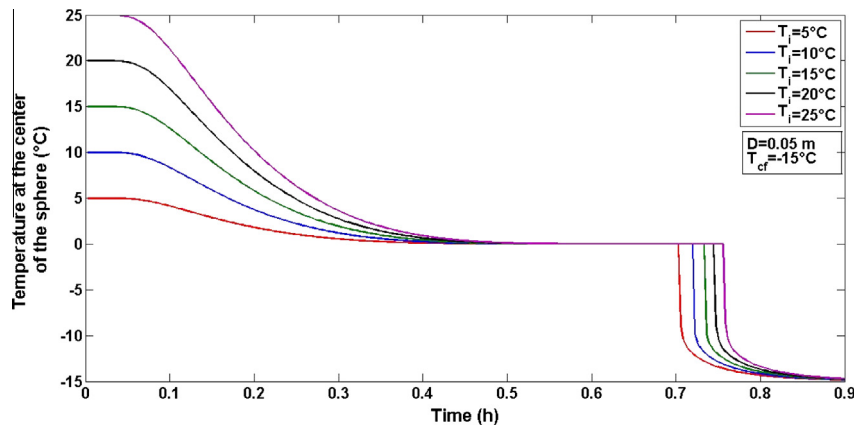


Figure 10 Variation of the temperature at the center of the sphere with respect to the time and the initial temperature.

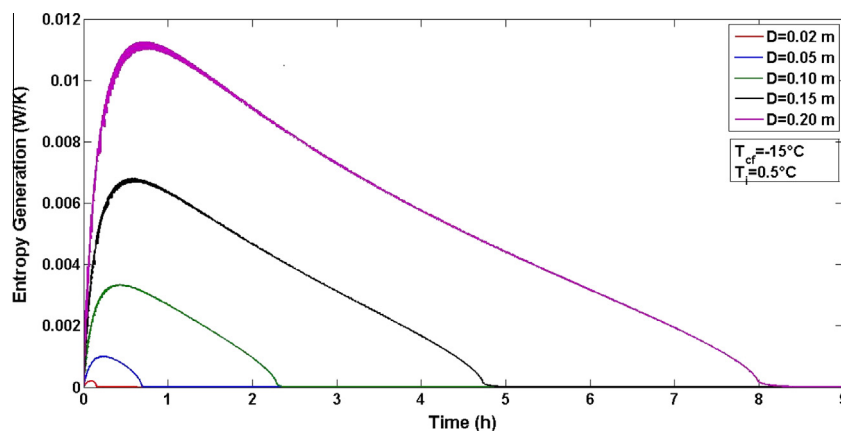


Figure 11 Effect of the diameter of sphere capsule on entropy generation.

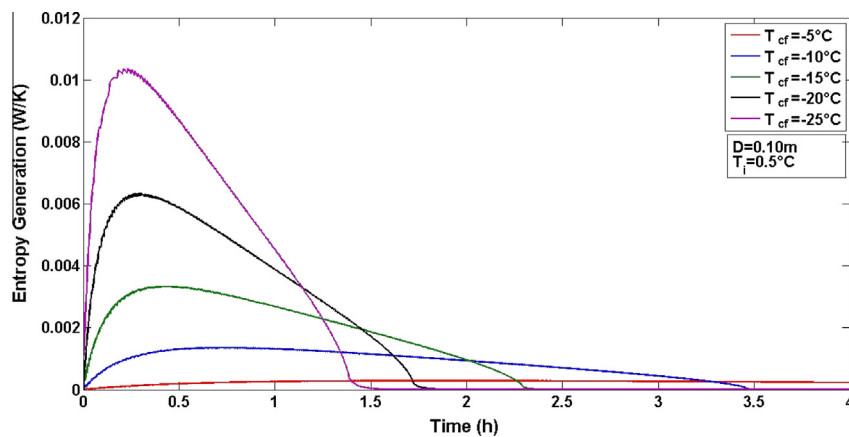


Figure 12 Effect of the coolant fluid temperature on entropy generation.

5. Conclusions

The inward solidification of a phase change material inside spherical capsule is analyzed numerically by using temperature transforming method. Furthermore, a parametric study is conducted to evaluate the effect of the varying selected design parameters such as sphere diameter and coolant fluid temperature on solidification time and entropy generation. The model is validated with the experimental published data taken from the literature. The temperature at the center and at the surface of the sphere with respect to time is determined over a range of operating conditions. The following specific concluding remarks can be drawn:

- Small diameter of sphere capsules has significantly lower solidification time compared to larger diameters of sphere capsule.
- The diameter of the spherical capsule has a major effect on entropy generation as the entropy generation increases with increasing diameter of the sphere.
- Selecting a lower coolant fluid temperature results in considerable drop in entropy generation.
- In this context, the diameter of sphere capsule and coolant fluid temperature play an important role in the designing and assessing the thermal energy storage systems.

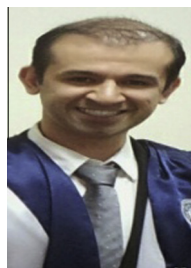
References

- [1] Sharma A, Tyagi VV, Chen CR, Buddhi D. Review on thermal energy storage with phase change materials and applications. *Renew Sustain Energy Rev* 2009;13(2):318–45.
- [2] Farid MM, Khudhai AM, Razack SAK, Al-Hallaj SA. Review on phase change energy storage: materials and applications. *Energy Convers Manage* 2004;45:1597–615.
- [3] Zalba B, Marin JM, Cabeza LF, Mehling H. Review on thermal energy storage whit phase change: materials, heat transfer analysis and applications. *Appl Therm Eng* 2003;23:251–83.
- [4] Abhat A. Low temperature latent heat thermal energy storage. *Heat storage materials*. *Sol Energy* 1983;30(4):313–32.
- [5] Regin AF, Solanki SC, Saini JS. Heat transfer characteristics of thermal energy storage system using PCM capsules: a review. *Renew Sustain Energy Rev* 2008;12(9):2438–58.
- [6] Verma P, Varun, Singal SK. Review of mathematical modeling on latent heat thermal energy storage systems using phase-change material. *Renew Sustain Energy Rev* 2008;12(4):999–1031.
- [7] Dutil Y, Rousse DR, Ben Salah N, Lassue S, Zalewski L. A review on phase-change materials: mathematical modeling and simulations. *Renew Sustain Energy Rev* 2011;15:112–30.
- [8] Ismail KAR, Moraes RIR. A numerical and experimental investigation of different containers and PCM options for cold storage modular units for domestic applications. *Int J Heat Mass Transf* 2009;52:4195–202.
- [9] Ismail KAR, Henriquez JR. Numerical and experimental study of spherical capsules packed bed latent heat storage system. *Appl Therm Eng* 2002;22:1705–16.
- [10] Adine HA, Qarnia HE. Numerical analysis of the thermal behaviour of a shell-and-tube heat storage unit using phase change materials. *Appl Mathemat Model* 2009;33:2132–44.
- [11] Lamberg P, Siren K. Approximate analytical model for solidification in a finite PCM storage with internal fins. *Appl Mathemat Model* 2003;27:491–513.
- [12] Tao LC. Generalized numerical solutions of freezing a saturated liquid in cylinders and spheres. *AIChE J* 1967;13(1):165–9.
- [13] Yang J, Zhao CY. Solidification analysis of a single particle with encapsulated phase change materials. *Appl Therm Eng* 2013;51:338–46.
- [14] Pedroso RI, Domoto GA. Perturbation solutions for spherical solidification of saturated liquids. *J Heat Transfer* 1973;95(1):42–6.
- [15] Riley DS, Smith FT, Poots G. The inward solidification of spheres and circular cylinders. *Int J Heat Mass Transf* 1974;17:1507–16.
- [16] Hill JM, Kucera A. Freezing a saturated liquid inside a sphere. *Int J Heat Mass Transfer* 1983;26:1631–7.
- [17] Prud'Homme M, Nguyen TH, Nguyen DL. A heat transfer analysis for solidification of slabs, cylinders and spheres. *J Heat Transf* 1989;111:699–705.
- [18] Ismail KAR, Henriquez JR. Solidification of PCM inside a spherical capsule. *Energy Convers Manage* 2000;41:173–87.
- [19] Ismail KAR, Henriquez JR, da Silva TM. A parametric study on ice formation inside a spherical capsule. *Int J Thermal Sci* 2003;42:881–7.
- [20] Eames IW, Adref KT. Freezing and melting of water in spherical enclosures of the type used in thermal (ice) storage systems. *Appl Therm Eng* 2002;22:733–45.
- [21] Bilir L, İken Z. Total solidification time of a liquid phase change material enclosed in cylindrical/spherical containers. *Appl Therm Eng* 2005;25:1488–502.
- [22] Chan CW, Tan FL. Solidification inside a sphere – an experimental study. *Int Commun Heat Mass Transf* 2006;33:335–41.

- [23] Veerappan M, Kalaiselvam S, Iniyan S, Goic R. Phase change characteristic study of spherical PCMs in solar energy storage. *Sol Energy* 2009;83:1245–52.
- [24] Shamsunder N, Sparrow EM. Analysis of multidimensional conduction phase change via the enthalpy model. *ASME J Heat Transf* 1975;97:333–40.
- [25] Crank J. *Free and moving boundary problems*. Oxford, U.K.: Clarendon Press; 1984.
- [26] Bonacina C, Comini G, Fasano A, Primicerio A. Numerical solution of phase change problems. *Int J Heat Mass Transf* 1973;16:1825–32.
- [27] Cao Y, Faghri A. A numerical analysis of phase-change problem including natural convection. *ASME J Heat Transf* 1990;112:812–6.
- [28] Faghri A, Zhang Y. *Transport phenomena in multiphase systems*. Elsevier Publications; 2006.
- [29] Zhang Y, Faghri A. Heat transfer enhancement in latent heat thermal energy storage system by using the internally finned tube. *Int J Heat Mass Transf* 1996;39(15):3165–73.
- [30] Cao Y, Faghri A. Performance characteristics of a thermal energy storage module: a transient pcm/forced convection conjugate analysis. *Int J Heat Mass Transf* 1991;34:93–101.
- [31] Cao Y, Faghri A. A study of thermal energy storage system with conjugate turbulent forced convection. *ASME J Heat Transf* 1992;114:1019–27.
- [32] Reid R, Poling BE, Prausnitz JM. *The properties of gases and liquids*. 4th ed. McGraw-Hill; 1987.
- [33] Churchill SW. Free convection around immersed bodies. In: Schlünder EU, editor. *Heat exchanger design handbook*. New York: Hemisphere; 1983, Section 2.5.7.
- [34] Versteeg HK, Malalasekera W. *An introduction to computational fluid dynamics: the finite volume method*. London: Prentice Hall; 1995.
- [35] Patankar SV. *Numerical heat transfer and fluid flow*. New York: McGraw-Hill; 1980.
- [36] Bejan A. *Entropy generation minimization*. New York: CRC Press; 1996.



Mustafa Asker has been working as an Assistant Professor in the Department of Mechanical Engineering at Engineering Faculty of Adnan Menderes University since 2014. He received B.Sc. and M.Sc. degrees in Mechanical Engineering from Al Nahrain Üniversitesi and his Ph.D. degree from Ege University.



Hadi Ganjehsarabi is currently an Assistant Professor in Mechanical Engineering at Erzurum University. He received his BS in Mechanical Engineering from Bou Ali Sina University in 2003, and his MS and PhD in Mechanical Engineering from Ege University in 2009 and 2014, respectively. He has also been a reviewer for several journals.



Mustafa Turhan Coban received his B.Sc. degree from Ege University Department of Mechanical Engineering in 1978, and his M.Sc. degree in Mechanical Engineering from Michigan Technological University (U.S.A.) in 1982. He graduated from University of Utah (U.S.A.) in 1986 with a Ph.D. in Mechanical Engineering. He is now Assistant Professor in Ege University. His research interests are fuel cell technologies, air conditioning and computer science.