# Azimuthal asymmetry in unpolarized $\pi N$ Drell-Yan process 

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#### Abstract

Taking into account the effect of final-state interaction, we calculate the non-zero (naïve) $T$-odd transverse momentum dependent distribution $h_{1}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ of the pion in a quark-spectator-antiquark model with effective pion-quark-antiquark coupling as a dipole form factor. Using the model result we estimate the $\cos 2 \phi$ asymmetries in the unpolarized $\pi^{-} N$ Drell-Yan process which can be expressed as $h_{1}^{\perp} \times \bar{h}_{1}^{\perp}$. We find that the resulting $h_{1 \pi}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ has the advantage to reproduce the asymmetry that agrees with the experimental data measured by NA10 Collaboration. We estimate the $\cos 2 \phi$ asymmetries averaged over the kinematics of NA10 experiments for 140,194 and $286 \mathrm{GeV} \pi^{-}$beam and compare them with relevant experimental data. © 2005 Elsevier B.V. Open access under CC BY license.


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## 1. Introduction

Recently it is demonstrated that the effect of final-state interaction (FSI) or initial-state interaction (ISI) can lead to significant azimuthal asymmetries in various high energy scattering processes involving hadrons [1,2]. Among these asymmetries, single spin asymmetry (SSA) in semi-inclusive deeply inelastic scattering (SIDIS) [1] and that in Drell-Yan

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processes [2] from FSI/ISI via the exchange of a gluon, have been explored and are recognized as previously known Sivers effect [3,4]. This effect, formerly thought to be forbidden by the time-reversal property of QCD [5], can be survived from time-reversal invariance due to the presence of the path-ordered exponential (Wilson line) in the gauge-invariant definition of the transverse momentum dependent parton distributions [6-8]. Along this direction some phenomenological studies [9-11] have been carried out on transverse single-spin asymmetries in SIDIS process, which is under investigation by current ex-
periment [12]. Analogously the exchange of a gluon can also lead to another leading twist (naive) $T$-odd distribution $h_{1}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ : the covariant transversely polarization density of quarks inside an unpolarized hadron. This chiral-odd partner of Sivers effect function, introduced first in Ref. [13] and is referred to as Boer-Mulders function, has been proposed [14] to account for the large $\cos 2 \phi$ asymmetries in the unpolarized pion-nucleon Drell-Yan process that were measured more than 10 years ago [15,16]. Recently $h_{1}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ of the proton has been computed in a quarkscalar diquark model $[9,17]$ and also used to analyze the consequent $\cos 2 \phi$ azimuthal asymmetries in both unpolarized ep SIDIS process [9] and unpolarized $p \bar{p}$ Drell-Yan process [17], respectively.

The same mechanism producing $T$-odd distribution functions can be applied to other hadrons such as mesons. In a previous paper [18] we reported that nonzero $h_{1}^{\perp}$ of the quark inside the pion (denoted as $h_{1 \pi}^{\perp}$ ) can also arise from final-state interaction, by applying a simple quark spectator-antiquark model. Among the phenomenological implications of the function $h_{1 \pi}^{\perp}$ is an important result for the $\cos 2 \phi$ azimuthal asymmetry in the unpolarized $\pi^{-} N$ Drell-Yan process [15,16], which can be produced by the product of $h_{1}^{\perp}$ of the pion and that of the nucleon. Therefore, one can investigate how the theoretical prediction of the asymmetry is comparable with the experimental result, as a test of the theory and the model. In the present Letter, based on $h_{1 \pi}^{\perp}$ from our model calculation, we analyze the $\cos 2 \phi$ azimuthal asymmetry in the unpolarized $\pi^{-} N$ Drell-Yan process by considering the kinematical region of NA10 experiments [15]. To obtain the right $Q_{T}$ dependence of the asymmetry, we recalculate $h_{1 \pi}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ in a spectator model similar to the model used in Ref. [18]. The difference is that here we treat the effective pion-quarkantiquark coupling $g_{\pi}$ as a dipole form factor $g_{\pi}\left(k^{2}\right)$, in contrary to the treatment in Ref. [18] where we take $g_{\pi}$ as a constant. We find that $h_{1 \pi}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ resulting from the new treatment together with $h_{1}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ for the nucleon in a similar treatment [10] can reproduce the $\cos 2 \phi$ asymmetry which agrees with NA10 data. We give the asymmetries predicted by our model averaged over the kinematics of NA10 experiments for 140,194 and $286 \mathrm{GeV} \pi^{-}$beam and find that the energy dependence of these asymmetries is not strong.

## 2. Non-zero $h_{1 \pi}^{\perp}$ of the pion in spectator model

In this section, we will show how to calculate $h_{1 \pi}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ in a quark-spectator antiquark model. We follow Ref. [1] to work in Abelian case at first and then generalize the result to QCD. There are pion-quarkantiquark interaction and gluon-spectator antiquark interaction in the model:
$\mathcal{L}_{I}=-g_{\pi} \bar{\psi} \gamma_{5} \psi \varphi_{\pi}-e_{2} \bar{\psi} \gamma^{\mu} \psi A_{\mu}+$ h.c.,
in which $g_{\pi}$ is the pion-quark-antiquark effective coupling, and $e_{2}$ is the charge of the antiquark. When the intrinsic transverse momentum of the quark is taken into account, as required by $T$-odd distributions, the quark correlation function of the pion in Feynman gauge (we perform calculation in this gauge) is [7,8]:

$$
\begin{align*}
& \Phi_{\alpha \beta}\left(x, \mathbf{k}_{\perp}\right) \\
& =\int \frac{d \xi^{-} d^{2} \xi_{\perp}}{(2 \pi)^{3}} e^{i k \cdot \xi}\left\langle P_{\pi}\right| \bar{\psi}_{\beta}(0) \mathcal{L}_{0}\left(0^{-}, \infty^{-}\right) \\
& \quad \times\left.\mathcal{L}_{\xi}^{\dagger}\left(\xi^{-}, \infty^{-}\right) \psi_{\alpha}(\xi)\left|P_{\pi}\right\rangle\right|_{\xi^{+}=0} \tag{2}
\end{align*}
$$

where $\mathcal{L}_{a}\left(a^{-}, \infty^{-}\right)$is the path-ordered exponential (Wilson line) accompanied with the quark field which has the form
$\mathcal{L}_{0}(0, \infty)=\mathcal{P} \exp \left(-i g \int_{0^{-}}^{\infty^{-}} A^{+}\left(0, \xi^{-}, \mathbf{0}_{\perp}\right) d \xi^{-}\right)$,
etc. The Wilson line has the importance to make the definition of the distribution/correlation function gauge-invariant. Without the constraint of timereversal invariance, in leading twist the quark correlation function of the pion can be parameterized into a set of leading twist transverse momentum dependent distribution functions as follows [13,19]

$$
\begin{align*}
& \Phi\left(x, \mathbf{k}_{\perp}\right) \\
& \quad=\frac{1}{2}\left[f_{1 \pi}\left(x, \mathbf{k}_{\perp}^{2}\right) \nprec+h_{1 \pi}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right) \frac{\sigma_{\mu \nu} \mathbf{k}_{\perp}^{\mu} n^{\nu}}{M_{\pi}}\right] \tag{4}
\end{align*}
$$

where $n$ is the light-like vector with components $\left(n^{+}, n^{-}, \mathbf{n}_{\perp}\right)=\left(1,0, \mathbf{0}_{\perp}\right), \sigma_{\mu \nu}=\frac{i}{2}\left[\gamma_{\mu}, \gamma_{\nu}\right]$ and $M_{\pi}$ is the pion mass. Knowing $\Phi_{\pi}\left(x, \mathbf{k}_{\perp}\right)$, one can obtain these distributions from equations
$f_{1 \pi}\left(x, \mathbf{k}_{\perp}^{2}\right)=\operatorname{Tr}\left[\Phi\left(x, \mathbf{k}_{\perp}\right) \gamma^{+}\right]$,
$\frac{2 h_{1 \pi}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right) \mathbf{k}_{\perp}^{i}}{M_{\pi}}=\operatorname{Tr}\left[\Phi\left(x, \mathbf{k}_{\perp}\right) \sigma^{i+}\right]$.

The calculation of unpolarized distribution function $f_{1 \pi}$ in the antiquark spectator model can be done [20] from the lowest order (without Wilson line) correlation function in Eq. (2). However it cannot lead to any $T$-odd distribution function such as $h_{1 \pi}^{\perp}$. As demonstrated in Ref. [1], the non-zero $T$-odd distribution requires final-state interaction from gluon exchange between the struck quark and target spectator. Here we follow the observation in Refs. [6,21] that final-state interaction in an initial hadron state can be taken into account effectively by introducing an appropriate Wilson line in the gauge-invariant definition of the transverse momentum dependent distribution function, or equivalently, quark correlation function of the hadron. The Wilson line can provide non-trivial phase needed for $T$-odd distribution functions. Since we have defined such a correlation function in Eq. (2), we can start from Eq. (2) to calculate $h_{1 \pi}^{\perp}$ with the explicit presence of the Wilson line. We expand the Wilson line to first order corresponding to one gluon exchange. Therefore, according to Eq. (6) and Eq. (2), $h_{1 \pi}^{\perp}$ can be calculated in the antiquark spectator model from the expression

$$
\begin{align*}
& \frac{2 h_{1 \pi}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right) \mathbf{k}_{\perp}^{i}}{M_{\pi}} \\
& \quad=\sum_{\bar{q}^{s}} \frac{1}{2} \int \frac{d \xi^{-} d \xi_{\perp}}{(2 \pi)^{3}} e^{i k \cdot \xi}\left\langle P_{\pi}\right| \bar{\psi}_{\beta}(0)\left|\bar{q}^{s}\right\rangle \\
& \quad \times\left\langle\bar{q}^{s}\right|\left(-i e_{1} \int_{\xi^{-}}^{\infty^{-}} A^{+}\left(0, \xi^{-}, \mathbf{0}_{\perp}\right) d \xi^{-}\right) \\
& \quad \times\left.\sigma_{\beta \alpha}^{i+} \psi_{\alpha}(\xi)\left|P_{\pi}\right\rangle\right|_{\xi^{+}=0}+\text { h.c. } \tag{7}
\end{align*}
$$

in which $\left|\bar{q}^{s}\right\rangle$ represents the antiquark spectator state with spin $s$, and $e_{1}$ is the charge of the struck quark.

Fig. 1 is the diagram equivalent to Eq. (7). The figure shows the effective correlation function in the antiquark spectator model with the Wilson line expanding to the first order. $h_{1 \pi}^{\perp}$ can be obtained from the diagram by inserting $\sigma^{i+}$, according to Eq. (6). Fig. 1 is similar to the diagram used by Ji and Yuan [21] to calculate $f_{1 T}^{\perp}$ of the proton in scalar diquark model. The $\gamma_{5}$ inside the circle denotes that the pion-quark-antiquark coupling is pseudoscalar coupling. The double line represents the eikonalized quark propagator (eikonal line), which is produced by the Wilson line along the


Fig. 1. Effective correlation function $\Phi$ in the antiquark spectator model with final-state interaction modeled by one gluon exchange.
light-cone vector $\bar{n}^{\mu}=\left(\bar{n}^{+}, \bar{n}^{-}, \overline{\mathbf{n}}_{\perp}\right)=\left(0,1, \mathbf{0}_{\perp}\right)$. The eikonal line gives rise the final-state interaction effect between the fast moving struck quark and the gluon field from target spectator system [6,21]. The Feynman rule for the eikonal line is $1 /\left(q^{+}+i \varepsilon\right)$ [22] (see also appendix in Ref. [17]), where $q$ is the momentum of the gluon attached to the eikonal line. The Feynman rule for the eikonal line-gluon vertex is $e_{1} \bar{n}^{\mu}$ [22]. The straight line cut by the vertical dashed line denotes the on-shell spectator antiquark state $v^{s}$ or $\bar{v}^{s}$.

Usually there are two choices of the pion-quarkantiquark coupling $g_{\pi}$ :

- Case 1: $g_{\pi}$ as a normalization constant which is used in Ref. [18]. A similar treatment for the proton-quark-diquark coupling $g$ has been adopted in Refs. [1,17] to estimate $f_{1 T}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ and $h_{1}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ of the proton.
- Case 2: $g_{\pi}$ as a dipole form factor [20]

$$
\begin{align*}
g_{\pi}\left(k^{2}\right) & =N_{\pi} \frac{k^{2}-m^{2}}{\left(\Lambda^{2}-k^{2}\right)^{2}} \\
& =N_{\pi}(1-x)^{2} \frac{k^{2}-m^{2}}{\left(\mathbf{k}_{\perp}^{2}+L_{\pi}^{2}\right)^{2}}, \tag{8}
\end{align*}
$$

with

$$
\begin{align*}
& L_{\pi}^{2}=(1-x) \Lambda^{2}+x m^{2}-x(1-x) M_{\pi}^{2}  \tag{9}\\
& \mathbf{k}_{\perp}^{2}=-(1-x) k^{2}-x m^{2}+x(1-x) M_{\pi}^{2} \tag{10}
\end{align*}
$$

and $N_{\pi}$ is the normalization constant, $m$ is the mass of the quark/antiquark inside the pion, $\Lambda$ is the cut off parameter of the quark momentum. This kind of treatment has been applied to model $T$-even nucleon distribution functions [20], and recently in the calculations of $f_{1 T}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ and $h_{1}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ of the proton [10] in order to eliminate
the divergences in the $k_{\perp}$-moments of these $k_{\perp}-$ dependent distribution functions.

In the previous paper [18] we performed a computation on $h_{1 \pi}^{\perp}$ in case 1 which yields:
$h_{1 \pi}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)=\frac{A_{\pi}}{\mathbf{k}_{\perp}^{2}\left(\mathbf{k}_{\perp}^{2}+B_{\pi}\right)} \ln \left(\frac{\mathbf{k}_{\perp}^{2}+B_{\pi}}{B_{\pi}}\right)$,
and the corresponding unpolarized distribution is
$f_{1 \pi}\left(x, \mathbf{k}_{\perp}^{2}\right)=C_{\pi} \frac{\mathbf{k}_{\perp}^{2}+D_{\pi}}{\left(\mathbf{k}_{\perp}^{2}+B_{\pi}\right)^{2}}$,
where
$A_{\pi}=\frac{g_{\pi}^{2}}{2(2 \pi)^{3}} \frac{\left|e_{1} e_{2}\right|}{4 \pi} m M_{\pi}(1-x)$,
$B_{\pi}=m^{2}-x(1-x) M_{\pi}^{2}$,
$C_{\pi}=(1-x) g_{\pi}^{2} /\left[2(2 \pi)^{3}\right]$,
$D_{\pi}=(1+x)^{2} m^{2}$.
An interesting result is that the transverse momentum dependence of $h_{1 \pi}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ in this model is the same as that of $h_{1}^{\perp}$ of the proton in the quark-scalar diquark model [17].

Now we perform the computation of $h_{1 \pi}^{\perp}$ in case 2, that is, in the situation of $g_{\pi}$ as a dipole form factor. According to Eq. (7), also with the help of Fig. 1 and the Feynman rules introduced above, we can calculate $h_{1 \pi}^{\perp}$ from the integral:

$$
\begin{align*}
& \frac{2 h_{1 \pi}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right) \mathbf{k}_{\perp}^{i}}{M_{\pi}} \\
& =\frac{i\left|e_{1} e_{2}\right|}{8(2 \pi)^{3}(1-x) P_{\pi}^{+}} \\
& \quad \times \sum_{s} \int \frac{d^{4} q}{(2 \pi)^{4}} \bar{v}^{s} g_{\pi}\left(k^{2}\right) \gamma_{5} \frac{\not k+m}{k^{2}-m^{2}} \sigma^{i+} \\
& \quad \times \frac{\not k+q+m}{(k+q)^{2}-m^{2}} g_{\pi}\left((k+q)^{2}\right) \gamma_{5} \\
& \quad \times \frac{\not k+q-\not p_{\pi}+m}{\left(k+q-P_{\pi}\right)^{2}-m^{2}+i \varepsilon} \\
& \quad \times \gamma^{+} v^{s} \frac{1}{q^{+}+i \varepsilon} \frac{1}{q^{2}-i \varepsilon}+\text { h.c. } \tag{15}
\end{align*}
$$

In above equation we have used $\left\langle P_{\pi}\right| \bar{\psi}(0)\left|\bar{q}^{s}\right\rangle=$ $\bar{v}^{s} g_{\pi}\left(k^{2}\right) \gamma_{5} i(k+m) /\left(k^{2}-m^{2}\right)$, etc., which is a result of the spectator model [20]. The $\gamma^{+}$in the last
line of Eq. (15) comes from the contraction of the eikonal line-gluon vertex and the gluon-antiquark vertex: $\bar{n}^{\mu} g_{\mu \nu} \gamma^{\nu}=\gamma^{+}$. The loop integral over the gluon momentum $q$ is similar to the integral for calculating $f_{1 T}^{\perp}$ in [17,21]. The $q^{-}$integral is realized from contour method, and $q^{+}$integral can be done by taking the imaginal part of the eikonal propagator: $1 /\left(q^{+}+i \varepsilon\right) \rightarrow-i \pi \delta\left(q^{+}\right)$, since the real part of the propagator is canceled by the Hermitian conjugate term. After performing the integral we yield $h_{1 \pi}^{\perp}$ with a form different from Eq. (11):
$h_{1 \pi}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)=\frac{\left|e_{1} e_{2}\right|}{4 \pi} \frac{N_{\pi}^{2}(1-x)^{3} m M_{\pi}}{2(2 \pi)^{3} L_{\pi}^{2}\left(\mathbf{k}_{\perp}^{2}+L_{\pi}^{2}\right)^{3}}$.
The corresponding unpolarized distribution is

$$
\begin{equation*}
f_{1 \pi}\left(x, \mathbf{k}_{\perp}^{2}\right)=\frac{N_{\pi}^{2}(1-x)^{3}\left(\mathbf{k}_{\perp}^{2}+D_{\pi}\right)}{2(2 \pi)^{3}\left(\mathbf{k}_{\perp}^{2}+L_{\pi}^{2}\right)^{4}} . \tag{17}
\end{equation*}
$$

To calculate the trace in the nominator of Eq. (15) we take the spin sum of the antiquark state as $\sum_{s} v^{s} \bar{v}^{s}=\left(\not 中_{\pi}-\nmid \nmid-m\right)$, which is a little different from the spin sum adopted in Ref. [20]. One can find that the form of Eq. (16) is similar to $h_{1}^{\perp}$ of the proton computed in Ref. [10]. We also calculate $\bar{h}_{1 \pi}^{\perp}$, the $T$-odd distribution of the valence antiquark inside the pion, and yield $\bar{h}_{1 \pi}^{\perp}=h_{1 \pi}^{\perp}$. Comparing the two versions of $h_{1 \pi}^{\perp}$ in Eq. (16) and Eq. (11), we find that each one has a significant magnitude, which means both of them can give unsuppressed $\cos 2 \phi$ asymmetry. However, the transverse momentum dependence of the two versions are very different, that is to say, the $Q_{T}$ behavior of the $\cos 2 \phi$ asymmetry predicted by the two cases should be different. One may expect experiments to make a discrimination between the two versions of $h_{1 \pi}^{\perp}$. We will give a further comparison with available experimental data in next section.

## 3. The $\cos 2 \phi$ asymmetry in the unpolarized $\pi^{-} N$ Drell-Yan process

The unpolarized Drell-Yan process cross section has been measured in muon pair production by pionnucleon collision: $\pi^{-} N \rightarrow \mu^{+} \mu^{-} X$, with $N$ denoting a nucleon in deuterium or tungsten and a $\pi^{-}$beam with energy of $140,194,286 \mathrm{GeV}$ [15] and 252 GeV [16]. The general form of the angular differential cross


Fig. 2. Angular definitions of the unpolarized Drell-Yan process in the lepton pair center of mass frame.
section for the unpolarized Drell-Yan process is

$$
\begin{align*}
\frac{1}{\sigma} \frac{d \sigma}{d \Omega}= & \frac{3}{4 \pi} \frac{1}{\lambda+3}\left(1+\lambda \cos ^{2} \theta+\mu \sin 2 \theta \cos \phi\right. \\
& \left.+\frac{v}{2} \sin ^{2} \theta \cos 2 \phi\right) \tag{18}
\end{align*}
$$

where $\phi$ is the angle between the lepton plane and the plane of the incident hadrons in the lepton pair center of mass frame (see Fig. 2). The definition of the lepton plane depends on the choice of axes $\hat{z}$ in the lepton pair center of mass system. In our calculation we choose $\hat{z}$ parallel to the bisector of $\overrightarrow{\mathbf{P}}_{\pi}$ and $-\overrightarrow{\mathbf{P}}_{N}$, which is referred to as Collins-Soper frame [23]. The experimental data show large value of $v$ near to $30 \%$ in the Collins-Soper frame. The asymmetry predicted by perturbative QCD is expected to be small $[15,24]$. Several theoretical approaches have been suggested to interpret the experimental data, such as hightwist effect [25,26] and factorization breaking mechanism [24]. A natural explanation has been proposed by Boer [14] that the product of two $T$-odd chiral-odd $h_{1}^{\perp}$ can give $\cos 2 \phi$ asymmetry without suppression by the momentum of the lepton pair. In that paper, a parametrization of $h_{1}^{\perp}$ in a similar form of Collins fragmentation function [5] has been given to fit the experiment data. Recently it is found that non-zero $h_{1}^{\perp}$ can arise from final-state interaction without violation of time-reversal invariance, and has been used to estimate $\cos 2 \phi$ asymmetry in the unpolarized $p \bar{p} \rightarrow l \bar{l} X$ Drell-Yan process [17].

Encouraged by this proposal, we apply $h_{1 \pi}^{\perp}$ given by our model calculation to estimate the consequent $\cos 2 \phi$ asymmetry in the unpolarized $\pi^{-} N$ Drell-Yan process measured by NA10 Collaboration. In case the vector boson that produces the lepton pair is a virtual photon, the leading order unpolarized Drell-Yan
cross section expressed in the Collins-Soper frame is [14]

$$
\begin{align*}
& \frac{d \sigma\left(h_{1} h_{2} \rightarrow l \bar{l} X\right)}{d \Omega d x_{1} d x_{2} d^{2} \mathbf{q}_{\perp}} \\
& =\frac{\alpha_{e m}^{2}}{3 Q^{2}} \sum_{a}\left\{A(y) \mathcal{F}\left[f_{1}^{a} \bar{f}_{1}^{\bar{a}}\right]+B(y) \cos 2 \phi \mathcal{F}\right. \\
& \left.\quad \times\left[\left(2 \hat{\mathbf{h}} \cdot \mathbf{p}_{\perp} \hat{\mathbf{h}} \cdot \mathbf{k}_{\perp}-\mathbf{p}_{\perp} \cdot \mathbf{k}_{\perp}\right) \frac{h_{1}^{\perp, a} \bar{h}_{1}^{\perp, \bar{a}}}{M_{1} M_{2}}\right]\right\} \tag{19}
\end{align*}
$$

where $Q^{2}=q^{2}$ is the invariance mass square of the lepton pair, $\mathbf{q}_{\perp}$ is the transverse momentum of the pair, and the vector $\hat{\mathbf{h}}=\mathbf{q}_{\perp} / Q_{T}$. We have used the notation

$$
\begin{align*}
\mathcal{F}\left[f_{1} \bar{f}_{1}\right]= & \int d^{2} \mathbf{p}_{\perp} d^{2} \mathbf{k}_{\perp} \delta^{2}\left(\mathbf{p}_{\perp}+\mathbf{k}_{\perp}-\mathbf{q}_{\perp}\right) \\
& \times f_{1}\left(x, \mathbf{p}_{\perp}^{2}\right) \bar{f}_{1}\left(\bar{x}, \mathbf{k}_{\perp}^{2}\right) . \tag{20}
\end{align*}
$$

From Eq. (19) one can give the expression for the asymmetry coefficient $v$ [14]:

$$
\begin{gather*}
\nu=2 \sum_{a} e_{a}^{2} \mathcal{F}\left[\left(2 \hat{\mathbf{h}} \cdot \mathbf{p}_{\perp} \hat{\mathbf{h}} \cdot \mathbf{k}_{\perp}-\mathbf{p}_{\perp} \cdot \mathbf{k}_{\perp}\right) \frac{h_{1}^{\perp, a} \bar{h}_{1}^{\perp, \bar{a}}}{M_{1} M_{2}}\right] \\
\times\left(\sum_{a} e_{a}^{2} \mathcal{F}\left[f_{1}^{a} \bar{f}_{1}^{\bar{a}}\right]\right)^{-1} \tag{21}
\end{gather*}
$$

The $\cos 2 \phi$ dependence as observed by the NA10 Collaboration does not show a strong dependence on $A$ [15], i.e., the asymmetry is unlikely associated with nuclear effect. The leading contribution which comes from the valence quarks is $\bar{h}_{1 \pi}^{\perp, \bar{u}} \times h_{1}^{\perp, u}$, therefore we can adopt the $u$-quark dominance, i.e., we do not include sea quark contribution which is expected to be small. We use $h_{1 \pi}^{\perp}$ given in Eq. (16) (case 2) to estimate the asymmetry. We also need $h_{1}^{\perp}$ of the nucleon in a similar treatment with effective coupling as a dipole form factor. This has been done in Ref. [10]. We use this version $h_{1}^{\perp}$ but only include contribution from the scalar diquark ( $h_{1}^{\perp, u}=h_{1}^{\perp, S}$, with $S$ denoting the scalar diquark). There are two considerations: the first is to reduce the number of the parameters, and the second is because of the $u$-quark dominance assumption. Based on Eq. (21) with the denominator from the same model result, we give the numerical estimation of the asymmetry at $\bar{x}=x=0.5 \mathrm{in}$ Fig. 3 (shown by the
solid curve) with experiment data from NA10 Collaboration. For the parameters in the expressions of $h_{1 \pi}^{\perp}$ and $h_{1}^{\perp}$ we choose: $\Lambda=0.6 \mathrm{GeV}, M_{\pi}=0.137 \mathrm{GeV}$, $m=0.1 \mathrm{GeV}, M_{N}=0.94 \mathrm{GeV}, \lambda_{S}=0.8 \mathrm{GeV}$, and $m_{N}=0.3 \mathrm{GeV}$, where $M_{N}, \lambda_{S}$ and $m_{N}$ are the nucleon mass, the scalar diquark mass and the mass of the quark inside the nucleon, respectively. For the coupling constant $\left|e_{1} e_{2}\right| / 4 \pi$ we extrapolate $\left|e_{1} e_{2}\right| / 4 \pi \rightarrow$ $C_{F} \alpha_{s}$, and take $\alpha_{s}=0.3$ and $C_{F}=4 / 3$ which are adopted in Ref. [1]. We choose the data at 194 GeV of Ref. [15], since the error bars of them are smallest


Fig. 3. The $\cos 2 \phi$ asymmetry (solid line) in the unpolarized $\pi^{-} N$ Drell-Yan process defined in the Collins-Soper frame at $\bar{x}=x=0.5$. The dashed line represents asymmetry given by the previous model result [18] (with $h_{1 \pi}^{\perp}$ calculated in case 1 ). The data are taken from Ref. [15] at 194 GeV .
(the error in $Q_{T}$ is chosen to be the bin size). We find that the estimated asymmetry agrees with the experiment data fairly well, although our estimation is crude since some approximations are adopted. In contrast, as shown by the dashed line in Fig. 3, the $Q_{T}$ shape of the asymmetry produced by $h_{1 \pi}^{\perp}$ denoted in Eq. (11) (case 1 with the pion-quark-antiquark coupling as a constant) is not consistent with experimental data (see Ref. [18]).

We further use $h_{1 \pi}^{\perp}$ given in Eq. (16) and the same parameters adopted above to estimate the asymmetries averaged over the kinematics of NA10 experiments. For $Q_{T}^{2} \ll Q^{2}$, the momentum fractions of the quarks inside the pion and the nucleon satisfy the relation: $\bar{x} x=Q^{2} / s$, where $\sqrt{s}$ is the center of mass energy of the pion-nucleon system, for instance, for 194 GeV beam $\sqrt{s}=\sqrt{(194+0.94)^{2}-194^{2}}=$ 19.1 GeV , and for $140 \mathrm{GeV}, 286 \mathrm{GeV}$ beam $\sqrt{s}=$ $16.2 \mathrm{GeV}, 23.2 \mathrm{GeV}$ respectively [15]. The dataselecting condition of the NA10 experiments is: $\bar{x}<$ $0.7,4.0 \mathrm{GeV} \leqslant Q \leqslant 8.5 \mathrm{GeV}(4.05 \mathrm{GeV} \leqslant Q \leqslant$ 8.5 GeV for the 194 GeV data) and $Q \geqslant 11 \mathrm{GeV}$. We use above kinematical constrains to evaluate the averaged asymmetries for $\pi^{-}$beam with different energy. In Fig. 4 we plot the asymmetries in the Collins-Soper frame versus $Q_{T}$ for 140,194 and 286 GeV beam together with the data of Ref. [15]. The estimated asymmetries for pion beam with different energy are still consistent with experimental data. Our estimation shows that the energy dependence of the asymmetries is not strong, and this agrees with the experimental observation.


Fig. 4. The $\cos 2 \phi$ asymmetries in the Collins-Soper frame for pion beam with different energy averaged over kinematics region: $\bar{x}<0.7$, $4.0 \mathrm{GeV} \leqslant Q \leqslant 8.5 \mathrm{GeV}(4.05 \mathrm{GeV} \leqslant Q \leqslant 8.5 \mathrm{GeV}$ for the 194 GeV data) and $Q \geqslant 11 \mathrm{GeV}$. The data are taken from Ref. [15].

## 4. Summary

The observed large $\cos 2 \phi$ azimuthal asymmetry in the unpolarized Drell-Yan process indicates a substantial non-zero value for leading twist $T$-odd distribution function $h_{1}^{\perp}\left(x, \mathbf{k}_{\perp}^{2}\right)$ (which is referred to as Boer-Mulders function) from phenomenological aspects. Theoretically it has been demonstrated that $h_{1}^{\perp}$ of the nucleon and the meson can arise from finalor initial-state interaction. In this connection, we have performed a calculation of $h_{1 \pi}^{\perp}$ of the pion in a simple antiquark spectator model by taking into account final-state interaction, and estimated the consequent $\cos 2 \phi$ azimuthal asymmetry in the unpolarized $\pi^{-} N$ Drell-Yan process which is then compared with experimental data measured by NA10 Collaboration. In the calculation we adopt the pion-quark-antiquark effective coupling as a dipole form factor. We find that the resulting $h_{1 \pi}^{\perp}$, together with $h_{1}^{\perp}$ of the nucleon resulting from a similar treatment with nucleon-quarkdiquark coupling as a dipole form factor, can give a good agreement of the estimated $\cos 2 \phi$ azimuthal asymmetry with experimental data from NA10 Collaboration. This provides a new indication on the role of $T$-odd distribution $h_{1}^{\perp}$ to the $\cos 2 \phi$ asymmetry in the unpolarized Drell-Yan process from initial-state interaction.

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