Physics Letters B 693 (2010) 36–43

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Inclusive decays of η_b into S- and P-wave charmonium states

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ARTICLE INFO

Article history:
Received 10 December 2009
Received in revised form 30 July 2010
Accepted 6 August 2010
Available online 11 August 2010
Editor: B. Grinstein

Keywords:
Color-octet
Heavy quarkonium
Decay
Production

ABSTRACT

Inclusive S- and P-wave charmonium productions in the bottomonium ground state η_b decay are calculated at the leading order in the strong coupling constant α_s and quarkonium internal relative velocity v in the framework of the NRQCD factorization approach. We find the contribution of $\eta_b \to \chi_{cJ} + gg$ followed by $\chi_{cJ} \to J/\psi + \gamma$ is also very important to inclusive J/ψ production in the η_b decays, which maybe helpful to the investigation of the color-octet mechanism in the inclusive J/ψ production in the η_b decays in the forthcoming LHCb and SuperB. As a complementary work, we also study the inclusive production of η_c, and χ_{cJ} in the η_b decays, which may help us understand the $X(3940)$ and $X(3872)$ states.

1. Introduction

The existence of the spin-singlet state η_b, which is the ground state of $b\bar{b}$ system, is a solid prediction of the non-relativistic quark model. Since the discovery of its spin-triplet partner Υ, people have make great efforts to search for it in various experimental environments, such as in e^+e^- collisions at CLEO [1], in $\gamma\gamma$ collisions at LEP II [2] and in $p\bar{p}$ collisions at Tevatron [3]. Unfortunately, no evident signal was seen in these attempts. Recently, a significant progress has been achieved by the BaBar Collaboration. After analyzing about 10^8 data samples, they observed η_b in the photon spectrum of $\Upsilon(3S) \to \gamma\eta_b$ [4] with a signal of 10σ significance. They found the hyperfine $\Upsilon(1S)$–η_b mass splitting is $71.4^{+2.3}_{-3.1}(\text{stat}) \pm 2.7(\text{syst})$ MeV. Soon after, another group in BaBar observed that $\Upsilon(2S) \to \gamma\eta_b$ [5], and they determined the mass splitting to be $67.4^{+4.8}_{-4.6}(\text{stat}) \pm 2.0(\text{syst})$ MeV. The η_b state has also been observed by the CLEO Collaboration in $\Upsilon(3S)$ radiative decay, and their measurement of the hyperfine mass splitting is $68.5 \pm 6.6 \pm 2.0$ MeV [6].

On the theoretical side, many works have been done to study its properties. The mass of η_b has been predicted with potential model [7], effective theory [8] and Lattice QCD [9]. Furthermore, the recent determinations of $\Upsilon(1S)$–η_b mass splitting in the range [illegible] η_b have also been [illegible] by about an order of magnitude. In Ref. [15], the authors calculated the production rates of η_b at Tevatron Run II and suggested detecting it through the decay of $\eta_b \to J/\psi J/\psi$, while the authors in Ref. [16] thought that the double J/ψ channel might be overestimated and suggested that the $\eta_b \to D^*D^{(*)}$ channel is the most promising channels. An explicit calculation of $\eta_b \to J/\psi J/\psi$ at NLO in v^2 [17] and NLO in α_s [18] shows that this branching fraction is on the order of 10^{-8}, which is about four orders of magnitude smaller than that given in Ref. [15]. Furthermore, the author in Ref. [19] argued the effect of final state interactions in $\eta_b \to D\bar{D}^* \to J/\psi J/\psi$ was also important. Some other exclusive decay modes, such as $\eta_b \to \gamma J/\psi$ [20,21] and η_b decays into double charmonia [22] and inclusive decays, e.g. $\eta_b \to c\bar{c}c\bar{c}$ [16] and $\eta_b \to J/\psi + X$ [23] have also been taken into account.

However, compared to the $c\bar{c}\,{}^1S_0$ state η_c, our knowledge about η_b is quite limited and further work is necessary. In this Letter, we will systemically study the inclusive decays of η_b into S- and P-wave charmonium states. The motivations of this work are fourfold.

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0370-2693
doi:10.1016/j.physletb.2010.08.013

First, in these processes, the typical energy scale m_b in the initial state and m_c in the final state are both much larger than the QCD scale Λ_{QCD},[1] [24], so we can calculate the decay widths perturbatively and the non-perturbative effect plays a minor role, which will reduce the theoretical uncertainties. Second, the branching fraction of the inclusive decay process is much larger than that of the exclusive process, which makes testing the theoretical prediction for the inclusive process more feasible. Third, in Ref. [23], the authors calculated the branching ratio of $\eta_b \to J/\psi + X$ and found that the contribution of the color-octet process $\eta_b \to c\bar{c}(^3S_1^{[8]}) + g$ is larger than that of the color-singlet process by about an order of magnitude. Because the color-octet process can also contribute to P-wave states χ_{cJ} production, in which the χ_{c1} and χ_{c2} have about 36% and 20% branching ratio to $J/\psi + \gamma$, respectively, we expect that the contribution of $\eta_b \to \chi_{cJ} + X$ process followed by $\chi_{cJ} \to J/\psi + \gamma$ might also be important for inclusive J/ψ production in η_b decay. Fourth, in recent years, many charmonium or charmonium-like states have been found at B-factories (see Refs. [25–27] for a review). In the forthcoming LHCb and Super-B, with enough data, it might be possible to observe the interesting decay of η_b to $X(3940)$ or $X(3872)$, etc.

The J/ψ inclusive production has already been studied in Ref. [23], and the $J/\psi(\eta_c, \chi_{cJ})$ production in association with $c\bar{c}$ pair has been discussed in our previous work [28]. Here we are going to consider the contribution of the $\eta_b \to \eta_c(\chi_{cJ}) + gg$ process in the non-relativistic limit at leading order in α_s.

2. NRQCD factorization formalism

Due to the non-relativistic nature of $b\bar{b}$ and $c\bar{c}$ systems, we adopt the non-relativistic QCD (NRQCD) effective theory [29] to calculate the inclusive decay widths of η_b to charmonium states. In NRQCD, the inclusive decay and production of heavy quarkonium are factorized into the product of the short distance coefficient and the corresponding long distance matrix element. The short distance coefficient can be calculated perturbatively through the expansion of the QCD coupling constant α_s. The non-perturbative matrix element, which describes the possibility of the $Q\bar{Q}$ pair transforming into the bound state, is weighted by the relative velocity v_Q of the heavy quarks in the heavy meson rest frame.

In the framework of NRQCD, at leading order in v_b and v_c, for the S-wave heavy quarkonium production and decay, only the $Q\bar{Q}$ pair in color-singlet contributes. For P-wave χ_{cJ} production, the color-singlet P-wave matrix elements and color-octet S-wave matrix element are both in the same order of v_c. Then, the factorization formulas for the processes considered in this work are given by:

$$\Gamma(\eta_b \to \eta_c + gg) = \hat{\Gamma}\big(b\bar{b}\big(^1S_0^{[1]}\big) \to c\bar{c}\big(^1S_0^{[1]}\big) + X\big)\langle\eta_b|\mathcal{O}_b\big(^1S_0^{[1]}\big)|\eta_b\rangle\big\langle\mathcal{O}_c^{\eta_c}\big(^1S_0^{[1]}\big)\big\rangle, \tag{1a}$$

$$\begin{aligned}\Gamma(\eta_b \to \chi_{cJ} + X) = {}& \hat{\Gamma}_1\big(b\bar{b}\big(^1S_0^{[1]}\big) \to c\bar{c}\big(^3P_J^{[1]}\big) + X\big)\langle\eta_b|\mathcal{O}_b\big(^1S_0^{[1]}\big)|\eta_b\rangle\big\langle\mathcal{O}_c^{\chi_{cJ}}\big(^3P_J^{[1]}\big)\big\rangle \\ & + \hat{\Gamma}_8\big(b\bar{b}\big(^1S_0\big) \to c\bar{c}\big(^3S_1^{[8]}\big) + X\big)\langle\eta_b|\mathcal{O}_b\big(^1S_0^{[1]}\big)|\eta_b\rangle\big\langle\mathcal{O}_c^{\chi_{cJ}}\big(^3S_1^{[8]}\big)\big\rangle,\end{aligned} \tag{1b}$$

where the $\hat{\Gamma}$s are the short-distance factors and $\langle\eta_b|\mathcal{O}_b(^1S_0^{[1]})|\eta_b\rangle$, $\langle\mathcal{O}_c^{\eta_c}(^1S_0^{[1]})\rangle$, $\langle\mathcal{O}_c^{\chi_{cJ}}(^3P_J^{[1]})\rangle$ and $\langle\mathcal{O}_c^{\chi_{cJ}}(^3S_1^{[8]})\rangle$ are the long-distance matrix elements. During our calculation of the short distance coefficients associated with the P-wave color-singlet matrix elements, the infrared divergence will appear. This divergence will be absorbed into the color-octet matrix element $\langle\mathcal{O}_c^{\chi_{cJ}}(^3S_1^{[8]})\rangle$.

3. $\eta_b \to \eta_c + gg$

We first consider the S-wave η_c production from η_b decay. At leading order in α_s, there are eight Feynman diagrams for $b\bar{b}(^1S_0^{[1]}) \to c\bar{c}(^1S_0^{[1]}) + gg$. A typical diagram is shown in Fig. 1a. The general form of the short distance coefficient can be expressed as:

$$\hat{\Gamma}\big(b\bar{b}\big(^1S_0^{[1]}\big) \to c\bar{c}\big(^1S_0^{[1]}\big) + gg\big) = \frac{\alpha_s^4}{m_b^5} f(r), \tag{2}$$

where $r = m_c/m_b$ is a dimensionless parameter. Because there is no infrared divergence, we calculate $f(r)$ directly using the standard covariant projection technique [30]. Given $m_b = 4.65$ GeV and $m_c = 1.5$ GeV, we get $f(r) = 23.1$. In Table 1, we also list the numerical results of $f(r)$ for different choices of r. The lower and upper boundaries of r are obtained by keeping m_b constant and setting $m_c = 1.3$ GeV and $m_c = 1.8$ GeV, respectively. In NRQCD, up to the v^4 order, the relations between the color-singlet matrix elements and the non-relativistic wave functions are[2]:

$$\langle\eta_b|\mathcal{O}_b\big(^1S_0^{[1]}\big)|\eta_b\rangle = \frac{1}{4\pi}\big|R_{1S}^b(0)\big|^2\big(1 + \mathcal{O}\big(v_b^4\big)\big), \qquad \big\langle\mathcal{O}_c^{\eta_c}\big(^1S_0^{[1]}\big)\big\rangle = \frac{1}{4\pi}\big|R_{1S}^c(0)\big|^2\big(1 + \mathcal{O}\big(v_c^4\big)\big). \tag{3}$$

To compare with our previous work, we choose the same numerical values of $m_b = 4.65$ GeV, $m_c = 1.5$ GeV, $\alpha_s = 0.22$, $|R_{1S}^c(0)|^2 = 0.81\ \text{GeV}^3$, and $|R_{1S}^b(0)|^2 = 6.477\ \text{GeV}^3$ [31]. Then we get

$$\Gamma(\eta_b \to \eta_c + gg) = 0.83\ \text{keV}. \tag{4}$$

The total width of η_b is estimated by using the two gluon decay, which at leading order in α_s and v_b is read to be:

$$\Gamma_{\text{Total}} \approx \Gamma(\eta_b \to gg) = \frac{2\alpha_s^2}{3m_b^2}\big|R_{1S}^b(0)\big|^2 = 9.67\ \text{MeV}. \tag{5}$$

[1] Strictly, the assumption of $m_c \gg \Lambda_{QCD}$ is only reasonably good.

[2] For the color-singlet four-fermion operators, there is an additional $\frac{1}{2N_c}$ factor compared to those in Ref. [29].

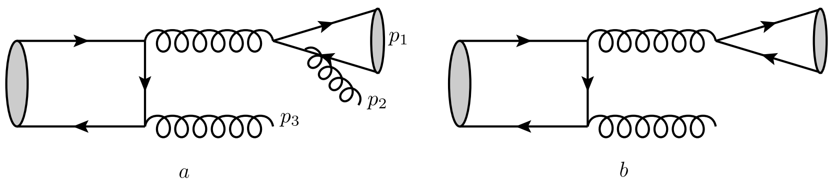

Fig. 1. Typical Feynman diagrams for the short distance process: (a) $b\bar{b}[^1S_0,1] \to c\bar{c}[^1S_0^{[1]}(^3P_J^{[1]})] + gg$; and (b) $b\bar{b}[^1S_0,1] \to c\bar{c}[^3S_1^{[8]}] + g$.

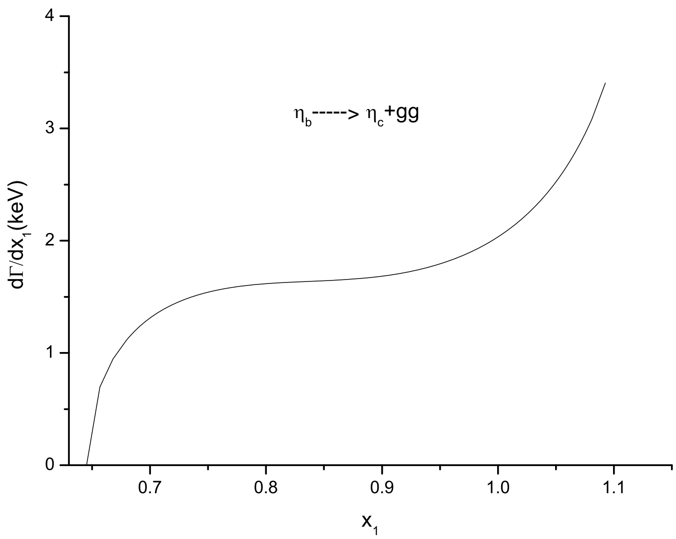

Fig. 2. The scaled energy distribution of η_c for the $\eta_b \to \eta_c + gg$ process.

Table 1
The values of $f(r)$ for $\eta_b \to \eta_c + gg$ with different inputs of $r = \frac{m_c}{m_b}$.

r	0.280	0.301	0.323	0.344	0.366	0.387
$f(r)$	39.1	29.9	23.1	18.0	14.1	11.1

In our previous work, we obtained $\Gamma(\eta_b \to \eta_c + c\bar{c}) \approx 0.27$ keV [28]. Thus, the branching ratio of inclusive decay of η_b into η_c is

$$\mathrm{Br}(\eta_b \to \eta_c + X) \approx 1.1 \times 10^{-4}, \tag{6}$$

in which the contribution of gg process is about 3 times larger than that of the $c\bar{c}$ process. The re-scaled energy distribution curve $d\Gamma/dx_1$ for $\eta_b \to \eta_c + X$ is shown in Fig. 2, where x_1 is the ratio of η_c energy E_{η_c} to m_b.

Recently the $X(3940)$ state was observed by the Belle Collaboration in the recoiling spectrum of J/ψ in e^+e^- annihilation [32]. It is most likely to be a $\eta_c(3S)$ state [33]. In the non-relativistic limit, the only difference between η_c and $\eta_c(3S)$ is the value of wave function. If $X(3940)$ is the $\eta_c(3S)$ state, we predict the branching ratio of $X(3940)$ production in η_b decay to be

$$\mathrm{Br}\big(\eta_b \to X(3940) + X\big) \simeq 0.62 \times 10^{-4}. \tag{7}$$

To obtain the prediction, we have chosen $|R^c_{3S}(0)|^2 = 0.455\ \mathrm{GeV}^3$ [31] to take the place of $|R^c_{1S}(0)|^2 = 0.81\ \mathrm{GeV}^3$.

4. $\eta_b \to \chi_{cJ} + gg$

As mentioned above, the color-singlet short distance coefficients are infrared divergent in the full QCD calculation. We adopt the dimensional regularization scheme to regularize the divergence. To absorb the divergence into the color-octet matrix elements $\langle \mathcal{O}_c^{\chi_{cJ}}(^3S_1^{[8]})\rangle$, it is necessary to calculate the color-octet short distance coefficient in $D = 4 - 2\epsilon$ dimensions. The $b\bar{b}(^1S_0^{[1]}) \to c\bar{c}(^3S_1^{[8]}) + g$ process includes two Feynman diagrams, one of which is shown in Fig. 1b. Using the D dimension spin projector expression [34], at leading order in α_s, the short distance factor is given by

$$\hat{\Gamma}\big(b\bar{b}(^1S_0) \to c\bar{c}(^3S_1^{[8]}) + g\big) = \frac{(4\pi\alpha_s)^3\mu^{6\epsilon}}{24 m_b^5 r^3}\Phi_2\frac{(D-2)(D-3)}{(D-1)}, \tag{8}$$

where

$$\Phi_2 = \left(\frac{\pi}{m_b^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)(1-r^2)}{8\pi\Gamma(2-2\epsilon)} \tag{9}$$

is the 2-body phase space in D dimensions.

The calculation of the color-singlet coefficient in full QCD is a little more complicated. The Feynman diagrams for $b\bar{b}({}^1S_0^{[1]})(P) \to c\bar{c}({}^3P_J^{[1]})(p_1) + g(p_2)g(p_3)$ are the same as those for the η_c production process. Such $1 \to 3$ processes can be described by the following invariants:

$$x_i = \frac{2P \cdot p_i}{M^2}, \qquad \sum_i x_i = 2, \tag{10}$$

where $M = 2m_b$. In $D = 4 - 2\epsilon$ dimensions, the three-body phase space is given by

$$d\Phi_{(3)} = K\big((a_1 + a_2 - x_2)(x_2 + a_1 - a_2)\big)^{-\epsilon}\big(1 + r^2 - x_1\big)^{-\epsilon}\delta(2 - x_1 - x_2 - x_3)\,dx_1\,dx_2\,dx_3, \tag{11}$$

where

$$r = \frac{m_c}{m_b}, \qquad a_1 = \sqrt{x_1^2 - 4r^2}\big/2, \qquad a_2 = (2 - x_1)/2, \qquad K = \frac{\pi^{2\epsilon} m_b^{2-4\epsilon}}{32\pi^3 \Gamma(2 - 2\epsilon)}. \tag{12}$$

The parton level process $b\bar{b}({}^1S_0^{[1]}) \to c\bar{c}({}^3P_J^{[1]}) + gg$ includes eight Feynman diagrams; a typical diagram is shown in Fig. 1a. The eight diagrams can be divided into two groups according to which gluon, p_2 or p_3, is attached to the charm quark line. The total amplitude of the four diagrams with p_2 gluon on the charm quark line, like the diagram in Fig. 1a, is denoted by M_2, and the total amplitude of the four diagrams with p_3 gluon on the charm quark line is denoted by M_3 similarly. Then, the total amplitude $M = M_2 + M_3$ and $|M|^2 = |M_2|^2 + |M_3|^2 + 2\,\mathrm{Re}(M_2^* M_3)$.

As illustrated in Ref. [29], for the P-wave case when p_i $(i = 2, 3)$ goes to zero, there will be singularities in M_i. However, due to the four-momentum conservation, p_2 and p_3 cannot be soft simultaneously in the phase space. Therefore, the integration of the interference term $2\,\mathrm{Re}(M_2^* M_3)$ is finite. We could perform it in 4 dimensions directly. Because of the symmetry of the two gluons, the phase space integration results for $|M_2|^2$ and $|M_3|^2$ are equal to each other, and we only need to calculate one of them. The total $\hat{\Gamma}_1$ could then be written as

$$\hat{\Gamma}_1 = 2\hat{\Gamma}_{M_2} + \hat{\Gamma}_{\mathrm{Int}}, \tag{13}$$

where $\hat{\Gamma}_{M_2}$ and $\hat{\Gamma}_{\mathrm{Int}}$ are the contributions related to $|M_2|^2$ and $2\,\mathrm{Re}(M_2^* M_3)$, respectively.

We now present how we calculate $\hat{\Gamma}_{M_2}$ in detail. The denominator of the charm-quark propagator in Fig. 1a is

$$(p_2 - p_{\bar{c}})^2 - m_c^2 = -2p_2 \cdot p_{\bar{c}}|_{q_c=0} \propto \big(1 + r^2 - x_1 - x_2\big), \tag{14}$$

where $p_{\bar{c}} = \frac{p_1}{2} - q_c$ is the momentum of the anti-charm quark and q_c is the relative momentum of c and $\bar{c}$. When $c\bar{c}$ is in P-wave configuration, we need to know the first derivative of the amplitude with respect to q_c. Then in the non-relativistic limit, three kinds of the divergences in $|M_2|^2$ exist that are proportional to

$$\frac{x_2^{n-2}}{(1 + r^2 - x_1 - x_2)^n} \quad (n = 2, 3, 4). \tag{15}$$

These terms, diverging at point $(x_1, x_2) = (1 + r^2, 0)$, are not easily to be integrated out. We introduce two new variables (x_1', x_2'), defined by

$$x_1' = x_1,\, x_2' = 1 - \frac{1 + r^2 - x_1}{x_2}. \tag{16}$$

In the variables x_1' and x_2', the phase space is re-expressed as:

$$d\Phi_{(3)} = \frac{\pi^{2\epsilon} m_b^{2-4\epsilon}}{32\pi^3 \Gamma(2 - 2\epsilon)} \int_{2r}^{1+r^2} dx_1' \int_{1-(a_2'+a_1')}^{1-(a_2'-a_1')} \frac{dx_2'}{(1 - x_2')^2} \big(1 + r^2 - x_1'\big)^{1-2\epsilon} \left(\big(a_1' + a_2' - \bar{x}\big)\left(\frac{1}{1 - x_2'} - \frac{1}{a_1' + a_2'}\right)\right)^{-\epsilon}, \tag{17}$$

where $a_1' = \frac{\sqrt{x_1'^2 - 4r^2}}{2}$, $a_2' = \frac{(2 - x_1')}{2}$ and $\bar{x} = \frac{1 + r^2 - x_1'}{1 - x_2'}$. And the three divergence structures are changed to be

$$\frac{1}{x_2'^n} \frac{(1 - x_2')^2}{(1 + r^2 - x_1')^2} \quad (n = 2, 3, 4) \tag{18}$$

respectively, which are all proportional to $\frac{1}{(1 + r^2 - x_1')^2}$. Then, $|M_2|^2$ could be expanded as

$$|M_2|^2 = \frac{f_1(1 + r^2, x_2', \epsilon)}{(1 + r^2 - x_1')^2} + f_2\big(x_1', x_2', \epsilon\big). \tag{19}$$

Accordingly,

$$\hat{\Gamma}_{M_2} = \hat{\Gamma}_{M_2}^{\mathrm{div}} + \hat{\Gamma}_{M_2}^{\mathrm{fin}}, \tag{20}$$

where $\hat{\Gamma}^{\rm fin}_{M_2}$ is finite and can be calculated in $D=4$ dimensions. The phase space integration of the first term in Eq. (19) is expressed as

$$\int d\Phi_{(3)} \frac{f_1(1+r^2, x_2', \epsilon)}{(1+r^2-x_1')^2} = K \int_{2r}^{1+r^2} \frac{dx_1'\, g(x_1', \epsilon)}{(1+r^2-x_1')^{1+2\epsilon}}, \tag{21}$$

where

$$g(x_1', \epsilon) = \int_{1-(a_1'+a_2')}^{1-(a_2'-a_1')} \frac{f_1(1+r^2, x_2', \epsilon)}{(1-x_2')^2} \left((a_1'+a_2'-\bar{x}) \left(\frac{1}{1-x_2'} - \frac{1}{a_1'+a_2'} \right) \right)^{-\epsilon} dx_2'. \tag{22}$$

Furthermore, the integrals in Eq. (21) can be written as the sum of two terms defined by:

$$\int_{2r}^{1+r^2} \frac{dx_1'\, g(x_1', \epsilon)}{(1+r^2-x_1')^{1+2\epsilon}} \equiv \int_{2r}^{1+r^2} \frac{dx_1'\, g(1+r^2, \epsilon)}{(1+r^2-x_1')^{1+2\epsilon}} + \int_{2r}^{1+r^2} \frac{dx_1'\, (g(x_1', \epsilon) - g(1+r^2, \epsilon))}{(1+r^2-x_1')^{1+2\epsilon}}. \tag{23}$$

The first term on the right side includes $\frac{1}{\epsilon}$ pole, and the second term is finite. Therefore, we only need to keep the $\mathcal{O}(\epsilon)$ contribution when calculating $g(1+r^2, \epsilon)$, and the second term can be evaluated directly by setting $\epsilon = 0$.

Putting Eqs. (11) and (16) together, we get

$$\hat{\Gamma}_1 = 2\left(\hat{\Gamma}^{\rm div}_{M_2} + \hat{\Gamma}^{\rm fin}_{M_2}\right) + \hat{\Gamma}_{\rm Int}. \tag{24}$$

$\hat{\Gamma}^{\rm div}_{M_2}$ is calculated analytically, and $\hat{\Gamma}^{\rm fin}_{M_2}$ and $\hat{\Gamma}_{\rm Int}$ are calculated numerically. For $J=0,1,2$, the divergence parts in $\hat{\Gamma}^{\rm div}_{M_2}$ are the same, which will be absorbed into the color-octet matrix element. Then, the results of $\hat{\Gamma}^{\rm div}_{M_2}$ for different J are given by

$$\hat{\Gamma}^{\rm div}_{M_2} = \frac{128(-1+r^2) C_A C_F (\alpha_s \pi \mu^{2\epsilon})^4 K}{81 m_b^9 r^5 \epsilon} + B_J, \tag{25}$$

where

$$B_0 = \frac{64 C_A C_F \pi^4 \alpha_s^4 K (4 - 4r^6 + 24(1 - r^2(3 - 3r^2 + r^4)) \log(1-r^2) + 12(1+r^6)\log(r))}{243 m_b^9 r^5 (-1+r^2)^2}, \tag{26a}$$

$$B_1 = \frac{128 C_A C_F \pi^4 \alpha_s^4 K (2 - 9r^2 + 9r^4 - 2r^6 + 3(2 - 3r^2 - 3r^4 + 2r^6)\log(r) + 12(1 - 3r^2 + 3r^4 - r^6)\log(1-r^2))}{243 m_b^9 r^5 (-1+r^2)^2}, \tag{26b}$$

$$B_2 = \frac{128 C_A C_F \pi^4 \alpha_s^4 K (10 - 27r^2 + 27r^4 - 10r^6 + 3(10 - 9r^2 - 9r^4 + 10r^6)\log(r) - 60(-1+r^2)^3 \log(1-r^2))}{1215 m_b^9 r^5 (-1+r^2)^2}. \tag{26c}$$

The $C_A = 3$ and $C_F = 4/3$ in the above equations are the color factors. And the finite part $2\hat{\Gamma}^{\rm fin}_{M_2} + \hat{\Gamma}_{\rm Int}$ can be expressed by

$$2\hat{\Gamma}^{\rm fin}_{M_2} + \hat{\Gamma}_{\rm Int} = \frac{\alpha_s^4}{m_b^7} A_J(r) \quad (\text{for } J = 0, 1, 2). \tag{27}$$

When $r = 1.5/4.65$, we obtain $A_0(r) \simeq -9.71 \times 10^2$, $A_1(r) \simeq -2.66 \times 10^2$ and $A_2(r) \simeq -6.06 \times 10^2$. The results of $A_J(r)$ for r varying from 0.323 to 0.376 are shown in Fig. 3. The lower and upper boundaries of r are obtained by choosing $m_c = \frac{m_{J/\psi}}{2} \simeq 1.5$ GeV and 1.75 GeV, which is approximately the c.o.g. (center of gravity) mass of χ_{cJ} states, respectively and fixing $m_b = \frac{m_{\eta_b}}{2} \simeq 4.65$ GeV.

To cancel the infrared divergence of $\hat{\Gamma}^{\rm div}_{M_2}$, we also need to take into account the renormalization of $\langle \mathcal{O}_c^{\chi_{cJ}}({}^3S_1^{[8]}) \rangle$. In the $\overline{MS}$ scheme, it is given by [29,34]

$$\left\langle \mathcal{O}_c^{\chi_{cJ}}\left({}^3S_1^{[8]}\right)\right\rangle^{(\Lambda)} = \left\langle \mathcal{O}_c^{\chi_{cJ}}\left({}^3S_1^{[8]}\right)\right\rangle^{\rm (Born)} - \frac{4\alpha_s C_F}{3\pi m_c^2} \left(\frac{1}{\epsilon} + \log 4\pi - \gamma_E \right) \left(\frac{\mu}{\mu_\Lambda} \right)^{2\epsilon} \sum_{J=0}^{2} \left\langle \mathcal{O}_c^{\chi_{cJ}}\left({}^3P_J^{[1]}\right)\right\rangle. \tag{28}$$

Combining the results of Eqs. (1b), (8), (25), (26), (27), (28), we finally obtain the infrared-safe expressions for inclusive decay of η_b into χ_{cJ} $(J = 0, 1, 2)$ states

$$\Gamma(\eta_b \to \chi_{cJ} + X) = \Gamma_8^J + \Gamma_1^J, \tag{29}$$

where Γ_8^J is

$$\frac{2\pi^2 \alpha_s^3 (1-r^2)}{9 m_b^5 r^3} \langle \eta_b | \mathcal{O}_b\left({}^1S_0^{[1]}\right) | \eta_b \rangle \left\langle \mathcal{O}_c^{\chi_{cJ}}\left({}^3S_1^{[8]}\right)\right\rangle, \tag{30}$$

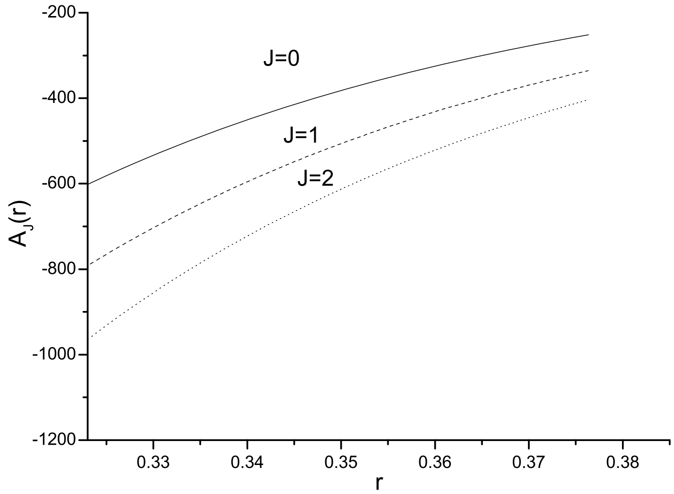

Fig. 3. The results of $A_J(r)$ defined in Eq. (23) as function of $r=\frac{m_c}{m_b}$. The solid line is for $J=0$ case; the dashed line is 3 times $A_1(r)$ and the dotted line is for $J=2$ case.

and Γ_1^J are

$$\Gamma_1^0=\frac{8\pi\alpha_s^4\langle\eta_b|\mathcal{O}_b(^1S_0^{[1]})|\eta_b\rangle\langle\mathcal{O}_c^{\chi_{cJ}}(^3P_0^{[1]})\rangle}{243m_b^7r^5(1-r^2)^2}\biggl(12(r^6+1)\log(r)+24(1-3r^2+3r^4-r^6)\log(1-r^2)$$
$$+2(1-r^2)\biggl((6\log 2-5)r^4-4(3\log 2-4)r^2+6\log 2-5+6(1-r^2)^2\log\biggl(\frac{m_b}{\mu_\Lambda}\biggr)\biggr)+\frac{243r^5(1-r^2)^2A_0(r)}{8\pi}\biggr),\tag{31a}$$

$$\Gamma_1^1=\frac{16\pi\alpha_s^4\langle\eta_b|\mathcal{O}_b(^1S_0^{[1]})|\eta_b\rangle\langle\mathcal{O}_c^{\chi_{cJ}}(^3P_1^{[1]})\rangle}{243m_b^7r^5(1-r^2)^2}\biggl(3(2r^6-3r^4-3r^2+2)\log(r)+12(1-r^2)^3\log(1-r^2)$$
$$+(1-r^2)\biggl((6\log 2-5)r^4+(7-12\log 2)r^2+6\log 2-5+6(1-r^2)^2\log\biggl(\frac{m_b}{\mu_\Lambda}\biggr)\biggr)+\frac{243r^5(1-r^2)^2A_1(r)}{16\pi}\biggr),\tag{31b}$$

$$\Gamma_1^2=\frac{16\pi\alpha_s^4\langle\eta_b|\mathcal{O}_b(^1S_0^{[1]})|\eta_b\rangle\langle\mathcal{O}_c^{\chi_{cJ}}(^3P_2^{[1]})\rangle}{1215m_b^7r^5(1-r^2)^2}\biggl(3(10r^6-9r^4-9r^2+10)\log r+60(1-r^2)^3\log(1-r^2)$$
$$+(1-r^2)\biggl(5(6\log 2-5)r^4+(53-60\log 2)r^2+5(6\log 2-5)+30(1-r^2)^2\log\biggl(\frac{m_b}{\mu_\Lambda}\biggr)\biggr)$$
$$+\frac{1215r^5(1-r^2)^2A_2(r)}{16\pi}\biggr).\tag{31c}$$

It can be seen that the contribution of the P-wave color-singlet depends on the factorization scale μ_Λ. When combined with the color-octet S-wave contribution, in which the matrix element also depends on μ_Λ, the μ_Λ-dependence will be canceled.

To give numerical predictions, we also need to know the values of the long-distance matrix elements. The color-octet matrix elements can be studied in lattice simulations, fitted to experimental data phenomenologically or determined through some other non-perturbative methods. Here, we determined their numerical values with the help of operator evolution equations. In the decay process, the solution of the operator evolution equations is [29]:

$$\langle\chi_{cJ}|\mathcal{O}_8(^3S_1;\mu_\Lambda)|\chi_{cJ}\rangle=\langle\chi_{cJ}|\mathcal{O}_8(^3S_1;\mu_{\Lambda_0})|\chi_{cJ}\rangle+\frac{8C_F}{3\beta_0m_c^2}\ln\frac{\alpha_s(\mu_{\Lambda_0})}{\alpha_s(\mu_\Lambda)}\langle\chi_{cJ}|\mathcal{O}_1(^3P_J)|\chi_{cJ}\rangle,\tag{32}$$

where $\beta_0=\frac{11N_c-2N_f}{6}$. We then naively relate the matrix element of production operator $\mathcal{O}_n^H$ to that of the decay operator $\mathcal{O}_n$ using

$$\langle\mathcal{O}_n^H\rangle\approx(2J+1)\langle H|\mathcal{O}_n|H\rangle.\tag{33}$$

When $\mu_\Lambda\gg\mu_{\Lambda_0}$, the evolution term will be dominant, and the contribution of the initial matrix elements can be neglected. Since the operator evolution hold only down to the energy scale of m_cv_c order, we set the lower bound $\mu_{\Lambda_0}=m_cv_c$ and choose $v_c^2=0.3$. Moreover, we set $\mu_\Lambda=2m_c$ because the divergence comes from the soft gluons linked with the $c\bar{c}$ pair. If we use the two-loop β function to evolve $\alpha_s(\mu)$ and choose $m_c=1.5$ GeV, we find the ratio $\frac{\langle\chi_{cJ}|\mathcal{O}_8(^3S_1;2m_c)|\chi_{cJ}\rangle}{\langle\chi_{cJ}|\mathcal{O}_1(^3P_J)|\chi_{cJ}\rangle}=0.39\ \mathrm{GeV}^{-2}$, which is consistent with the lattice result [35] and the result obtained by fitting experimental data [36]. The P-wave color-singlet matrix elements can be estimated by relating them to the first derivative of the non-relativistic wave function at the origin, which, in non-relativistic limit, is given by

$$\langle\mathcal{O}_c^{\chi_{cJ}}(^3P_J^{[1]})\rangle\approx\frac{3(2J+1)}{4\pi}|R_c'(0)|^2.\tag{34}$$

Setting $N_f=3$, $\Lambda_{QCD}=390$ MeV and $|R_c'(0)|^2=0.075\ \mathrm{GeV}^5$ [31], we obtain

$$\Gamma(\eta_b \to \chi_{cJ} + gg) = (0.17, 1.55, 1.76)\ \text{keV} \quad (\text{for } J = 0, 1, 2). \tag{35}$$

The $\eta_b \to \chi_{cJ} + c\bar{c}$ processes have been considered in our previous work; both the color-singlet and color-octet contributions were included but with different values of the color-octet matrix elements [28]. If we use the color-octet matrix elements determined in this work, the decay widths of $\eta_b \to \chi_{cJ} + c\bar{c}$ are $\Gamma(\eta_b \to \chi_{cJ} + c\bar{c}) = (4.54, 4.21, 4.28) \times 10^{-2}$ keV (for $J = 0, 1, 2$), which are about an order of magnitude less than the widths of $\eta_b \to \chi_{cJ} + gg$ processes, respectively. Including the contribution of the associate processes, we then predict that the branching ratios for η_b inclusive decay into χ_{cJ} are

$$\text{Br}(\eta_b \to \chi_{cJ} + X) = (0.22, 1.65, 1.87) \times 10^{-4} \quad (\text{for } J = 0, 1, 2). \tag{36}$$

The $X(3872)$ state was discovered in $p\bar{p}$ collisions at Tevatron [37] and B decay at Belle [38]. Until now, a convincing explanation has not been proposed yet. The authors in [39] suggest that it is a $\chi_{c1}(2P)$ state. If it is a $\chi_{c1}(2P)$ state, we roughly predict

$$\text{Br}\big(\eta_b \to X(3872) + X\big) = 2.25 \times 10^{-4}, \tag{37}$$

where we have chosen $|R_c'(0)|^2 = 0.102\ \text{GeV}^5$ for the $2P$ state and assumed that the ratio between color-singlet and color-octet matrix elements does not change for the $2P$ state.

$$\frac{\langle \mathcal{O}_c^{\chi_{c1}}(^3S_1^{[8]})\rangle}{\langle \mathcal{O}_c^{\chi_{c1}}(^3P_1^{[1]})\rangle} = \frac{\langle \mathcal{O}_c^{X(3872)}(^3S_1^{[8]})\rangle}{\langle \mathcal{O}_c^{X(3872)}(^3P_1^{[1]})\rangle}. \tag{38}$$

The χ_{cJ} states can also decay into $J/\psi + \gamma$ with $\text{Br}(\chi_{c1} \to J/\psi + \gamma) = 36\%$ and $\text{Br}(\chi_{c2} \to J/\psi + \gamma) = 20\%$ [40], then we find the contribution of χ_{cJ} feed-down to J/ψ production in η_b decay is

$$\text{Br}\big(\eta_b \to (J/\psi + \gamma)_{\chi_{cJ}} + X\big) = 0.97 \times 10^{-4}. \tag{39}$$

Here, we have neglected the feed-down contribution of χ_{c0}, because the branching ratio of $\chi_{c0} \to J/\psi + \gamma$ is very small. If we set $m_c = 1.5 \pm 0.1$ GeV and keep the other parameters unchanged, the branching ratio becomes $0.97^{-0.30}_{+0.46} \times 10^{-4}$. In the process of J/ψ production in Υ decay, the energy scale of α_s is proposed to be $2m_c$ instead of m_b [41]. If we make the same choice, where $\alpha_s(2m_c) = 0.249^{-0.05}_{+0.07}$ for $m_c = 1.5 \pm 0.1$ GeV, the branching ratio becomes $0.91^{-0.28}_{+0.43} \times 10^{-4}$. It can be seen that the dependence of our prediction on the energy scale of α_s is not strong. This is because we estimate both the total and partial widths theoretically, therefore, their ratio reduces the dependence on α_s. Our numerical prediction also depends on the inputs of the long distance matrix elements. In this work, we use the potential model result calculated with the B–T type potential in Ref. [31] for the color-singlet long distance matrix elements.

In [23], the authors studied the $\eta_b \to J/\psi + X$ process with $\Gamma(\eta_b \to J/\psi + X) = 2.29$ keV. They found the contribution of the color-octet process $\eta_b \to J/\psi_{\text{color-octet}} + X$ is more than one order of magnitude larger than that of the color-singlet contribution. If we choose the same values for the parameters as those in Ref. [23], we find the χ_{cJ} feed-down contribution to the decay of η_b into J/ψ is:

$$\Gamma\big(\eta_b \to (J/\psi + \gamma)_{\chi_{cJ}} + X\big) = 0.71\ \text{keV}, \tag{40}$$

which is about three times larger than that of the color-singlet process. Therefore, we conclude that in future experiments, when measuring the J/ψ production in η_b decay, the contribution of η_b decays into χ_{cJ} followed by $\chi_{cJ} \to J/\psi + \gamma$ is also important.

5. Summary

In this Letter, we have studied the inclusive production of charmonium state η_c, χ_{cJ} in the decay of ground bottomonium state η_b within the framework of NRQCD factorization formula. We find for the P-wave states χ_{cJ} case, the color-singlet processes $b\bar{b}(^1S_0^{[1]}) \to c\bar{c}(^3P_J^{[1]}) + gg$ include infrared divergence. We show that such divergence can be absorbed into the S-wave color-octet matrix element. To give numerical predictions, we use the potential model results to determine the color-singlet matrix elements and estimate the color-singlet matrix elements with the help of operator evolution equations naively. We find that the branching ratios of η_b decay into η_c or χ_{cJ} plus anything are all on the order of 10^{-4}. Furthermore, we give the branching ratios of $\eta_b \to X(3940) + X$ and $\eta_b \to X(3872) + X$, if the $X(3940)$ and $X(3872)$ are the excited $\eta_c(3S)$ and $\chi_{c1}(2P)$ states respectively. In Ref. [23], the authors investigated the color-octet mechanism for J/ψ production in η_b decay. Our results show that the J/ψ production from χ_{cJ} feed-down is also important, because it is about three times larger than the direct J/ψ production via color-singlet channel. These theoretical predictions may not be observed in experiment for the time being, but they are very helpful when studying η_b's properties in future experiments, such as LHCb and Super-B.

Acknowledgements

We would like to thank Yu Jia for helpful discussions. The author Zhi-Guo He also thanks the organization of the “Effective Field Theories in Particle and Nuclear Physics” by KITPC Beijing. The author Zhi-Guo He is currently supported by the CPAN08-PD14 contract of the CSD2007-00042 Consolider-Ingenio 2010 program, and by the FPA2007-66665-C02-01/ project (Spain).

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