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Non-commutative power-law inflation: mode equation and spectra index

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Abstract

Following an elegant approach that merges the effects of the stringy spacetime uncertainty relation into primordial perturbations suggested by Brandenberger and Ho, we show the mode equation up to the first order of non-commutative parameter. A new approximation is provided to calculate the mode functions analytically in the non-commutative power-law inflation models. It turns out that non-commutativity of spacetime can provide small corrections to the power spectrum of primordial fluctuations as the first-year results of WMAP indicate. Moreover, using the WMAP data, we obtain the value of expansion parameter, non-commutative parameter and find the approximation is viable. In addition, we determined the string scale $l_s \simeq 2.0 \times 10^{-29}$ cm.

The cosmological parameters and the properties of inflationary models are tightly constraint by the recent result from Wilkinson Microwave Anisotropy Probe (WMAP) [1], Sloan Digital Sky Survey (SDSS) and Two degree Field (2dF) galaxy clustering analyses [2], and from the latest SNIa data [3]. The standard inflationary ΛCDM model provides a good fit to the observed cosmic microwave background (CMB) anisotropies. The first-year results of WMAP also bring us something intriguing. Some analyses [4–7] show that the new data of CMB suggest an anomalously low quadrupole and octupole and a larger running of the spectral index of the power spectrum than that predicted by standard single scalar field inflation models satisfying the slow-roll conditions.

On the other hand, it is well known that during the period of inflation, the classical gravitational theory, general relativity, might break down due to the very high energies at that time and the correction from string theory may take effect. In the non-perturbative string/M theory, any physical process at the very short distance takes an uncertainty relation, called stringy spacetime uncertainty relation (SSUR),

$$\Delta t_p \Delta x_p \geqslant l_s^2, \tag{1}$$

where t_p and x_p are the physical time and space, l_s is the string length scale. It is suggested that the SSUR is

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a universal property for strings as well as D-branes [8]. Unfortunately, we now have no ideas to derive cosmology directly from string/M theory. Brandenberger and Ho [9] have proposed a variation of spacetime non-commutative field theory to realize the stringy spacetime uncertainty relation without breaking any of the global symmetries of the homogeneous isotropic universe. If the inflation is affected by physics at a scale close to string scale, one expects that spacetime uncertainty must leave vestiges in the CMB power spectrum [10–12]. It is found that, in the non-commutative inflation context, IR modes are created on scales larger than the Hubble radius and thus are not as squeezed as they would be in the commutative case. Cai [13] shows that the choice of initial vacuum has a significant effect on the power spectrum of density fluctuation in a non-commutative spacetime. Following Ref. [9], the scalar fluctuations of tachyon inflation were discussed in non-commutative spacetime [14].

While the standard model is observationally well justified, successful non-commutative models predict that there should be observable deviations from it. Undoubtedly, we should expect that the effect of the non-commutativity of the spacetime may only provide a small correction to the prediction of the standard model.

The primary observational test of inflation is observation of CMB. Temperature fluctuations in the CMB are related to perturbations in the metric at the surface at last scattering. During the inflationary epoch, metric perturbations are created by field fluctuation, and quantum fluctuations on small scales are rapidly redshifted to scales much larger than the Hubble radius. The metric perturbations can be decomposed according to their spin with respect to a local rotation of the spatial coordinates on hypersurfaces of constant time. This leads to two types: scalar perturbations which couple to the stress-energy of matter in the universe and form "seeds" for structure formation, and tensor perturbations which do not couple to matter.

In this Letter, we show the equation of mode functions up to the first order of non-commutative parameter λ beyond the slow-roll approximation. When the string scale $l_s \to 0$, mode equation can be reduced to one in ordinary commutative spacetime. The mode equation of non-commutative inflation is complicated, and computing the power spectrum will in general require numerical evaluation. However, the spectrum can be evaluated analytically in power-law inflation. There are corrections to the primordial power spectrum which arise in the non-commutative power-law models. These corrections lead to a blue tilt ($n_s > 1$) for small wavenumber and a red one ($n_s < 1$) for large wavenumber which accords with the first-year result of WMAP [1].

Following the scenario proposed by Brandenberger and Ho [9], the model incorporating the SSUR can be written as

$$S = V \int_{k<k_0} d\tilde{\tau}\, d^3k\, z_k^2(\tilde{\tau})\big(\zeta'_{-k}\zeta'_k - k^2\zeta_{-k}\zeta_k\big), \tag{2}$$

where V denotes the total spatial coordinate volume and the primes represent derivatives with respect to the time variable $\tilde{\tau}$, which is related to the conformal time τ via

$$d\tilde{\tau} = \left(\frac{a}{a_{\rm eff}}\right)^2 d\tau, \tag{3}$$

where a is the scale factor, and $a_{\rm eff}$ is defined as

$$a_{\rm eff} \equiv \left(\frac{\beta_k^+}{\beta_k^-}\right)^{1/4}. \tag{4}$$

Here, $\beta_k^\pm$ are determined by

$$\beta_k^\pm = \frac{1}{2}\big[a^{\pm 2}\big(\hat{\tau} + kl_s^2\big) + a^{\pm 2}\big(\hat{\tau} - kl_s^2\big)\big], \tag{5}$$

in which the new time variable $\hat{\tau}$ is defined as $d\hat{\tau} = a^2\, d\tau$. z_k in Eq. (2) is some smeared version of the "Mukhanov variable" z over a range of time of characteristic scale $\Delta\tau = l_s^2 k$,

$$z_k = \big(\beta_k^+\beta_k^-\big)^{1/4} z = \frac{a\dot{\phi}}{H}\big(\beta_k^+\beta_k^-\big)^{1/4}, \tag{6}$$

where H and ϕ are Hubble rate and inflaton field, respectively, and overdot denotes derivative with respect to cosmic time t.

From the action (2), the equation of motion of the scalar perturbations mode equation can be written as

$$u_k'' + \left(k^2 - \frac{z_k''}{z_k}\right)u_k = 0, \tag{7}$$

where the mode function is defined by $u_k = z_k\zeta_k$.

Apparently, if the string length scale l_s goes to zero, the action (2) will reduce to the action for the fluctuations in the classical spacetime, which leads to the equation of motion of perturbations

$$\frac{d^2u_k}{d\tau^2} + \left(k^2 - \frac{1}{z}\frac{d^2z}{d\tau^2}\right)u_k = 0, \tag{8}$$

where u_k now is reduced to $z\zeta_k$. Using the slow-roll parameters

$$\epsilon = \frac{M_{\rm pl}^2}{4\pi}\left(\frac{H'(\phi)}{H(\phi)}\right)^2, \tag{9}$$

$$\eta = \frac{M_{\rm pl}^2}{4\pi}\frac{H''(\phi)}{H(\phi)}, \tag{10}$$

$$\xi = \frac{M_{\rm pl}^2}{4\pi}\left(\frac{H'(\phi)H'''(\phi)}{H^2(\phi)}\right)^{1/2}, \tag{11}$$

the expression for $\frac{1}{z}\frac{d^2z}{d\tau^2}$ can be written as [15]

$$\frac{1}{z}\frac{d^2z}{d\tau^2} = 2(aH)^2 \times \left(1 + \epsilon - \frac{3}{2}\eta + \epsilon^2 - 2\epsilon\eta + \frac{1}{2}\eta^2 + \frac{1}{2}\xi^2\right). \tag{12}$$

The non-local coupling in time between the background and the fluctuation is manifested in Eq. (5). As mentioned above, we assume that the effect of SSUR only provides a small correction to the prediction of standard scenario that produce the primordial fluctuation. This is equivalent to suppose that $kl_s^2 \ll |\hat{\tau}|$ in Eq. (5). This condition is crucial because SSUR takes effects only via $\beta_k^{\pm}$ and it will be showed that the computations in the following are all based on this assumption. In order to calculate the non-commutative power spectrum correctly, we introduce a non-commutative parameter λ as

$$\lambda(k,t) \equiv \frac{H^2k^2}{a^2M_s^4}, \tag{13}$$

where k is the comoving wavenumber of a perturbation mode, and $M_s = l_s^{-1}$ is the string mass scale. There exists a great difference between the slow-roll parameters ϵ, η and the non-commutative parameter λ. According to the definition, slow-roll parameters do not involve a which increases rapidly during inflation. Note that λ contains scale factor a, which is in contrast to the slow-roll parameters. We note the general picture of fluctuations during inflation: for a given fluctuation whose initial wavelength $\sim a/k$ is within the Hubble radius, it oscillates till the wavelength becomes of the order of the Hubble radius scale; when the wavelength crosses the Hubble radius, the fluctuation ceases to oscillate and gets frozen in. After a prolix but straightforward calculation, we obtain

$$\frac{z_k''}{z_k} = \frac{1}{z}\frac{d^2z}{d\tau^2}\left[1 - 2(1+\epsilon)\lambda\right] + 2a^2H^2\lambda\left[3\epsilon + \eta + 3\epsilon\eta + \epsilon^2 + \epsilon\eta(\epsilon - \eta)\right], \tag{14}$$

up to the first order of λ, where $\frac{1}{z}\frac{d^2z}{d\tau^2}$ is defined in Eq. (12). Clearly, when $l_s \to 0$ or $M_s \to \infty$, the quantity z_k''/z_k and $\tilde{\tau}$ will be reduced to $\frac{1}{z}\frac{d^2z}{d\tau^2}$ and τ, respectively, and then the mode equation (7) in non-commutative spacetime will recover the one in ordinary commutative spacetime (8).

Brandenberger and Ho [9] have shown that, for each mode k of fluctuation, there is a critical time $\tilde{\tau}_0$ at which the spacetime uncertainty relation is saturated, and k and $\tilde{\tau}_0$ have the relation

$$k = \frac{a_{\rm eff}(\tilde{\tau}_0)}{l_s}. \tag{15}$$

$\tilde{\tau}_0$ is also the time when the mode is generated. Before the critical time $\tilde{\tau}_0$, the fluctuations do not contain the mode k.

Let us now consider power law inflation models where the scale factor can be given by $a(\tau) = l_0|\tau|^{1+\beta}$ where β is a number such that $\beta \leqslant -2$ and the coefficient l_0 has the dimension of a length. In order for slow-roll parameter ϵ to be a little number, we may assume β is close to -2. In the limit case $\beta = -2$, which corresponds to exponential expansion, the length l_0 is nothing but the Hubble radius, $l_0 = l_H$. Supposing that $l_s \ll l_0$, according to Eq. (15), we obtain that $-k\tilde{\tau}_0 \approx l_0/l_s \gg 1$ provided that β is close to -2, which means that the mode k is generated on scales inside the Hubble radius in the local vacuum state.

In power law models, the slow-roll parameters can all be determined exactly,

$$\epsilon = \eta = \xi = \frac{2+\beta}{1+\beta}, \tag{16}$$

which is a virtue of this class of models, and the mode equation (7) thus is reduced to

$$\frac{d^2u_k}{d\tilde{\tau}^2}+\left[k^2-\beta(1+\beta)\frac{1}{\tau^2}+4\alpha k^2(1+\beta)^2\left(5+5\beta+\beta^2\right)\times\frac{1}{|\tau|^{8+4\beta}}\right]u_k=0, \tag{17}$$

where the parameter $\alpha\equiv(l_s/l_0)^4$, and the relation between conformal time τ and $\tilde{\tau}$ can be rewritten as

$$\tilde{\tau}=\tau+\alpha k^2\frac{(1+\beta)(3+2\beta)}{(5+4\beta)}\frac{1}{|\tau|^{5+4\beta}}. \tag{18}$$

In principle, we can solve Eq. (18), then insert the solution $\tau(\tilde{\tau})$ into Eq. (17) and finally obtain the solution $u_k(\tilde{\tau})$ of Eq. (17). However, this procedure is too complicated to implement directly in practice. The situation here are in many ways equivalent to the modified dispersion relations considered by Brandenberger and Martin in Ref. [16]. Note that Eq. (17) is a linear equation and we have assumed $\alpha\ll 1$. Therefore, we can use perturbation method to solve Eqs. (17) and (18). For this purpose, let

$$\tau=\tau^{(0)}+\alpha\tau^{(1)}+\alpha^2\tau^{(2)}+\cdots, \tag{19}$$

$$u_k=u_k^{(0)}+\alpha u_k^{(1)}+\alpha^2u_k^{(2)}+\cdots. \tag{20}$$

Inserting them into Eqs. (17) and (18), we obtain that

$$\tau=\tilde{\tau}-\alpha k^2\frac{(1+\beta)(3+2\beta)}{(5+4\beta)}\frac{1}{|\tilde{\tau}|^{5+4\beta}}+O\left(\alpha^2\right), \tag{21}$$

and then

$$u_k''^{(0)}+\left[k^2-\frac{\beta(1+\beta)}{\tilde{\tau}^2}\right]u_k^{(0)}=0, \tag{22}$$

$$u_k''^{(1)}+\left[k^2-\frac{\beta(1+\beta)}{\tilde{\tau}^2}\right]u_k^{(1)}=h(\tilde{\tau}), \tag{23}$$

where

$$h(\tilde{\tau})=\frac{2k^2(1+\beta)^2}{|\tilde{\tau}|^{8+4\beta}}\times\left[\frac{\beta(3+2\beta)}{(5+4\beta)}-2\left(5+5\beta+\beta^2\right)\right]u_k^{(0)}. \tag{24}$$

If the non-commutative spacetime effects are ignored, the mode function u_k is reduced to $u_k^{(0)}$ which obeys Eq. (22). It is easy to find that Eq. (22) is nothing but Eq. (8) with τ replaced by $\tilde{\tau}$. Thus, if we impose that in the ultraviolet regime ($-\tilde{\tau}_0>-\tilde{\tau}\gg 1/k$), the solution of Eq. (22) matches the plane-wave solution we expect in flat spacetime and obey the Wronskian condition

$$u_k^*\frac{du_k}{d\tilde{\tau}}-u_k\frac{du_k^*}{d\tilde{\tau}}=-i, \tag{25}$$

the exact solution of Eq. (22) becomes

$$u_k(\tilde{\tau})^{(0)}=\frac{\sqrt{\pi}}{2}\exp\left[i\frac{\pi}{2}\left(\nu+\frac{1}{2}\right)\right]\times(-\tilde{\tau})^{1/2}H_\nu^{(1)}(-k\tilde{\tau}), \tag{26}$$

where, the parameter $\nu=-\frac{1}{2}-\beta$. For solving Eq. (23), we can use the general methods of both homogeneous and inhomogeneous linear ordinary differential equations. The general solutions of the second-order equation can be written as

$$u_k(\tilde{\tau})^{(1)}=\varphi_2\int\frac{\varphi_1 h}{W}d\tilde{\tau}-\varphi_1\int\frac{\varphi_2 h}{W}d\tilde{\tau}, \tag{27}$$

where two linearly independent solutions of the homogeneous equation

$$\varphi_1=(-\tilde{\tau})^{1/2}H_\nu^{(1)}(-k\tilde{\tau}), \tag{28}$$

$$\varphi_2=(-\tilde{\tau})^{1/2}H_\nu^{(2)}(-k\tilde{\tau}), \tag{29}$$

and the Wronskian of the two solutions of homogeneous equation

$$W=\varphi_2\frac{d\varphi_1}{d\tilde{\tau}}-\varphi_1\frac{d\varphi_2}{d\tilde{\tau}}. \tag{30}$$

Fortunately, the integral in Eq. (27) can be explicitly integrated.

On the subhorizon scales, i.e. for $k^2\tilde{\tau}_0^2\gg k^2\tilde{\tau}^2\gg 1$, since

$$\varphi_1\sim\sqrt{\frac{2}{\pi k}}e^{-i(k\tilde{\tau}+\frac{\pi}{2}\nu+\frac{\pi}{4})},$$

$$\varphi_2\sim\sqrt{\frac{2}{\pi k}}e^{i(k\tilde{\tau}+\frac{\pi}{2}\nu+\frac{\pi}{4})}$$

and

$$u_k^{(0)}\sim\frac{1}{\sqrt{2k}}e^{-ik\tilde{\tau}},$$

inserting these expressions into Eq. (27), we obtain that

$$u_k^{(1)}\approx ik\frac{(1+\beta)^2(50+87\beta+48\beta^2+8\beta^3)}{(5+4\beta)(7+4\beta)}\times(-\tilde{\tau})^{-7-4\beta}\frac{e^{-ik\tilde{\tau}}}{\sqrt{2k}}. \tag{31}$$

We are specially interested in solution on the superhorizon scales, i.e., for $k^2\tilde{\tau}^2 \ll 1$. For these scales, since $H_\nu^{(1)}(x \ll 1) = \sqrt{\frac{2}{\pi}} e^{-i\pi/2} 2^{\nu-3/2} \frac{\Gamma(\nu)}{\Gamma(3/2)} x^{-\nu}$, we obtain that

$$u_k^{(0)} \approx e^{i(\nu-\frac{1}{2})\frac{\pi}{2}} 2^{\nu-\frac{3}{2}} \frac{\Gamma(\nu)}{\Gamma(3/2)} \frac{1}{\sqrt{2k}} (-k\tilde{\tau})^{\frac{1}{2}-\nu}, \tag{32}$$

and

$$u_k^{(1)} \approx -\frac{(1+\beta)^2(50+87\beta+48\beta^2+8\beta^3)}{\sqrt{\pi}(75+140\beta+84\beta^2+16\beta^3)} \times 2^{-\frac{3}{2}-\beta} e^{-i\frac{\pi}{2}(\beta+1)} \times \Gamma(-1/2-\beta) k^{\frac{5}{2}+\beta} (-\tilde{\tau})^{-5-3\beta}. \tag{33}$$

Therefore, we can express the power spectrum on superhorizon scales of the comoving curvature as

$$\begin{aligned} P_R(k) &= \frac{k^3}{2\pi^2} \left| \frac{u_k(\tilde{\tau}_c)}{z_k(\tilde{\tau}_c)} \right|^2 \\ &\simeq \frac{k^3}{2\pi^2} \frac{u_k^{(0)} u_k^{(0)*} + \alpha(u_k^{(0)} u_k^{(1)*} + u_k^{(0)*} u_k^{(1)})}{z_k^2} \bigg|_{\tilde{\tau}=\tilde{\tau}_c} \\ &\simeq \frac{2^{-2\beta-2}\Gamma(\nu)^2 k^{4+2\beta}}{\pi^2 M_{\rm pl}^2 l_0^2 \epsilon} \left[1 + 2\alpha f(\beta) k^2 (-\tilde{\tau}_c)^{-6-4\beta}\right], \end{aligned} \tag{34}$$

where

$$f(\beta) = -(1+\beta)^2 \times \left(1 + \frac{50+87\beta+48\beta^2+8\beta^3}{75+140\beta+84\beta^2+16\beta^3}\right), \tag{35}$$

and $\tilde{\tau}_c$ is the time when fluctuation mode k comes across the Hubble radius, (i.e., for $-\tilde{\tau}_c \approx 1/k$). Just as Lidsey et al. have pointed in Ref. [15] that, in spite of the appearance of spectrum equation, the calculated value for the spectrum is not the value at which the scale crosses outside the Hubble radius. Rather, it is the asymptotic value as $k/aH \to 0$, but rewritten in terms of the values the quantities had when the Hubble radius was crossed.

We may now compute the spectra index n_s of the scalar metric perturbation on superhorizon scales

$$n_s - 1 \equiv \frac{d\ln P_R(k)}{d\ln k} \approx 2(2+\beta)\left[1 + 4\alpha f(\beta) k^{8+4\beta}\right]. \tag{36}$$

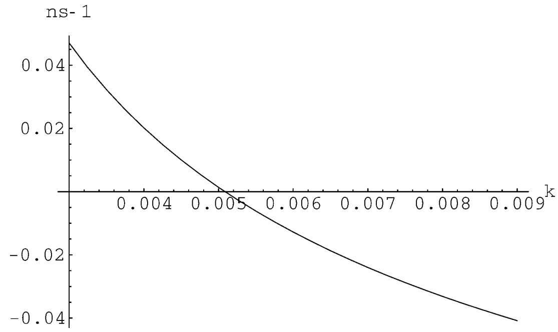

Fig. 1. The spectral index $n_s - 1$ for different wavenumber k, where the parameter $\alpha = 10^{-2}$ and $\beta = -2.1$.

The running of the spectrum index is

$$\frac{dn_s}{d\ln k} \approx 32\alpha(2+\beta)^2 f(\beta) k^{8+4\beta}. \tag{37}$$

Obviously, when the parameter $\alpha \to 0$ (i.e., $M_s \to \infty$), the contribution from the non-commutativity of spacetime to the spectral index and its running will also vanish. Note that in the vicinity of $\beta = -2$, $f(\beta)$ is negative. Thus, the spectrum has a negative spectral index for small scales and a positive one for large scales (see Fig. 1), while the running is always negative. Since the slope of the power spectrum decreases as β goes towards to -2, the more rapidly the universe is accelerating, the closer the power spectrum is to being scale-invariant. In the limit case $\beta = -2$, the results for commutative and non-commutative spacetimes converge at a completely scale-invariant spectrum.

In the non-commutative inflationary spacetime, there are corrections to the primordial power spectrum which arise in a model of power-law inflation. These corrections lead to a blue tilt ($n_s > 1$) for small wavenumber and a red one ($n_s < 1$) for large wavenumber which accords with the first-year results of WMAP [1]. The origin of the suppressions in the power spectrum of the fluctuations is that the non-commutativity of the spacetime delayed the generation of the fluctuation modes and then postponed the time when they crossing the Hubble radius. However, in the de Sitter limit, i.e., for $\epsilon = \eta = \xi = 0$, the non-commutativity of the spacetime has no influence in the spectrum, this is because no time delay can be generated in this special case. According to the analysis of the results of WMAP [1,10], for the scalar modes, the mean and the 68% error level of the 1d marginalized likelihood for the power spectrum slope $n_s = 0.93^{+0.02}_{-0.03}$, $dn_s/d\ln k = -0.031^{+0.016}_{-0.017}$ at $k = 0.05\ \mathrm{Mpc}^{-1}$ and $n_s = 1.20^{+0.12}_{-0.11}$, $dn_s/d\ln k = -0.077^{+0.050}_{-0.052}$ at $k = 0.002\ \mathrm{Mpc}^{-1}$. Using the data at $k = 0.05\ \mathrm{Mpc}^{-1}$, the parameters β and α should be constraint by $\beta \simeq -2.08$ and $\alpha \simeq 0.0186$, respectively. The parameter α is so small that ensure that our treatments, i.e., Eqs. (19) and (20), are suitable. Using the values of parameters β and α gained above, we predict that $n_s \simeq 1.11$, $dn_s/d\ln k \simeq -0.089$ at the scale of $k = 0.002\ \mathrm{Mpc}^{-1}$. This results are in good agreement with those obtained in Ref. [10]. Although the predicted central values of the spectra index and its running have small deviations from the corresponding WMAP data, but they both fall within the error bar. The differences exist due to the fact that at the large scales, the effect of higher orders in α is not completely negligible. As these higher-order effects is taken into account, the result would be improved. In addition, using the WMAP data that $P_R(k = 0.002\ \mathrm{Mpc}^{-1}) = 2.09 \times 10^{-9}$ and the parameters obtained above, we estimate the string scale $l_s \simeq 1.2 \times 10^4 l_p \simeq 2.0 \times 10^{-29}$ cm, which is also consistent with the result obtained in Refs. [11,12].

In summary, following the elegant idea that merge the effects of the stringy spacetime uncertainty relation into primordial perturbations proposed by Brandenberger and Ho, we obtain the mode equation up to the first order of non-commutative parameter. Moreover, we also provide a new analytical approximation to calculate the mode functions in the power-law inflation models. It turns out that non-commutativity of spacetime can provide small corrections to the power spectrum of primordial fluctuations and our results are consistent with the previous results and WMAP data.

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