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Incompressibility of strange matter [☆]

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Abstract

Strange stars (ReSS) calculated from a realistic equation of state (EOS), that incorporate chiral symmetry restoration as well as deconfinement at high density [Phys. Lett. B 438 (1998) 123; Phys. Lett. B 447 (1999) 352, Addendum; Phys. Lett. B 467 (1999) 303, Erratum; Indian J. Phys. B 73 (1999) 377] show compact objects in the mass radius curve. We compare our calculations of incompressibility for this EOS with that of nuclear matter. One of the nuclear matter EOS has a continuous transition to ud-matter at about five times normal density. Another nuclear matter EOS incorporates density dependent coupling constants. From a look at the consequent velocity of sound, it is found that the transition to ud-matter seems necessary.

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Keywords: Compact stars; Realistic strange stars; Dense matter; Elementary particles; Equation of state

1. Introduction

An exciting issue of modern astrophysics is the possible existence of a family of compact stars made entirely of deconfined u , d , s quark matter or “strange

matter” (SM) and thereby denominated strange stars (SS). They differ from neutron stars, where quarks are confined within neutrons, protons and eventually within other hadrons (hadronic matter stars). The possible existence of SS is a direct consequence of the so-called strange matter hypothesis [2], according to which the energy per baryon of SM would be less than the lowest energy per baryon found in nuclei, which is about 930 MeV for Fe⁵⁶. Also, the ordinary state of matter, in which quarks are confined within hadrons, is a metastable state. Of course, the hypothesis does not conflict with the existence of atomic nuclei as conglomerates of nucleons, or with the stability of ordinary matter [3–5].

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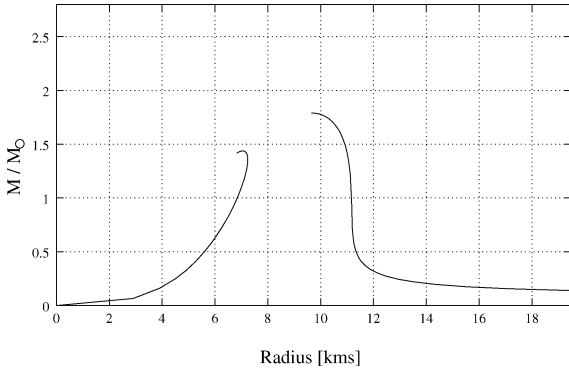


Fig. 1. Mass and radius of stable stars with the strange star EOS (left curve) and neutron star EOS (right curve), which are solutions of the Tolman–Oppenheimer–Volkoff (TOV) equations of general relativity. Note that while the self sustained strange star systems can have small masses and radii, the neutron stars have larger radii for smaller masses since they are bound by gravitation alone.

The best observational evidence for the existence of quark stars come from some compact objects, the X-ray burst sources SAX J1808.4–3658 (the SAX in short) and 4U 1728–34, the X-ray pulsar Her X-1 and the superburster 4U 1820–30. The first is the most stable pulsating X-ray source known to man as of now. This star is claimed to have a mass $M_* \sim 1.4M_\odot$ and a radius of about 7 km [6]. Coupled to this claim are the various other evidences for the existence of SS, such as the possible explanation of the two kHz quasi-periodic oscillations in 4U 1728–34 [7] and the quark-nova explanation for γ -ray bursts [8].

The expected behaviour of SS is directly opposite to that of a neutron star as Fig. 1 shows. The mass of 4U 1728–34 is claimed to be less than $1.1M_\odot$ in Li et al. [7], which places it much lower in the M–R plot and thus it could be still gaining mass and is not expected to be as stable as the SAX. So for example, there is a clear answer [9] to the question posed by Franco [10]: why are the pulsations of SAX not attenuated, as they are in 4U 1728–34?

From a basic point of view the equation of state for SM should be calculated solving QCD at finite density. As we know, such a fundamental approach is presently not feasible even if one takes recourse to the large colour philosophy of 't Hooft [11]. A way out was found by Witten [12] when he suggested that one can borrow a phenomenological potential from the meson sector and use it for baryonic matter. Therefore,

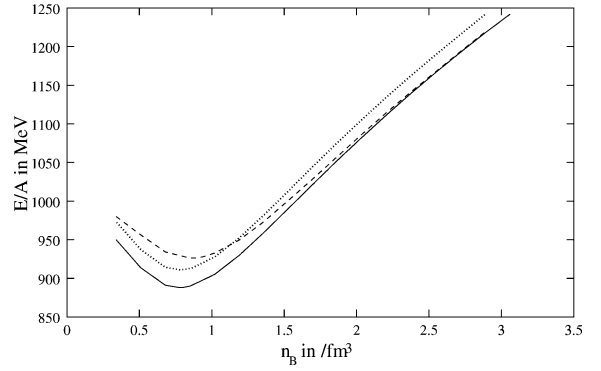


Fig. 2. Strange matter EOS employed by D98 show respective stable points. The solid line for is EOS1, the dotted line for EOS2 and the dashed line for EOS3. All have the minimum at energy per baryon less than that of Fe^{56} .

one has to rely on phenomenological models. In this work, we use different equations of state (EOS) of SM proposed by Dey et al. [1] using the phenomenological Richardson potential. Other variants are now being proposed, for example, the chromo dielectric model calculations of Malheiro et al. [13].

Fig. 2 shows the energy per baryon for the EOS of [1]. One of them (EOS1, SS1 of [6]) has a minimum at $E/A = 888.8$ MeV compared to 930.4 of Fe^{56} , i.e., as much as 40 MeV below. The other two have this minimum at 911 MeV and 926 MeV, respectively, both less than the normal density of nuclear matter. The pressure at this point is zero and this marks the surface of the star in the implementation of the well-known TOV equation. These curves clearly show that the system can fluctuate about this minimum, so that the zero pressure point can vary.

2. Incompressibility: its implication for Witten's Cosmic separation of phase scenario

In nuclear physics incompressibility is defined as [14]

$$K = 9 \frac{\partial}{\partial n} \left(n^2 \frac{\partial \varepsilon}{\partial n} \right), \quad (1)$$

where $\varepsilon = E/A$ is the energy per particle of the nuclear matter and n is the number density. The relation of K with bulk modulus B is

$$K = \frac{9B}{n}. \quad (2)$$

Table 1
Parameters for the three EOS

EOS	ν	α_s	Λ	M_Q (MeV)	M_G/M_\odot	R (km)	n_s/n_0
EOS1	0.333	0.20	100	310	1.437	7.055	4.586
EOS2	0.333	0.25	100	310	1.410	6.95	4.595
EOS3	0.286	0.20	100	310	1.325	6.5187	5.048

K has been calculated in many models. In particular, Bhaduri et al. [14] used the non-relativistic constituent quark model, as well as the bag model, to calculate K_A as a function of n for the nucleon and the delta. They found that the nucleon has an incompressibility K_N of about 1200 MeV, about six times that of nuclear matter. They also suggested that at high density K_A matches onto quark gas incompressibility.

The velocity of sound in units of light velocity c is given by

$$v = \sqrt{K/9\varepsilon}. \quad (3)$$

The simple models of quark matter considered in [1] use a Hamiltonian with an interquark potential with two parts, a scalar component (the density dependent mass term) and a vector potential originating from gluon exchanges. In the absence of an exact evaluation from QCD, this vector part is borrowed from meson phenomenology [15]. In common with the phenomenological bag model, it has built in asymptotic freedom and quark confinement (linear). In order to restore the approximate chiral symmetry of QCD at high densities, an ansatz is used for the constituent masses, viz.,

$$M(n) = M_Q \operatorname{sech}\left(\nu \frac{n}{n_0}\right), \quad (4)$$

where n_0 is normal nuclear matter density and ν is a parameter. There may be several EOS's for different choices of parameters employed to obtain the EOS. Some of them are given in the Table 1. There α_s is perturbative quark gluon coupling and Λ is the scale parameter appeared in the vector potential. Changing other parameters one can obtain more EOS's. The Table 1 also shows the masses (M_G), radii (R) and the number density (n_s) at star surface of maximum mass strange stars obtained from the corresponding EOS's. The surface of the stars occur at the minimum ε where pressure is zero.

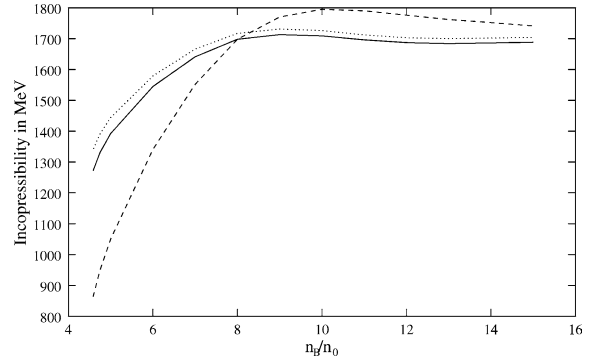


Fig. 3. Incompressibility as a function of density ratio for EOS's for different EOS's given in Table 1. The solid curve for EOS1, the dotted curve for EOS2 and the dashed curve for EOS3.

The general behaviour of the curves is relatively insensitive to the parameter ν in $M(n)$ as well as the gluon mass, as evident from the Fig. 3.

In Fig. 4, K with three values of M_Q implying different running masses, $M(n)$, is plotted as a function of the density expressed by its ratio to n_0 . Given for comparison, is the incompressibility K_q of a perturbative massless three flavour quark gas consisting of zero mass current quarks [14] using the energy expression given in [16] to order α_s^2 .

It can be seen that as M_Q decreases, the nature of the relation approaches the perturbative case of [14]. At high density our incompressibility and that due to Baym [16] matches, showing the onset of chiral symmetry restoration. In EOS1 for uds-matter, the minimum of ε occurs at about $4.586n_0$. Nucleation may occur at a density less than this value of n . This corresponds to a radius of about 0.67 fm for a baryon. For EOS1 we find K to be 1.293 GeV per quark at the star surface.

It is encouraging to see that this roughly matches with the compressibility K_N so that no 'phase expands explosively'. In the Cosmic separation of phase scenario, Witten [12] had indicated at the outset that he had assumed the process of phase transition to occur

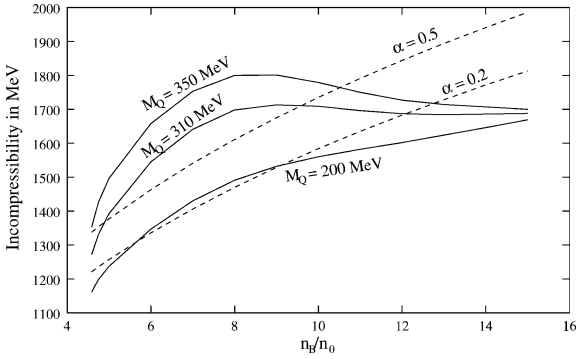


Fig. 4. Incompressibility as a function of density ratio for EOS's with different constituent mass as parameter. Dashed lines correspond to perturbative massless three flavour quark gas with different values of α_s (see [14,16]).

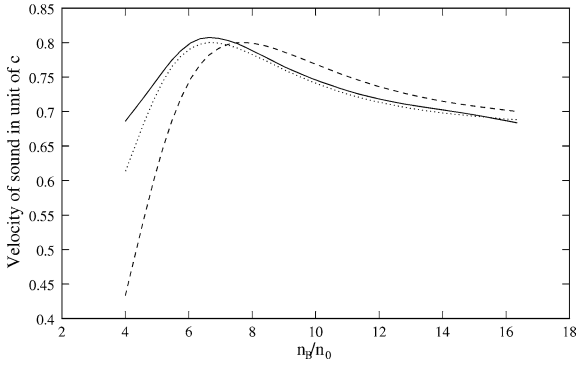


Fig. 5. Velocity of sound, v_s as a function of density ratio. The solid line is for EOS1, the dotted for EOS2 and the dashed for EOS3.

smoothly without important departure from equilibrium. If the two phases were compressed with significantly different rates, there would be inhomogeneities set up.

But near the star surface at $n \sim 4$ to $5n_0$ the matter is more incompressible showing a stiffer surface. This is in keeping with the stability of strange stars observed analytically with the Vaidya–Tikekar metric by Sharma et al. [17].

The velocity of sound, v_s peaks somewhere around the middle of the star and then falls off. We show it for three different EOS in Fig. 5 with parameters given in Table 1.

Next we turn to the model of Zimanyi and Moszkowski [18], where the coupling constants are density dependent. It was shown [19] that the quark condensate derived from this model via the Hellmann–

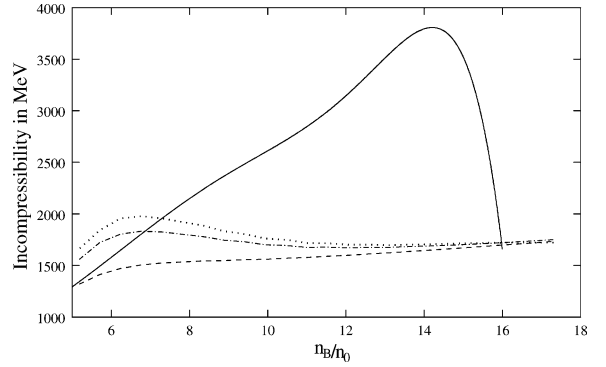


Fig. 6. Incompressibility of neutron matter (ZM model) with three strange matter EOS's for different constituent mass as a function of density ratio. The solid line is for neutron matter, the dotted for $M_Q = 350$ MeV, the dash-dot for $M_Q = 310$ MeV and the dashed for $M_Q = 200$ MeV.

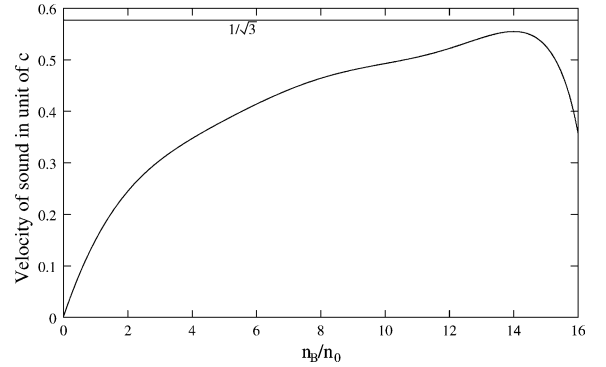


Fig. 7. Velocity of sound, v_s as a function of density ratio for ZM neutron matter.

Feynman theorem is physically acceptable in this model. Details may be found in [19] but the essence is that in the more conventional Walecka model the condensate increases with density whereas in QCD a decrease is expected. The incompressibility and the velocity of sound in the ZM model are shown in Figs. 6 and 7, respectively. At density 14 times normal, the velocity is close to that of a non-interacting massless gas.

In a recent paper [20] Krein and Vizcarra (KV in short) have put forward an EOS for nuclear matter which exhibits a transition from hadronic to quark matter. KV start from a microscopic quark–meson coupling Hamiltonian with a density dependent quark–quark interaction and construct an effective quark–hadron Hamiltonian which contains interactions that

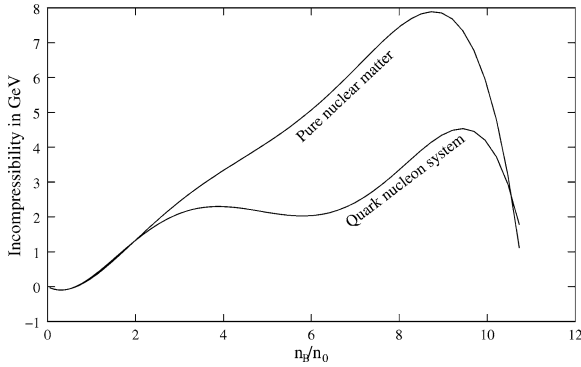


Fig. 8. Incompressibility as a function of density ratio for pure nuclear matter and quark–nucleon system.

lead to quark deconfinement at sufficiently high densities. At low densities, their model is equivalent to a nuclear matter with confined quarks, i.e., a system of non-overlapping baryons interacting through effective scalar and vector meson degrees of freedom, while at very high densities it is *ud* quark matter. The K_{NM} at the saturation density is fitted to be 248 MeV. This EOS also gives a smooth phase transition of quark into nuclear matter and thus, conforms to Witten’s assumption. Interestingly enough, the transition takes place at about $\sim 5n_0$.

The KV model does not incorporate strange quarks so that comparison with our EOS is not directly meaningful. However, it is quite possible that the signal results of the KV calculations mean that quark degrees of freedom lower the energy already at the *ud* level and once the possibility of strange quarks is considered the binding exceeds that of Fe^{56} . An extension of KV with strange quarks is in progress.⁴

Results obtained from the KV calculation are presented simultaneously in Fig. 8. The incompressibility shows softening and the velocity of sound decreases when quark degrees of freedom open up, as expected. At $\sim 5n_0$ K_{qN} is about 2 GeV in qualitative agreement with our value.

Note that in our EOS we also have strange quarks reducing the value of K . Fig. 9 shows the v_s as a function of n_B/n_0 for both kinds of EOS. The EOS with quarks shows a lowering of v_s .

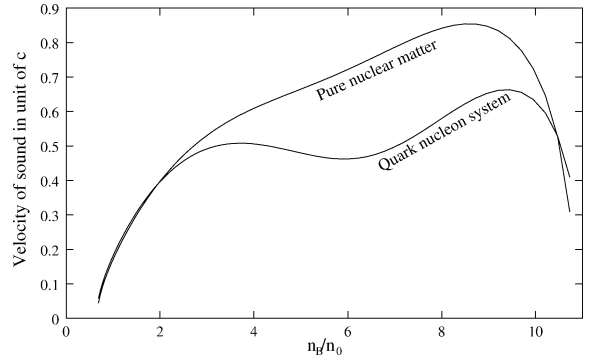


Fig. 9. The velocity of sound in pure nuclear matter and in quark–nucleon system as a function of density ratio. For pure nuclear matter at $9n_0$ the sound velocity is too close to that of light c , whereas for quark–nucleon system it is much less, about $0.5c$.

3. Discussions

In our model the density is large even at the surface ($4.586n_0$ for EOS1) so that the quarks in the Fermi sea have high momentum on the average. Therefore, they often come close together. It was shown by Bailin and Love that in certain colour channels [21], the short range one gluon exchange spin–spin interaction can produce extra attraction, forming diquarks. In *s*-state the pair can be formed only in flavour antisymmetric state. The flavour antisymmetric diquark may form in colour symmetric (6) and spin symmetric (triplet) state, or in colour antisymmetric ($\bar{3}$) and spin antisymmetric (singlet) state. The potential strength of the latter is 6 times the former. The N – Δ mass splitting is obtainable from the one gluon exchange potential, the delta function part of which is smeared to a Gaussian,

$$H_{i,j} = -80.51\sigma^3(\lambda_i \cdot \lambda_j)(S_i \cdot S_j)e^{-\sigma^2 r_{ij}^2}. \quad (5)$$

In the above, the strength of this spin dependent part is taken from Dey and Dey (1984) [22]. The pairing energy of *ud* pairs in the spin singlet and colour $\bar{3}$ state is -3.84 MeV [23].⁵ Our model does not predict diquarks that are permanent—since the quarks must have high momentum transfer if they are to interact strongly with a force that is short-range. The formation and breaking of pairs give rise

⁴ G. Krein, by e-mail.

⁵ With this pairing energy superbursts observed from some astrophysical X-ray sources has been explained very well.

to endothermic and exothermic processes leading to fast cooling and superbursts, respectively.

We also note that the lowering of energy is not very large, as compared to the approximately 42 MeV energy difference per baryon (930.6 MeV for ${}^{56}\text{Fe}$ to 888 MeV), seen in EOS1. So we do not expect any drastic change in the incompressibility or the velocity of sound due to diquark pairing, nor do we expect a phase transition to a colour-flavour locked state.

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