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A Monte Carlo study of erraticity behavior in nucleus–nucleus collisions at high energies ☆

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Abstract

It is demonstrated using Monte Carlo simulation that in different nucleus–nucleus collision samples, the increase of the fluctuation of event factorial moments with decreasing phase space scale, called erraticity, is still dominated by the statistical fluctuations. This result does not depend on the Monte Carlo models. Nor does it depend on the concrete conditions, e.g., the collision energy, the mass of colliding nuclei, the cut of phase space, etc. This means that the erraticity method is sensitive to the appearance of novel physics in the central collisions of heavy nuclei. © 2001 Elsevier Science B.V.

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It is generally believed that through the collision of heavy nuclei at ultrahigh energies big systems with very high energy density [1] might be produced. In these systems novel phenomena, such as colour deconfinement [2], chiral-symmetry restoration [3], discrete-symmetry spontaneous-breaking [4], etc., are expected to be present and different events might be governed by different dynamics. With this goal in mind, the event-by-event (E-by-E) study of high energy collisions has attracted more and more attention [5].

A well-known example of E-by-E fluctuation is the dynamics of self-similar cascade, which results in a fractal system, and the dynamical probability-distribution fluctuates E-by-E [6]. Such kind of self-similar dynamical fluctuations can be studied by means of the method of normalized factorial moments (NFM) [6]. The latter are defined as

$$F_q(M) = \frac{1}{M}\sum_{m=1}^{M}\frac{\langle n_m(n_m-1)\cdots(n_m-q+1)\rangle}{\langle n_m\rangle^q}, \quad (1)$$

where a region Δ in 1-, 2- or 3-dimensional phase space is divided into M cells, n_m is the multiplicity in the mth cell, and $\langle\cdots\rangle$ denotes vertically averaging over the event sample,

$$\langle\cdots\rangle = \frac{1}{\mathcal{N}}\sum_{i=1}^{\mathcal{N}}(\cdots), \quad (2)$$

$\mathcal{N}$ is the number of events in the sample. If self-similar dynamical fluctuations exist, the NFM will possess an anomalous scaling property with the diminishing of phase space scale (or increasing of partition number M),

$$F_q(M) \propto (M)^{\phi_q} \quad (M \to \infty). \quad (3)$$

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Recently the predicted anomalous scaling of NFM, Eq. (3), has been successfully observed in experiments [7,8]. (For a review, see [9].)

In Eq. (1) the *vertical* average $\langle\cdots\rangle$ over the event sample precedes the *horizontal* average $(1/M) \times \sum_{m=1}^{M}(\cdots)$ over the M bins. The NFM defined in this way is sometimes referred to as *vertically averaged factorial moment* and denoted by $F_q^{(\mathrm{v})}(M)$

$$F_q^{(\mathrm{v})}(M) = \frac{1}{M}\sum_{m=1}^{M}\frac{\langle n_m(n_m-1)\cdots(n_m-q+1)\rangle}{\langle n_m\rangle^q}. \tag{4}$$

Alternatively, one can also reverse the order of the two average processes, i.e., doing the horizontal average first, and define *horizontally averaged factorial moment* as

$$F_q^{(\mathrm{h})}(M) = \left\langle \frac{\frac{1}{M}\sum_{m=1}^{M} n_m(n_m-1)\cdots(n_m-q+1)}{\left(\frac{1}{M}\sum_{m=1}^{M} n_m\right)^q}\right\rangle. \tag{5}$$

It can be shown that if the vertical NFM has the anomalous scaling property, Eq. (3), then the horizontal NFM will have the same property.

Note that in the definition Eq. (5) of horizontal NFM an average over the event sample has been made for the *event normalized factorial moment* $F_q^{(\mathrm{e})}(M)$ (EFM) defined as

$$F_q^{(\mathrm{e})}(M) = \frac{\frac{1}{M}\sum_{m=1}^{M} n_m(n_m-1)\cdots(n_m-q+1)}{\left(\frac{1}{M}\sum_{m=1}^{M} n_m\right)^q}, \tag{6}$$

where n_m is the multiplicity in the mth cell of that event. Therefore, it is natural to ask the question: how about the E-by-E fluctuation of EFM $F_q^{(\mathrm{e})}$?

Cao and Hwa [10] propose to quantify this fluctuation by the normalized moments

$$C_{p,q} = \left\langle \left(\Phi_q^{(\mathrm{e})}\right)^p\right\rangle, \quad \Phi_q^{(\mathrm{e})} = F_q^{(e)} \big/ \left\langle F_q^{(e)}\right\rangle \tag{7}$$

of $F_q^{(\mathrm{e})}$. If $C_{p,q}$ has a power law behavior as the division number M goes to infinity

$$C_{p,q}(M) \propto M^{\psi_q}, \quad M \to \infty, \tag{8}$$

then the phenomenon is referred to as *erraticity*, and is characterized by the slope μ_q of $\psi_q(p)$ at $p=1$

$$\mu_q = \frac{d}{dp}\psi_q\bigg|_{p=1}, \tag{9}$$

which is called *entropy index*. Define

$$\Sigma_q = \frac{\partial C_{p,q}}{\partial p}\bigg|_{p=1} = \left\langle \Phi_q^{(\mathrm{e})} \ln \Phi_q^{(\mathrm{e})}\right\rangle, \tag{10}$$

then the entropy index μ_q can be calculated through

$$\mu_q = \frac{\partial \Sigma_q}{\partial \ln M}. \tag{11}$$

The usefulness of erraticity, or entropy index, in the study of E-by-E fluctuation is limited by the fact that this behaviour is dominated by statistical fluctuations when the multiplicity is low [11]. Only for high multiplicity events, as for example in the central collisions of heavy nuclei, the “entropy index” coming from statistical fluctuations becomes very small and the dynamical effect can be expected to show up [12].

In the present Letter this problem is studied in some detail using the Monte Carlo generators Fritiof and Venus. It will be shown that within the framework of these models the statistical fluctuations still dominate the erraticity behaviour of central nuclear collisions, even though the multiplicity is as high as several hundreds to several thousands. What is interesting is that this dominance of statistical fluctuations does not depend on the model used. Neither does it depend on any physical condition, e.g., the collision energy, the mass of the colliding nuclei, the cut of phase space, etc. This means that the erraticity method has the peculiar property that it is able to filter out all the concrete physical conditions used in data analysis and therefore may be used as a sensitive signal for the appearance of novel physics.

We start from the study of Pb–Pb collisions. Two samples are generated using Fritiof for the incident energies 158 and 500 A GeV, each consisting of 10000 events. The phase space regions used for the study of erraticity behaviour are listed in the first 3 rows of Table 1. The collisions are central in the sense that the impact parameters lie between 0 and 0.5 fm.

In order to eliminate the effect of non-flat average distribution, the phase space variables y, p_{t}, φ are transformed into the corresponding cumulant forms [13] X_y, $X_{p_{\mathrm{t}}}$, X_φ as usual. After the transformation, the phase space regions of all three X_a ($a = y, p_{\mathrm{t}}, \varphi$) become [0,1].

In calculating the EFM, the phase space region in each direction is divided into M subcells. The total number of subcells in the 3-D phase space region Δ

Table 1
The phase space region, average multiplicity $\langle N \rangle$ and entropy index μ_2 in Fritiof Monte Carlo of Pb–Pb collisions

	Incident energy (A GeV)				
	158				500
y	[1, 2]	[0, 1]	[0, 2]	[−2, 2]	[−2, 2]
p_t (GeV/c)	[0, 10]	[0, 10]	[0, 10]	[0, 10]	[0, 10]
φ	$[-\pi, \pi]$	$[-\pi, \pi]$	$[-\pi, \pi]$	$[-\pi, \pi]$	$[-\pi, \pi]$
$\langle N \rangle$	286.1	407.2	693.2	1397.9	1677.7
μ_2	0.487	0.273	0.0857	0.0167	0.00856

is $M_{3D} = M^3$. The log–log plots of the event-space moment $C_{p,2}$ of EFM versus M_{3D} are shown in the left column of Figs. 1 and 2 for $p = 0.5$, 0.7, 0.9, 1.0, 1.1, 1.5, 2.0, respectively.

The derivatives Σ_2 of $C_{p,2}$ at $p = 1$ versus log M_{3D} are plotted in the right column of Figs. 1 and 2. The entropy indices μ_2 are then obtained as the slope of Σ_2 versus log M_{3D} at large M. The results are listed in the last row of Table 1.

It can be seen from the figures that the log–log plots of $C_{p,2}$ versus M_{3D} have similar shape for all the cases but only with different scales. This means that erraticity exists in all the cases with

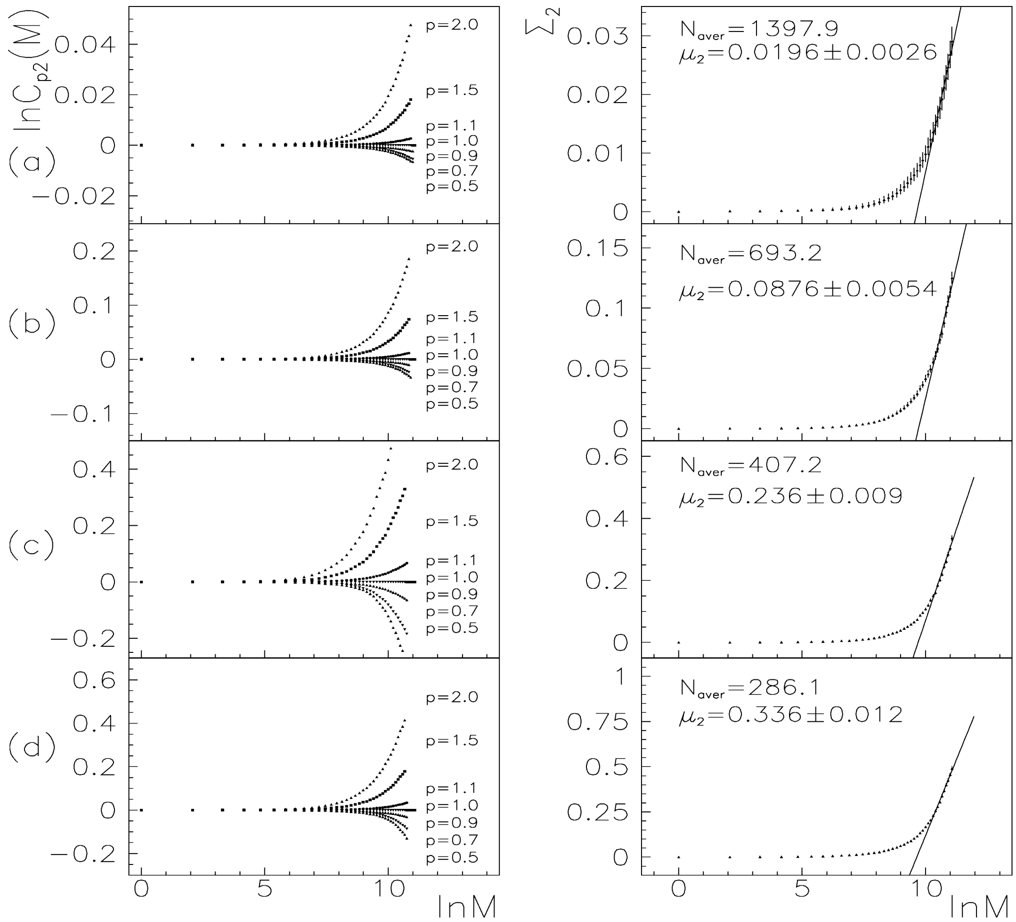

Fig. 1. Log $C_{p,2}$ and Σ_2 versus log M for Pb–Pb collisions at 158 A GeV obtained by Fritiof generator. The rapidity regions (in c.m.s.) in (a), (b), (c), (d) are: $y \in [-2, 2]$, [0, 2], [0, 1], [1, 2], respectively. The transverse momentum region is $p_t \in [0, 10\ \text{GeV}/c]$ and the azimuthal region is $[\varphi \in -\pi, \pi]$.

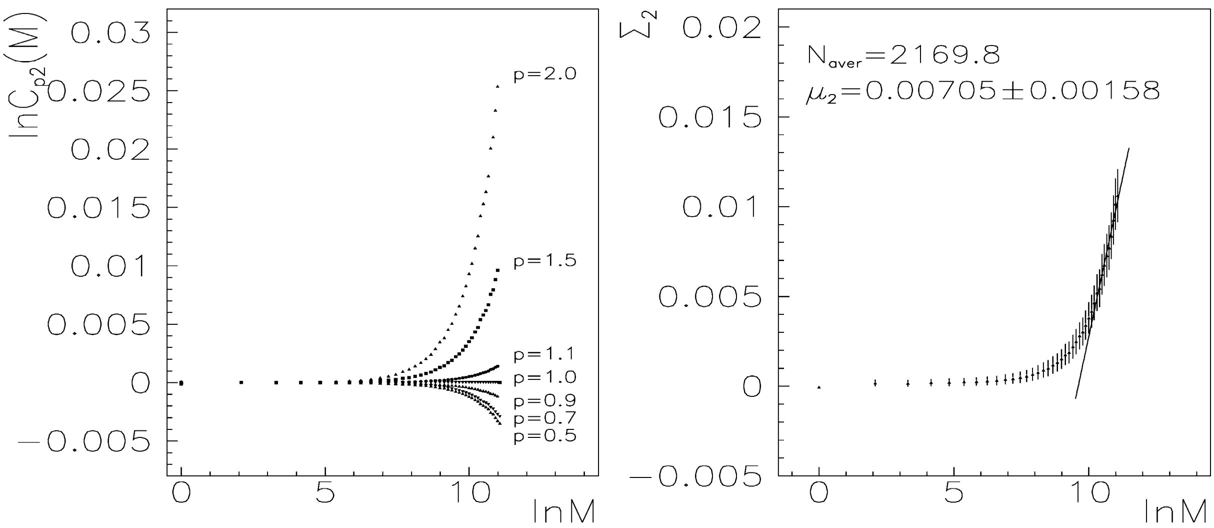

Fig. 2. The same as Fig. 1 but at incident energy 500 *A* GeV.

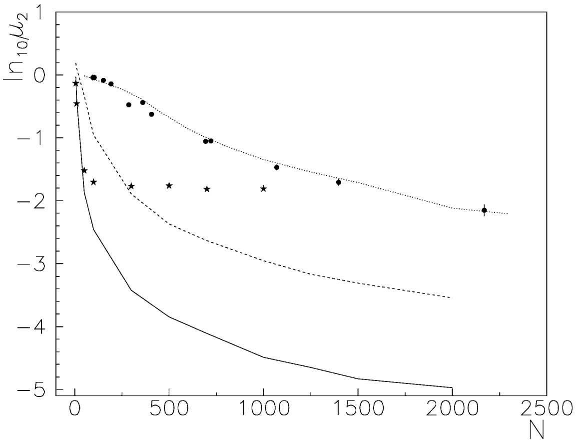

Fig. 3. The dependence of $\log \mu_2$ on $\langle N\rangle$. Full circles are from Fritiof Monte Carlo. Full stars are from Gaussian-alpha model. Full, dashed and dotted lines are the results of pure statistical fluctuations in 1-, 2- and 3-D, respectively.

different strength, characterized by the different values of entropy index μ. A regularity that can easily be observed from Table 1 is that the entropy index μ decreases with increasing average mutiplicity $\langle N\rangle$.

The dependence of μ_2 on $\langle N\rangle$ is plotted in Fig. 3. The full line in this figure is the result of pure statistical fluctuations taken from Ref. [12]. Our results lie well above this line, which seems to indicate that some dynamical effect shows up. However, this conclusion cannot be drawn because the full line was obtained from the pure-statistical-fluctuation model in one-dimensional phase space [12], while our results are for 3-dimensional case.

In order to make a faithful comparison between the results from the Fritiof generator and the pure-statistical-fluctuation case, we construct models of pure statistical fluctuations in 1-, 2- and 3-dimensions, respectively. For illustration, consider the 2-D model. Let X_a and X_b denote the two (cumulant) variables. For each particle in an event take two random numbers distributed uniformly in the region [0, 1] as the values of X_a and X_b of this particle. Repeating N times, the X_a and X_b values of all the N particles in the event are determined and a Monte Carlo event, containing only statistical fluctuations, is obtained. Constructing in this way $\mathcal{N}$ events, the $C_{p,q}$ and Σ_q can be calculated. Note that, by construction, for the characterization of each particle in the 1-, 2-, 3-D models we need 1, 2, 3 random numbers, respectively. Therefore, the "degree of randomness" is higher and the entropy index μ_q should be larger for the 3-D (2-D) model than for the 2-D (1-D) ones.

The results of the calculation shown in Fig. 3 as full (1-D), dashed (2-D) and dotted (3-D) lines confirm the expectation. A striking fact which can be seen from the figure is that the results of the

Fritiof Monte Carlo for Pb–Pb collisions at 158 and 500 A GeV all lie on the dotted line, which means that the erraticity phenomena observed in the Fritiof Monte Carlo simulation of Pb–Pb collisions at these two energies are dominated by statistical fluctuations, inspite of the high multiplicities.

In order to check whether this conclusion depends on the projectile and target nuclei and/or on the event generator used, similar analysis is carried out for various colliding systems at different incident energies using both Fritiof and Venus event generators.

The resulting average multiplicity $\langle N\rangle$ and entropy index μ_2 are listed in Tables 2 and 3 and Fig. 4. Also listed in the tables are the colliding nuclei, the incident energy, the particle type and the rapidity region used in the analysis. The p_t and φ regions in all cases are [0,10] and $[0, 2\pi]$, respectively. The impact parameter takes a value between 0 and 0.5 fm.

It can be seen from Fig. 4 that μ_2 versus $\langle N\rangle$ from both Fritiof and Venus Monte Carlo simulations fits very well to that expected from the 3-D pure-statistical-fluctuation model, independent of the event generator, colliding nuclei, incident energy, particle type and phase space region used in the calculation. This means that, in the framework of Fritiof and/or Venus event generators, even in the central collision of heavy nuclei at energies up to 200 A GeV, the statistical fluctuations still dominate the erraticity behaviour. No dynamical fluctuation can be observed through erraticity analysis.

This disappointing fact, however, provides us a possibility to signal the appearance of novel physics. The point is that, within the framework of traditional high energy nuclear physics the dominance of statistical fluctuations in a given physical process does not depend on the concrete conditions, e.g., the collision energy, the mass of colliding nuclei, the cut of phase space, etc. This dominance will disappear and the observed erraticity will deviate from that of pure statistical fluctuations only if the events of the studied sample are coming from some new kind of physical processes. For illustration, we plot in Figs. 3 and 4 the results from the Gaussian-alpha model proposed in Ref. [12] as stars. It can clearly be seen that they do not lie on any of the three curves in these figures. Therefore, we conclude that erraticity method has the peculiar property that it is able to filter out all the concrete physical conditions used in data analysis and is sensitive to the

Table 2
The average multiplicity and entropy index of nuclear collisions obtained from Fritiof Monte Carlo for different projectile-targets, incident energies, rapidity regions and particle types

Colliding nuclei	E_{inc} (A GeV)	Rapidity region	Particle type	Average multiplicity	Entropy index μ_2
O–Au	200	$[-1, 1]$	Charged	104.1	0.908
S–Au	200	$[-1, 1]$	Charged	152.4	0.825
S–S	158	$[0, 2]$	Charged	96.3	0.908
S–S	158	$[-2, 2]$	Charged	192.5	0.718
Pb–Pb	158	$[1, 2]$	Charged	286.1	0.336
Ag–Ag	158	$[0, 2]$	Charged	360.2	0.365
Pb–Pb	158	$[0, 1]$	Charged	407.1	0.236
Pb–Pb	158	$[0, 2]$	Charged	693.2	0.0876
Ag–Ag	158	$[-2, 2]$	Charged	721.4	0.0891
Pb–Pb	500	$[0, 3]$	Charged	1069.9	0.0338
Pb–Pb	158	$[-2, 2]$	Charged	1397.9	0.0196
Pb–Pb	500	$[-3, 3]$	Charged	2169.2	0.0071

Table 3
The average multiplicity and entropy index of nuclear collisions obtained from Venus Monte Carlo for different projectile-targets, incident energies, rapidity regions and particle types

Colliding nuclei	E_{inc} (A GeV)	Rapidity region	Particle type	Average multiplicity	Entropy index μ_2
H–H	650	$[-4, 4]$	All	14	1.8499
Pb–Pb	158	$[0, 1]$	Negative	21	1.509
Pb–Pb	158	$[0, 2]$	Negative	23	1.507
Pb–Pb	200	$[1, 2]$	All	26	1.519
O–Au	200	$[-1, 1]$	Negative	57	1.277
S–Au	200	$[-1, 1]$	Negative	80	1.122
Pb–Pb	200	$[0, 1]$	All	154	0.8787
Pb–Pb	200	$[0, 2]$	All	180	0.7673
Pb–Pb	158	$[-2, 2]$	Negative	310	0.42
Pb–Pb	200	$[-0.85, 1]$	All	509	0.174
Pb–Pb	200	$[-1, 1]$	All	601	0.1208
Pb–Pb	200	$[-1.3, 2]$	All	846	0.0560
Pb–Pb	200	$[-1.7, 2]$	All	1214	0.0267
Pb–Pb	200	$[-2, 2]$	All	1542	0.01186

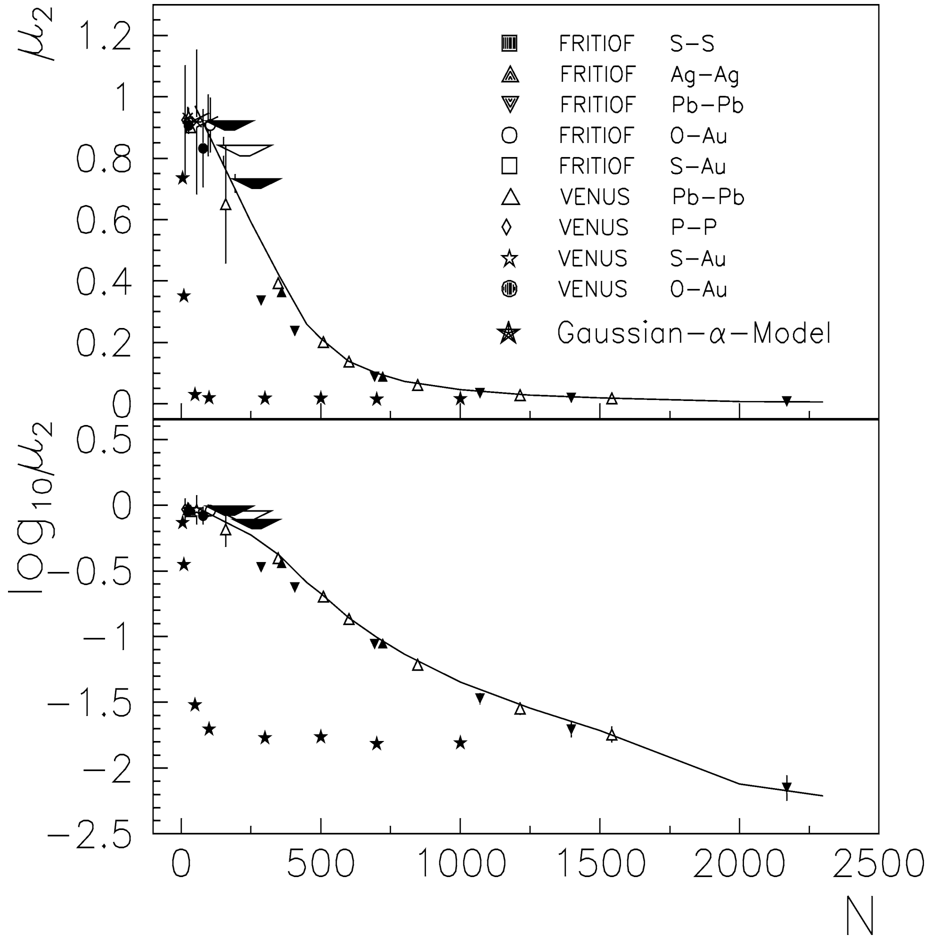

Fig. 4. The dependence of μ_2 on $\langle N \rangle$ from Fritiof and Venus Monte Carlo compared with the 3-D pure-statistical-fluctuation model. The phase space regions used are listed in Tables 2 and 3. Full stars are from Gaussian-alpha model.

appearance of novel physics in the central collisions of heavy nuclei.

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