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Can the curvaton paradigm accommodate a low inflation scale?

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Abstract

The cosmological curvature perturbation may be generated when some ‘curvaton’ field, different from the inflaton, oscillates in a background of unperturbed radiation. In its simplest form the curvaton paradigm requires the Hubble parameter during inflation to be bigger than 10^7 GeV, but this bound may be evaded if the curvaton field (or an associated tachyon) is strongly coupled to a field which acquires a large value at the end of inflation. As a result the curvaton paradigm might be useful in improving the viability of low-scale inflation models, in which the supersymmetry-breaking mechanism is the same as the one which operates in the vacuum.

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1. Introduction

In some of the most interesting inflation models, the inflationary potential comes from the same SUSY-breaking mechanism that operates in the vacuum, giving a Hubble parameter is of order the gravitino mass [1]. Inventing some of the terminology [2], these are: (i) non-hybrid modular inflation [3,4], (ii) hybrid modular inflation [5,6], (iii) μ -field inflation invoking either gauge-mediated [7], gravity-mediated [8] or gaugino-mediated [9] SUSY breaking, (iv) modular thermal inflation [10,11], (v) locked inflation [12].

In gravity-mediated SUSY breaking the gravitino mass is of order TeV, and in the other schemes it is some orders of magnitude lower except for anomaly-mediated [13] where it is up to 100 TeV. An inflation-

ary Hubble parameter of order the gravitino mass is therefore low compared with the maximum value of order 10^{14} GeV allowed by the CMB anisotropy [14], and low compared with the value in other sensible-looking models of inflation [1,2].

One of the most important constraints on models of the very early Universe is the existence of a curvature perturbation ζ , known from observation to be present on cosmological scales a few Hubble times before such scales start to enter the horizon. At that epoch, the earliest one at which it can be directly observed, ζ is almost time-independent, with an almost scale-independent spectrum \mathcal{P}_ζ given by [15] $\mathcal{P}_\zeta^{1/2} \simeq 5 \times 10^{-5}$. This curvature perturbation is supposed to be generated by some field which is light during inflation, because indeed inflation converts the vacuum fluctuation of every such field into an almost scale-invariant classical perturbation. The question is, which light field does the job?

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The usual answer is the inflaton [14]. Unfortunately, this ‘inflaton paradigm’ tends to make life difficult for the low-scale models [1]. In its original form, non-hybrid modular inflation predicts a curvature perturbation that is far too small, and so do the μ -field models, unless one admits extreme fine-tuning.¹ Hybrid modular inflation fares better, but some fine-tuning is still required [5] unless the inflaton mass is allowed to run [4,6] with the attendant danger of a running of the spectral index in conflict with observation. Thus, it may reasonably be said that the inflaton paradigm makes life difficult for low-scale inflation models.

According to the inflaton paradigm, the curvature perturbation has already reached its observed value at the end of inflation and does not change thereafter. The simplest alternative is to suppose that the curvature perturbation is negligible at the end of inflation, being generated later from the perturbation of some ‘curvaton’ field different from the inflaton [16] (see also [17,18]). This curvaton paradigm has attracted a lot of attention [10,17–54] because it opens up new possibilities both for observation and for model-building.²

It is attractive to suppose [11] that the curvaton paradigm can be implemented in conjunction with low-scale inflation models, so as to liberate them from the troublesome requirement that the inflaton generate the curvature perturbation. In this Letter I show that in the simplest version of the curvaton paradigm this will not work, because the curvaton can generate the observed curvature perturbation only if the inflationary Hubble parameter exceeds 10^7 GeV. I go on to consider possible variants of the curvaton paradigm.

¹ The fine-tuning in model of [8] might be removed if the curvature perturbation is generated during preheating, from the decay products of the perturbed Higgs field [9]. This is an alternative to the curvaton mechanism that we are about to discuss.

² According to the scenario developed in the above papers, the curvature perturbation is generated by the oscillation of the curvaton field. A different idea [55] is that the field causing the curvature perturbation does so because its value determines the epoch of reheating, and another is that it does so through a preheating mechanism [9]. In all these cases one might reasonably call the relevant field the curvaton, but the present Letter deals only with the original scenario.

2. The simplest curvaton model

In the simplest model [16], the curvaton field is practically frozen, from the epoch of horizon exit during inflation to the epoch when the Hubble parameter H falls below the curvaton mass m . Also, the curvaton potential in the early Universe is not appreciably modified, and in particular the mass m is not modified.

With this setup, the value of the curvaton field σ when the oscillation begins is practically the same as its value σ_* at the epoch when the observable Universe leaves the horizon during inflation. (Throughout, I will denote the latter epoch by star.) The curvaton energy density is then $\rho_\sigma \sim m^2 \sigma_*^2$, and the total energy density is $\rho \sim M_{\text{P}}^2 m^2 \sim M_{\text{P}}^2 H^2$, making the ratio

$$\left. \frac{\rho_\sigma}{\rho} \right|_{H=m} \sim \frac{\sigma_*^2}{M_{\text{P}}^2}. \quad (1)$$

This ratio is less than 1 by definition, corresponding to $\sigma_* \lesssim M_{\text{P}}$ which is a reasonable requirement. Afterwards it may grow, to achieve some final value which we denote by r . Such growth takes place during any era when the non-curvaton energy density is radiation-dominated. Let us assume for the moment that the growth is continuous, and denote the radiation density by ρ_r . Discounting any variation in the effective number of species, ρ_σ/ρ_r is proportional to the temperature, and curvaton decay increases this temperature by a factor of order $(\rho/\rho_r)^{1/4}$. (Complete thermalisation is assumed after curvaton decay.) It follows [40] that

$$\frac{r}{(1-r)^{3/4}} \sim \frac{\sqrt{m M_{\text{P}} \sigma_*^2}}{T_{\text{dec}} M_{\text{P}}^2}. \quad (2)$$

Remembering now that the growth may not actually be continuous we arrive at the inequality

$$r \lesssim \frac{\sqrt{m M_{\text{P}} \sigma_*^2}}{T_{\text{dec}} M_{\text{P}}^2}, \quad (3)$$

which will be crucial in bounding H_* .

To obtain rather precise results in a simple way, existing treatments of the curvaton scenario assume that ρ_σ/ρ does grow significantly. We will use these results, while noting that our rough order of magnitude estimates should be valid in the limiting case where there is no growth. Once significant growth has taken

place, the curvature perturbation is given by [16,23]

$$\zeta(t) \simeq \frac{1}{3} \frac{\rho_\sigma}{\rho} \frac{\delta\rho_\sigma}{\rho_\sigma}. \quad (4)$$

In this expression the fractional curvaton density perturbation is evaluated on spatially flat slices of space-time so that it is time-independent. It may therefore be evaluated at the beginning of the oscillation, when at each comoving point ρ_σ is proportional to σ^2 , and to first order $\delta\rho_\sigma/\rho_\sigma = 2\delta\sigma/\sigma$. After the curvaton decays ζ is supposed to remain constant until horizon entry, so that the observed curvature perturbation is equal to the one just before curvaton decay,

$$\zeta \simeq \frac{2}{3} \delta\sigma/\sigma \quad (5)$$

$$\simeq \frac{2}{3} \delta\sigma_*/\sigma_* \quad (6)$$

Since the spectrum of $\delta\sigma_*$ is $(H_*/2\pi)^2$, the spectrum of the observed curvature perturbation is therefore predicted to be [16,23]

$$\mathcal{P}_\zeta^{1/2} \simeq \frac{2r}{3} \frac{H_*}{2\pi\sigma_*}. \quad (7)$$

Using the observed value $\mathcal{P}_\zeta^{1/2} = 5 \times 10^{-5}$ one finds that

$$\sigma_* \simeq (5 \times 10^{-5} \times 3\pi)^{-1} r H_*. \quad (8)$$

Combining Eqs. (3) and (8) leads to the bound [40]

$$\frac{\sqrt{m M_{\text{P}} H_*^2}}{T_{\text{dec}} M_{\text{P}}^2} \gtrsim (5 \times 10^{-5} \times 3\pi)^2. \quad (9)$$

Imposing the BBN bound $T_{\text{dec}} > 1$ MeV and the constraint $m < H_*$ gives the advertised bound

$$H_* \gtrsim 10^7 \text{ GeV}. \quad (10)$$

Another bound comes from the fact that the curvaton decay rate Γ will be at least of order m^3/M_{P}^2 , corresponding to gravitational-strength interactions. Since the Hubble parameter at decay is of order Γ this implies

$$T_{\text{dec}} \sim \sqrt{M_{\text{P}} \Gamma} \gtrsim M_{\text{P}} (m/M_{\text{P}})^{3/2}, \quad (11)$$

and hence

$$H_* \gtrsim (10^{11} \text{ GeV}) \left(\frac{m}{H_*} \right). \quad (12)$$

This is stronger than Eq. (10) if $m \gtrsim 10$ TeV.

3. Evolution of the curvaton field

As the simplest model is incompatible with low-scale inflation, we need to explore alternatives. One possibility is to allow significant evolution of the curvaton field, with the curvaton potential either unmodified in the early Universe, or else altered only through a modification $\Delta m^2 \sim \pm H_*^2$ of the effective mass-squared that might be expected to come from supergravity. (In the latter case, the actual modification should be at least an order of magnitude or so below the expected one during inflation, and preferably also afterwards [39].)

Since we are dealing with super-horizon scales, the evolution of the curvaton field at each comoving position is given by the same equation as for the unperturbed Universe,

$$\ddot{\sigma} + 3H\dot{\sigma} + V'(\sigma) = 0, \quad (13)$$

with the initial condition $\sigma = \sigma_*$ and $\dot{\sigma} \simeq 0$. Evaluated at the epoch $H = m$ this will give some value $\sigma = g(\sigma_*)$, and a first-order perturbation

$$\delta\sigma \simeq g' \delta\sigma_*. \quad (14)$$

The effect of the evolution is to replace σ_* by g in Eqs. (10) and (8), and to multiply H_* in Eq. (8) by the factor g' corresponding to the evolution of $\delta\sigma$. The bound on H_* is affected only by the latter change, causing it to be multiplied by a factor $1/g'$. Unfortunately, the evolution typically goes the wrong way, *decreasing* both the value and the perturbation of the curvaton [39]. The opposite can be true if the evolving curvaton field almost reaches a maximum of its potential [36], but this happens only for a narrow range of σ_* , which at least for the model of [36] can be achieved only if there has not been too much inflation before our Universe leaves the horizon.

In addition to being difficult to achieve, a strong increase in the curvaton field brings with it the danger of generating too much non-Gaussianity. Indeed, extending Eqs. (5) and (14) to second order one finds that the

perturbation in the curvaton density when the oscillation begins is given by³

$$\frac{\delta\rho_\sigma}{\rho_\sigma} = 2\frac{g'}{g}\delta\sigma_* + \left(\frac{g'^2}{g^2} + \frac{g''}{g}\right)(\delta\sigma_*)^2. \quad (15)$$

Repeating the argument in [23] (which implicitly assumed $g'' = 0$) the non-linearity parameter becomes

$$f_{\text{NL}} = \frac{5}{4r} \left(1 + \frac{gg''}{g'^2}\right). \quad (16)$$

In the absence of evolution the current observational bound $f_{\text{NL}} \lesssim 100$ is achieved for any $r \gtrsim 0.01$, but strong evolution might violate the bound even for $r = 1$. It would be worth checking that the bound is not violated for the example of [39].

4. The heavy curvaton

The curvaton may have an unsuppressed coupling $\lambda\sigma^2\chi^2$ to some field χ , which is close to zero during inflation but moves quickly to a large VEV at the end of inflation. (Candidates for χ are the inflaton field in the case of non-hybrid inflation, and the waterfall field in the case of hybrid inflation.) In that case, the effective curvaton mass-squared increases by an amount $\lambda\langle\chi\rangle^2$ just after the end of inflation, allowing the true curvaton mass to be bigger than H_* . One may call this kind of curvaton a heavy curvaton.

The heavy curvaton begins to oscillate as soon as its mass is generated at the end of inflation. At this stage, its energy density is of order $m^2\sigma_*^2$ while the total density is of order $M_{\text{P}}^2H_*^2$, so that Eq. (1) is replaced by

$$\frac{\rho_\sigma}{\rho} \Big|_{H=m} \sim \frac{m^2\sigma_*^2}{M_{\text{P}}^2H_*^2}. \quad (17)$$

This is less than 1 by definition, and using Eq. (8) this requires

$$m/M_{\text{P}} \lesssim 5 \times 10^{-4}/r < 5 \times 10^{-2}, \quad (18)$$

where the second inequality comes from the current [56] non-Gaussianity bound $r > 0.01$.

Using Eq. (8) and repeating the arguments leading to Eqs. (10) and (12), one finds that Eq. (10) is replaced by

$$H_* \gtrsim (10^7 \text{ GeV})^5/m^4, \quad (19)$$

while Eq. (12) is replaced by

$$H_* \gtrsim (10^{11} \text{ GeV})^2/m. \quad (20)$$

In the physical range $H_* < M_{\text{P}}$ the second bound is always the stronger. Imposing Eq. (18) gives

$$H_* \gtrsim (10^7 \text{ GeV})r \gtrsim 10^5 \text{ GeV}. \quad (21)$$

This is marginally compatible with low-scale inflation models, though it will become incompatible if future non-Gaussianity observations require $r \simeq 1$.

5. Expansion of the curvaton field scale after inflation

The last possibility that I consider applies only if the curvaton is a PNGB corresponding to a symmetry which acts on the phase of one or more complex fields [16,40]. Taking the simplest case of a single complex field Σ , the potential will be of the form

$$V(\Sigma) \simeq V_0 - m_\Sigma^2|\Sigma|^2 + \lambda M_{\text{P}}^{-n}|\Sigma|^{4+n} + \dots, \quad (22)$$

with $n \geq 0$. The third term is the term mainly responsible for the stabilisation of $|\Sigma|$, which gives it a VEV

$$v \sim (m_\Sigma^2 M_{\text{P}}^n/\lambda)^{1/(2+n)}, \quad (23)$$

and defines the curvaton field through $\Sigma = v \exp(i\sigma/\sqrt{2}v)$. The dots indicate higher powers of Σ , which in general are expected to break the symmetry and generate the curvaton potential, as well as any quantum effects which do the same thing.

We now suppose that there is a coupling $\lambda|\Sigma|^2\chi^2$ with *negative* λ , to a field χ which suddenly acquires a large VEV after inflation. The negative coupling is un-typical, especially in the context of supersymmetry, but it can be achieved as discussed for instance in [57].

With such a coupling, the tachyonic mass m_Σ will suddenly increase at the end of inflation. This will suddenly increase the VEV v by some factor, and increase both σ and $\delta\sigma$ by the same factor. As we saw earlier, the increase in $\delta\sigma$ has the effect of weakening the bound Eq. (10) on H_* , which may

³ In this and the preceding formulas one can proceed more rigorously if the epoch $H = m$ is replaced by a somewhat later one, such that the harmonic oscillation of the curvaton field is well under way and σ is understood to be the amplitude of the oscillation [23].

allow low-scale inflation. Of course, there will in general also be a sudden change in the effective mass-squared of the curvaton, by an amount which depends on the mechanism which explicitly breaks the global symmetry. This change may or may not cancel out the beneficial effect of the increase in the value of the curvaton field perturbation.

6. Conclusion

In the simplest form of the curvaton paradigm, neither the curvaton field nor the form of the curvaton potential change appreciably before the curvaton begins to oscillate. This form is incompatible with low-scale inflation. On the other hand, the curvaton paradigm may become compatible with low-scale inflation if the mass of the curvaton increases sharply at the end of inflation. The same may be true if the curvaton field is the angular part of a complex field, whose tachyonic mass increases sharply.

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