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PHYSICS LETTERS B
Physics Letters B 619 (2005) 313–321
www.elsevier.com/locate/physletb

$B \to \chi_{c0,2}K$ decays: A model estimation

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Received 18 April 2005; received in revised form 3 June 2005; accepted 3 June 2005

Available online 13 June 2005

Editor: N. Glover

Abstract

In this Letter, we investigate the vertex corrections and spectator hard scattering contributions to $B \to \chi_{c0,2}K$ decays, which has no leading contribution from naive factorization scheme. A non-zero binding energy $b = 2m_c - M$ is introduced to regularize the infrared divergence of the vertex part. The spectator diagrams also contain logarithmic and linear infrared divergences, for which we adopt a model dependent parametrization. If we neglect possible strong phases in the hard spectator contributions, we obtain a too small branching ratio for $\chi_{c0}K$ while too large one for $\chi_{c2}K$, as can be seen from the ratio of the branching ratio of $B^+ \to \chi_{c2}K^+$ to that of $B^+ \to \chi_{c0}K^+$, which is predicted to be $2.15^{+0.63}_{-0.76}$ in our model, while experimentally it should be about 0.1 or even smaller. But a closer examination shows that, assuming large strong phases difference between the twist-2 and twist-3 spectator terms, together with a slightly larger spectator infrared cutoff parameter Λ_h, it is possible to accommodate the experimental data. This shows that, for $B \to \chi_{c0,2}K$ decays with no factorizable contributions, QCDF seems capable of producing decay rates close to experiments, in contrast to the $B \to J/\psi K$ decay which is dominated by the factorizable contributions.

PACS: 13.25.Hw; 12.38.Bx; 14.40.Gx

1. Introduction

Hadronic B decays attract a lot of attention because of its role in determining the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements, extracting CP-violating angles and even revealing physics beyond the Standard Model (SM). However in most cases, a deep understanding on the strong dynamics in hadronic B decays is prerequisite for the above purposes.

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0370-2693
doi:10.1016/j.physletb.2005.06.006

Phenomenologically the naive factorization ansatz (NF) [1], supported by color transparency argument [2], is widely used in hadronic two-body B decays. However the unphysical dependence of the decay amplitude on renormalization scale indicates a prominent role of QCD corrections to NF. In this respect, $B \to \chi_{c0,2} K$ decays are of special interest as these channels vanish in the approximation of NF, due to the spin-parity and vector current conservation. Therefore they provide a good opportunity to study the QCD corrections to NF. It was generally believed that the branching ratios of these channels should be quite small as the QCD corrections are either suppressed by strong coupling α_s or Λ_{QCD}/m_b. But BaBar [3] and Belle [4] have found a surprisingly large branching ratio of $B^+ \to \chi_{c0} K^+$ decay,

$$\mathcal{B}\big(B^+ \to \chi_{c0} K^+\big) = \begin{cases} (6.0^{+2.1}_{-1.8} \pm 1.1) \times 10^{-4} & \text{(Belle)}, \\ (2.7 \pm 0.7) \times 10^{-4} & \text{(BaBar)}. \end{cases} \tag{1}$$

Actually this large branching ratio is even comparable, for example, to that of $B \to \chi_{c1} K$ decay which is not forbidden in NF. Another surprising observation is that, the upper limit of $B \to \chi_{c2} K$ decay is roughly an order of magnitude smaller than the observed branching ratio of $B^+ \to \chi_{c0} K^+$ decay [5],

$$\mathcal{B}\big(B^+ \to \chi_{c2} K^+\big) < 3.0 \times 10^{-5} \quad \text{(BaBar)}, \tag{2}$$

while naively the branching ratios of $B \to \chi_{c0,2} K$ decays are expected to be at the same order.

In the following we shall discuss these decay channels using the QCD factorization (QCDF) approach [6]. In this framework, the final state light meson is described by the light-cone distribution amplitude(LCDA), while for the P-wave charmonium $\chi_{c0,2}$, we shall adopt the covariant projection method of non-relativistic QCD [7]. It is well known that, for the inclusive decay and production of P-wave charmonia, the color-octet mechanism must be introduced to guarantee the infrared safety. However it is still unclear how to incorporate this mechanism in a model-independent way into exclusive processes. Thus the decay amplitudes $\mathcal{A}(B \to \chi_{c0,2} K)$ would be inevitably infrared divergent when only the color-singlet picture is adopted for χ_c, which is shown explicitly in [8]. Thus strictly speaking, the QCDF approach is not applicable for $B \to \chi_{c0,2} K$ decays due to the breakdown of factorization.

In this Letter, to get a model estimation, we will introduce the binding energy $b = 2m_c - M$ [9] as an effective cutoff to regularize the infrared divergence appearing in the diagrams of vertex corrections (see Fig. 1). In fact, the logarithmic divergence $\ln(b)$ term in the limit $b \to 0$ for the vertex corrections in $B \to \chi_{c0,2} K$ decays is similar to the $\ln(b)$ term found in Ref. [9] for the production of P-wave charmonium in e^+e^- collisions. As for the spectator scattering contributions, there appears logarithmic divergence at twist-2 level and linear divergence at twist-3 level. Phenomenologically we shall parameterize these divergence as $\ln[m_B/\Lambda_h]$ and m_B/Λ_h respectively, where the non perturbative parameter $\Lambda_h = 500$ MeV again acts as an effective cutoff to regularize the endpoint divergence [10]. According to the QCDF approach, all other contributions are power suppressed by Λ_{QCD}/m_b.

We find that, with the above method, the branching ratio of $B^+ \to \chi_{c0} K^+$ decay is about 0.78×10^{-4}, which is several times smaller than the experimental measurements. At the same time, we also get the branching ratio of $B^+ \to \chi_{c2} K^+$ decay at about 1.68×10^{-4}, which is significantly larger than the upper limit 3×10^{-5} observed by BaBar [5]. But the above estimation is very crude in that the strong phases effects are completely ignored. Notice further that for the spectator contributions, there contains only logarithmic divergence at twist-2 level, while linear divergence appears at the twist-3 level, the strong phases of the twist-2 and twist-3 spectator terms could be quite different. We then briefly discuss the potential strong phases effects and argue that very different strong phases between twist-2 and twist-3 spectator terms together with a slightly larger Λ_h seems to be able to reproduce the experimental hierarchy $\mathcal{B}(B^+ \to \chi_{c0} K^+) \gg \mathcal{B}(B^+ \to \chi_{c2} K^+)$.

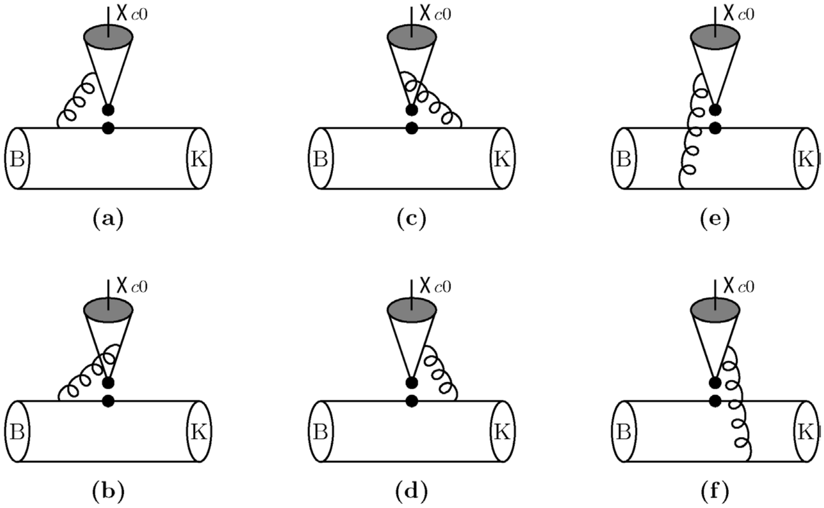

Fig. 1. Order of α_s contributions to $B \to \chi_{cJ} K$ decay. (a)–(d) and (e)–(f) are called vertex corrections and spectator scattering diagrams, respectively.

2. Vertex and spectator corrections

In the QCDF approach, K meson is described by the following light-cone projection operator in momentum space [6]

$$M^K_{\alpha\beta} = \frac{i f_K}{4}\left\{\not l \gamma_5 \Phi(x) - \mu_K \gamma_5 \frac{\not l_2 \not l_1}{l_2 \cdot l_1} \Phi_P(x)\right\}_{\alpha\beta}, \tag{3}$$

where l is the momentum of K meson and l_1 (l_2) is the momentum of quark (antiquark) in K meson. $\Phi(x)$ and $\Phi_P(x)$ are leading twist and twist-3 distribution amplitudes of K meson, respectively. It is understood that only after the factor $l_2 \cdot l_1$ in the denominator is canceled, may we take the collinear limit $l_1 = xl$, $l_2 = (1-x)l$. Notice that in principle we could also start directly from the original light-cone projector of K meson in coordinate space [11], and the physical results should be the same. But in this case care must be taken that, only with a proper regularization, can one do the relevant convolution integrals correctly. The readers may refer to the appendix of [12] for further details.

Since P-wave charmonium χ_{cJ} is involved, we shall use covariant projection method [7,9] to calculate the decay amplitude

$$\mathcal{A}(B \to \chi_{c0,2} K) = \mathcal{E}^{(0,2)}_{\alpha\beta} \frac{\partial}{\partial q_\beta} \mathrm{Tr}\big[\Pi^\alpha_1 \mathcal{C}_1 \mathcal{A}\big]\Big|_{q=0}. \tag{4}$$

Here $\mathcal{A}$ is the standard QCD amplitude for $c\bar{c}$ production, amputated of the heavy quark spinors, $\mathcal{C}_1 = \delta_{ij}/\sqrt{3}$ is the color singlet projector. While Π^α_1 is the $S = 1$ heavy quark spinor projector

$$\Pi^\alpha_1 = \frac{1}{\sqrt{8m_c^3}}\left(\frac{\not P}{2} - \not q - m\right)\gamma^\alpha\left(\frac{\not P}{2} + \not q + m\right), \tag{5}$$

where P is the momentum of charmonium and $2q$ is the relative momentum between the $c\bar{c}$ pair in χ_{cJ}. $\mathcal{E}^{(0,2)}_{\alpha\beta}$ is the polarization tensor of $\chi_{c0,2}$ which satisfies the following sum over polarization relation [7]

$$\mathcal{E}^{(0)}_{\alpha\beta}\mathcal{E}^{(0)}_{\alpha'\beta'} = \frac{1}{D-1}\Pi_{\alpha\beta}\Pi_{\alpha'\beta'}, \qquad \mathcal{E}^{(2)}_{\alpha\beta}\mathcal{E}^{(2)}_{\alpha'\beta'} = \frac{1}{2}(\Pi_{\alpha\alpha'}\Pi_{\beta\beta'} + \Pi_{\alpha\beta'}\Pi_{\beta\alpha'}) - \frac{1}{D-1}\Pi_{\alpha\beta}\Pi_{\alpha'\beta'}, \tag{6}$$

with

$$\Pi_{\alpha\beta} = -g_{\alpha\beta} + \frac{P_\alpha P_\beta}{M^2}. \tag{7}$$

Here M is the mass of χ_{cJ}.

For charmonium B decays, we shall start with the effective Hamiltonian [13]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} V_{cs}^* \big(C_1(\mu) Q_1^c(\mu) + C_2(\mu) Q_2^c(\mu) \big) - V_{tb} V_{ts}^* \sum_{i=3}^{6} C_i(\mu) Q_i(\mu) \right\}, \tag{8}$$

where C_i are Wilson coefficients which are perturbatively calculable and $Q_{1,2}$ $(Q_{3\text{–}6})$ are the effective tree (QCD penguin) operators. Notice that we have dropped the electroweak penguin contributions here which are numerically negligible. The four-quark effective operators are defined as

$$\begin{aligned} &Q_1^c = (\bar{q}_\alpha b_\alpha)_{V-A} (\bar{c}_\beta c_\beta)_{V-A}, && Q_2^c = (\bar{s}_\alpha b_\beta)_{V-A} (\bar{c}_\beta c_\alpha)_{V-A}, \\ &Q_{3,5} = (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V\mp A}, && Q_{4,6} = (\bar{s}_\beta b_\alpha)_{V-A} \sum_q (\bar{q}_\alpha q_\beta)_{V\mp A}. \end{aligned} \tag{9}$$

Here q denotes all the active quarks at the scale $\mu = \mathcal{O}(m_b)$, i.e., $q = u, d, s, c, b$. While α and β are color indices.

It is then straightforward to get the decay amplitude of $B \to \chi_{c0,2} K$ decay by considering the vertex and spectator corrections drawn in Fig. 1,

$$\begin{aligned} \mathcal{A}(B \to \chi_{c0,2} K) = {} & \frac{i G_F}{\sqrt{2}} \frac{6|R_1'(0)|}{\sqrt{\pi M}} \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} \big(V_{cb} V_{cs}^* C_1 - V_{tb} V_{ts}^* (C_4 + C_6) \big) \\ & \times F_0^{B\to K} \left(f_{(0,2)}^{\mathrm{I}} + \frac{4\pi^2}{N_c} \frac{f_B f_K}{F_0^{B\to K}} \big(f_{(0,2)}^{\mathrm{II2}} + f_{(0,2)}^{\mathrm{II3}} \big) \right), \end{aligned} \tag{10}$$

where $R_1'(0)$ is the derivative of the χ_{cJ} wave function at the origin and $F_0^{B\to K}$ the form factor of $B \to K$. The function f^{I} represents the contributions from vertex corrections while f^{II2} (f^{II3}) arising from the twist-2 (twist-3) spectator contributions. The vertex function f^{I} is actually infrared divergent and therefore depends on the binding energy $b = 2m_c - M$. In the following we shall keep $\ln(b/M)$ term and drop the terms suppressed by b/M. The explicit expressions of $f_{(0,2)}^{\mathrm{I}}$ are as follows

$$\begin{aligned} f_0^{\mathrm{I}} = {} & \frac{2m_B((1+12a)(1-4a)+16a\ln[4a])}{(1-4a)^2\sqrt{3a}} \ln\left[\frac{-b}{M}\right] + f_{0\,\text{fin}}^{\mathrm{I}} + \mathcal{O}(b/M), \\ f_2^{\mathrm{I}} = {} & \frac{32\mathcal{E}_{\alpha\beta}^{(2)*} p_B^\alpha p_B^\beta \sqrt{a}((1+12a)(1-4a)+16a\ln[4a])}{m_B(1-4a)^3} \ln\left[\frac{-b}{M}\right] + f_{2\,\text{fin}}^{\mathrm{I}} + \mathcal{O}(b/M), \end{aligned} \tag{11}$$

where $a = m_c^2/m_b^2$, and $f^{\mathrm{I}}_{\mathrm{fin}}$ is the finite part of the function f^{I} in the limit $b/M \to 0$. The explicit expressions of $f^{\mathrm{I}}_{\mathrm{fin}}$ are as follows,

$$
\begin{aligned}
f^{\mathrm{I}}_{0\,\mathrm{fin}} = {} & \frac{-m_B}{2(1-4a)^2(1-2a)^3\sqrt{3a}}\Big\{-6 - 22\ln 2 + 4a\big(26 + (15 - 56\ln 2)\ln 2 \\
& + 8a^2\big(65 + 52\ln 2 - 84\ln^2 2\big) + 384a^4(1 + 2\ln 2) + 2a\big(-85 + 28\ln 2(-1 + 6\ln 2)\big) \\
& + 32a^3\big(-23 - 32\ln 2 + 14\ln^2 2\big)\big) - 8\ln a + 4a\bigg(-(1-4a)^2\big(5 - 24(1-a)a\big)\ln\left[\frac{-1+4a}{a}\right] \\
& + 9\ln a + 2\big(a(-3+4a)\big(13 - 46a + 56a^2\big) - 4(1-2a)^3\ln[64a]\big)\ln a \\
& + 16(1-2a)^3\ln 2\ln[-1+4a]\bigg) \\
& - 64a(1-2a)^3\left(\mathrm{Li}\,2\left[\frac{2-4a}{1-4a}\right] + \mathrm{Li}\,2[1-4a] - \mathrm{Li}\,2\left[\frac{1-2a}{1-4a}\right]\right)\Big\}, \\
f^{\mathrm{I}}_{2\,\mathrm{fin}} = {} & \frac{-\mathcal{E}^{(2)*}_{\alpha\beta} p_B^\alpha p_B^\beta}{4m_B\sqrt{a}}\bigg\{\frac{32a}{(1-2a)^3(1-4a)}\big(4\ln 2(1-2a)^2 + (1-4a)\big(4a^2(1+2\ln 2) - 1\big) \\
& - 8a(3a-1)\big(\ln a - \ln[4a-1]\big)\big) + V_{AB}[a]\bigg\},
\end{aligned}
\tag{12}
$$

where the function $V_{AB}[a]$ denotes the finite part of vertex corrections from Fig. 1(a)–(b). The analytical form of $V_{AB}[a]$ is too complicated to be shown here, but numerically it has a very mild dependence on the parameter a, for example,

$$
V_{AB}[0.1] = 11.3, \qquad V_{AB}[0.15] = 11.9.
$$

As for the spectator functions, we have

$$
\begin{aligned}
f_0^{\mathrm{II}2} &= \frac{1}{m_b(1-4a)\sqrt{3a}}\int_0^1 d\xi\,\frac{\phi_B(\xi)}{\xi}\int_0^1 dy\,\frac{\phi_K(y)}{\bar{y}^2}\big(-8a + (1-4a)\bar{y}\big), \\
f_0^{\mathrm{II}3} &= \frac{2}{m_b(1-4a)^2\sqrt{3a}}\frac{\mu_K}{m_b}\int_0^1 d\xi\,\frac{\phi_B(\xi)}{\xi}\int_0^1 dy\,\frac{\phi_P(y)}{\bar{y}^2}\big(8a - (1-4a)\bar{y}\big), \\
f_2^{\mathrm{II}2} &= \frac{16\mathcal{E}^{(2)*}_{\alpha\beta} p_B^\alpha p_B^\beta\sqrt{a}}{m_b^3(1-4a)^3}\int_0^1 d\xi\,\frac{\phi_B(\xi)}{\xi}\int_0^1 dy\,\frac{\phi_K(y)}{\bar{y}^2}\big(4a + (1-4a)\bar{y}\big), \\
f_2^{\mathrm{II}3} &= \frac{32\mathcal{E}^{(2)*}_{\alpha\beta} p_B^\alpha p_B^\beta\sqrt{a}}{m_b^3(1-4a)^4}\frac{\mu_K}{m_b}\int_0^1 d\xi\,\frac{\phi_B(\xi)}{\xi}\int_0^1 dy\,\frac{\phi_P(y)}{\bar{y}^2}\big(8a - (1+8a)\bar{y}\big).
\end{aligned}
\tag{13}
$$

Here ξ is the momentum fraction of the light spectator quark in the B meson, and $\bar{y} = 1 - y$ the light-cone momentum fraction of the quark in the K meson which is from the spectator quark of B meson. Notice that our expressions for twist-2 spectator function $f^{\mathrm{II}2}_{(0,2)}$ are consistent with those of [8].

3. Numerical results and discussion

To get a numerical estimation on the branching ratios of $B \to \chi_{c0,2} K$ decays, several parameters appearing in Eqs. (10)–(13) should be first decided on. The derivative of χ_{cJ} wave function at the origin $|R'(0)|$ may be either estimated by QCD-motivated potential models [14], or extracted from χ_{cJ} decays [15]. $|R'(0)|^2$ varies from 0.075 to 0.131 GeV^5 in different potential models [14] while using χ_{cJ} decays, for instance [16],

$$\Gamma(\chi_{c2} \to \gamma\gamma) = \frac{36}{5} e_Q^4 \alpha_{\mathrm{em}}^2 \frac{|R'(0)|^2}{m_c^4}\left(1 - \frac{16}{3}\frac{\alpha_s}{\pi}\right), \tag{14}$$

it is easy to get $|R'(0)|^2 = (0.062 \pm 0.007)\ \mathrm{GeV}^5$ if we take $m_c = 1.5$ GeV. This result is a little bit lower than, but still consistent with the potential model calculations, especially considering that it is very sensitive to the choice of charm quark mass. In this Letter, we shall take $|R'(0)|^2 = (0.10 \pm 0.03)\ \mathrm{GeV}^5$ as input.

For the binding energy, if we take $m_c = 1.5$ GeV, the ratio b/M is about -0.11 (-0.16) for χ_{c0} (χ_{c2}), while $a = m_c^2/m_b^2 \simeq 0.1$. The QCD scale μ should be order of $\sqrt{m_b \Lambda}$, as in charmless B decays, which is about (1–1.5) GeV. In the following we shall fix the scale $\mu = 1.3$ GeV with $\alpha_s = 0.36$. Notice also that the Wilson coefficients should be evaluated at leading order, to be consistent with the leading order formula of Eq. (10),

$$C_1 = 1.26, \qquad C_4 = -0.049, \qquad C_6 = -0.074. \tag{15}$$

The relevant CKM parameters are chosen to be $A = 0.83$ and $\lambda = 0.224$.

As for the spectator contributions, we adopt the following LCDAs for the final K meson,

$$\phi_K(y) = 6y(1-y)\left(1 + \sum_{n \geqslant 1} a_n C_n^{(3/2)}(2y-1)\right), \qquad \phi_P(y) = 1, \tag{16}$$

where $C_n^{(3/2)}(x)$ are Gegenbauer polynomials. The parameters a_n are set to be [17]

$$a_1 = 0.17, \qquad a_2 = 0.115, \qquad a_4 = 0.015, \qquad a_3 = a_{n>4} = 0. \tag{17}$$

Then logarithmic and linear divergences appear in Eq. (13), which may be phenomenologically parameterized as [6]

$$\int \frac{dy}{y} = \ln \frac{m_B}{\Lambda_h}, \qquad \int \frac{dy}{y^2} = \frac{m_B}{\Lambda_h}, \tag{18}$$

with $\Lambda_h = 500$ MeV. Notice that the above parametrization of linear divergence would violate the power counting of QCDF, but we do not have better way yet to deal with it. This is clearly a very rough estimation, for example, we do not consider here the strong phase effect. We also know little about B wave function, but fortunately only the following integral is involved which may be parameterized as

$$\int d\xi \frac{\phi_B(\xi)}{\xi} = \frac{m_B}{\lambda_B} \tag{19}$$

and we shall simply fix $\lambda_B = 350$ MeV in our calculation. The chirally enhanced ratio $r_K = \mu_K/m_b$ is chosen to be $0.43^{+0.11}_{-0.08}$, which corresponds to taking $m_s(2\ \mathrm{GeV}) = (90 \pm 20)$ MeV and $(m_u + m_d)(2\ \mathrm{GeV}) = 9$ MeV. The form factor $F_0^{B\to K}(m_{\chi_c}^2)$ may be read from [17], in which as stated, the uncertainty of form factor at $q^2 \neq 0$ is likely to be smaller than that of $q^2 = 0$, which is about 12%. Therefore we will cite $F_0^{B\to K}(m_{\chi_c}^2) = 0.48 \pm 0.06$ as our input. The decay constants are set as $f_K = 160$ MeV and $f_B = (210 \pm 25)$ MeV. With the above input, we get

$$\mathcal{B}\big(B^+ \to \chi_{c0} K^+\big) = \big(0.78^{+0.46}_{-0.35}\big) \times 10^{-4}, \qquad \mathcal{B}\big(B^+ \to \chi_{c2} K^+\big) = \big(1.68^{+0.78}_{-0.69}\big) \times 10^{-4}. \tag{20}$$

We also show separately the contributions from vertex corrections and hard spectator scattering diagrams in Table 1, with all the input parameters taken at their central values. For the case of $\chi_{c0} K$ channel, our results are

Table 1
The numerical estimations of vertex corrections and hard spectator scattering contributions, with all the parameters taken at their central values. The constant $C \equiv \frac{4\pi^2}{N_c}\frac{f_B f_K}{F_0^{B\to K}}$

Decay channels	f^{I}	$C * f^{\mathrm{II}2}$	$C * f^{\mathrm{II}3}$
$\chi_{c0}K$	$46.3 - 33.6i$	-43.1	80.7
$\chi_{c2}K$	$1.7 + 14.1i$	69.3	68.3

approximately four times smaller than the average of BaBar and Belle measurements, $(3.0 \pm 0.7) \times 10^{-4}$, while our prediction on $B \to \chi_{c2}K$ decay is obviously too large compared with the experimental upper limit, 3.0×10^{-5}. This is a little bit surprising, because for charmonium B decays, the theoretical results are normally a few times *smaller* than the experimental measurements.

The careful reader may have noticed that in the above analysis we did not consider the uncertainty related to the parameter $a = m_c^2/m_b^2$. In fact a larger a could enhance the branching ratio of $B \to \chi_{c0}K$ decay significantly, but unfortunately it would also enhance that of $B \to \chi_{c2}K$ decay with similar magnitude. Notice that $B \to \chi_{c0,2}K$ share many common inputs, the ratio of branching ratios of these two channels should have mild dependence on the input parameters, for example it is independent on the parameter $|R'(0)|$. Our numerical analysis shows that this is indeed the case, with $a = 0.10 \pm 0.03$:

$$\mathcal{R} = \frac{\mathcal{B}(B^+ \to \chi_{c2}K^+)}{\mathcal{B}(B^+ \to \chi_{c0}K^+)} = 2.15^{+0.26}_{-0.55}{}^{+0.33}_{-0.31}{}^{+0.36}_{-0.28}{}^{+0.30}_{-0.31}, \tag{21}$$

where the uncertainties arise from the parameters a, r_K, $F_0^{B\to K}$ and f_B, respectively. The above ratio is clearly in strong contradiction with the experimental hierarchy $\mathcal{R} \lesssim 0.1 \ll 1$.

Notice that the chirally enhanced power corrections, namely twist-3 spectator contributions in this case, have been included in the above estimation. For the rest part of power corrections, there is no systematic way to estimate them yet. But since the power corrections are suppressed by $\Lambda_{\mathrm{QCD}}/m_b$, intuitively they might lead to an uncertainty of about 20% to the decay amplitude, which is unlikely to be able to change our estimation Eq. (21) dramatically.

In our model, the parameters Λ_h and λ_B will introduce additional uncertainties to $B \to \chi_{c0,2}K$ decays. It is very unlikely that we could reproduce the experimental observations by fine tuning λ_B, because although a larger λ_B would lead to a smaller branching ratio for $\chi_{c2}K$ decay, it would also make the already too small branching ratio of $\chi_{c0}K$ decay even smaller. However a larger Λ_h does help to close the gap between our predictions and the experimental data, due to the fact that a larger Λ_h will lead to a significantly smaller branching ratio for $\chi_{c2}K$ decay while $\chi_{c0}K$ decay does not change much. Of course we cannot choose a too large Λ_h, say larger than 1 GeV, because it is anyway a non-perturbative parameter. As an illustration, we take $\Lambda_h = 700$ MeV and get

$$\mathcal{B}\big(B^+ \to \chi_{c0}K^+\big) = 0.78 \times 10^{-4}, \qquad \mathcal{B}\big(B^+ \to \chi_{c2}K^+\big) = 0.74 \times 10^{-4}. \tag{22}$$

Although it seems to be on the right way, this effort alone is still not enough to accommodate the experimental data.

Let us take a closer look at the decay amplitudes. From Table 1, it is clear that the spectator hard scattering mechanism is dominant in $B \to \chi_{c2}K$ decay and also very important for $\chi_{c0}K$ channel. Furthermore there is significant destructive (constructive) interference between the twist-2 spectator term and the twist-3 one for $\chi_{c0}K$ ($\chi_{c2}K$) mode. This is probably the reason that we get too small $\chi_{c0}K$ decay as well as too large $\chi_{c2}K$ decay in our model. Notice that there are logarithmic and linear divergences appear in the spectator contributions, which are parameterized by Eq. (18). It is obviously a very rough model estimation and for example, strong phases effects are completely ignored. It is also reasonable to assume that the strong phase of twist-2 spectator term could be different from that of twist-3 part. As an illustration, the endpoint divergences could be parameterized as [6]:

$$\int \frac{dy}{y} = \ln\frac{m_B}{\Lambda_h}\big(1 + \rho_{2,3}e^{i\theta_{2,3}}\big), \qquad \int \frac{dy}{y^2} = \frac{m_B}{\Lambda_h}\big(1 + \rho_3 e^{i\theta_3}\big), \tag{23}$$

with $0 \leqslant \rho \leqslant 1$ and the phase θ completely free. In the above equations, (ρ_2, θ_2) denotes the parameters for twist-2 spectator term and (ρ_3, θ_3) for twist-3 one. In this case, the interference effects and therefore the predictions of the branching ratios, could be changed dramatically. For example, if we take a somewhat extreme case

$$\rho_2 = 0.6, \qquad \theta_2 = \pi, \qquad \rho_3 = 0, \qquad \theta_3 = 0,$$

with $\Lambda_h = 600$ MeV while keep all other input parameters fixed at their central values, we will get

$$\mathcal{B}\big(B^+ \to \chi_{c0} K^+\big) = 3.3 \times 10^{-4}, \qquad \mathcal{B}\big(B^+ \to \chi_{c2} K^+\big) = 1.7 \times 10^{-5}, \tag{24}$$

which are in good agreement with the experimental observations. Certainly, due to the non-perturbative nature of the above strong phases, there is strong model dependence of our predictions. Therefore it is not so meaningful to fine tune the parameters to get the best fit of the experimental data. The key point here is that, different strong phases between twist-2 and twist-3 spectator terms might be able to account for the experimental hierarchy that $\mathcal{B}(B \to \chi_{c0} K)$ is at least an order of magnitude larger than $\mathcal{B}(B \to \chi_{c2} K)$.

The authors of Ref. [18] also studied the $B \to \chi_{c0} K$ decay with the same QCDF method. But they used the gluon mass and gluon momentum cutoff, instead of binding energy adopted in this paper, to regularize the infrared divergences of the vertex corrections. Another difference is that they calculated the spectator contributions directly from the original light-cone projector of K meson in coordinate space and got a different result from this Letter. They claimed that the difference was due to the light-cone projector adopted in this paper which is inappropriate for $\chi_{cJ} K$ channels: to get the projector Eq. (3) from the original one in coordinate space [11], the integration by parts has been used and the boundary terms were dropped. However because of the linear singularities appeared in the above calculations, the boundary terms seem to be divergent and thus the justification of using the integration by parts is in doubt in this case. But Beneke has elaborated on this subtle point in the appendix of Ref. [12] and it is shown there that the boundary terms are indeed zero provided the propagators are regularized carefully when they go close to the mass-shell. Therefore the integration by parts can be used here and the light-cone projector adopted in this paper is justified. Certainly, with a proper regularization, the calculation starting directly from the coordinate space projector should give the same results as this Letter.

Most recently, the $B \to \chi_{c0} K$ decay was discussed by using the PQCD method [22]. Notice that the vertex corrections were not included in their calculations, and the spectator contributions alone are enough in their paper to account for the experimental data. It would be very interesting to see whether they could also reproduce the very small branching ratio of $\chi_{c2} K$ channel observed by BaBar, which has not been done yet.

The $B \to \chi_{c0} K$ decay was also analyzed with light-cone sum rules [19,20]. Although there are some discrepancies in their papers, they agreed on the point that their results were too small to accommodate the experimental data. A large charmed meson rescattering effects $B \to D_s^{(*)} D^{(*)} \to \chi_{c0,2} K$ could account for the surprisingly large $B \to \chi_{c0} K$ decay [21], but generally it will also lead to a large branching ratio for $\chi_{c2} K$ mode.

In summary, we discuss in this Letter the vertex corrections and spectator hard scattering contributions to $B \to \chi_{c0,2} K$ decays. Since there is no model independent way yet to estimate the color-octet contribution to exclusive processes, it is no wonder that the vertex corrections here are infrared divergent. The non-zero binding energy $b = 2m_c - M_{\chi_{cJ}}$ makes the charm quark slightly off-shell inside χ_{cJ}, and effectively acts as a cutoff to regularize the vertex part. There are also less serious logarithmic and linear endpoint divergences which appears in the spectator contributions and are parameterized in a model-dependent way as usually done in charmless B decays. This means that the spectator diagrams are actually dominated by soft gluon exchange, which in a sense could be viewed as a model estimation of color-octet contributions. Then our numerical analysis predicts the branching ratio of $B^+ \to \chi_{c0} K^+$ decay to be about 0.78×10^{-4}, about four times smaller than the experimental observations, while for $B^+ \to \chi_{c2} K^+$ decay, we get 1.68×10^{-4}, which is about five times larger than the experimental upper limit. But concerning the large theoretical uncertainties, it is more interesting to consider the ratio $R = \mathcal{B}(B^+ \to \chi_{c2} K^+)/\mathcal{B}(B^+ \to \chi_{c0} K^+)$, in which a large part of the theoretical uncertainty can be eliminated. Numerically we find the ratio to be $R = 2.15^{+0.63}_{-0.76}$, in sharp contrast to the experimental observation that this ratio should be about 0.1 or even smaller, if the BaBar analysis of the upper limit of $\mathcal{B}(B^+ \to \chi_{c2}K^+)$ decay will be confirmed by further measurements. We then have a closer look at the decay amplitudes. One observation is that, $\chi_{c0}K$ channel is not very sensitive to the spectator infrared cutoff parameter Λ_h while a larger Λ_h could reduce the branching ratio of $\chi_{c2}K$ decay significantly. Another observation is that, in our model there is large destructive (constructive) interference between the twist-2 and twist-3 spectator terms for $\chi_{c0}K$ ($\chi_{c2}K$) mode. But notice that the twist-2 spectator contributions contain only logarithmic endpoint divergence while twist-3 ones contain more severe linear endpoint divergence, it should be reasonable to assume that their strong phases could be quite different. Since the interference effects are very sensitive to the strong phases difference, this might change our model predictions dramatically. As an illustration, we then show in an explicit case that, with a slightly larger Λ_h and large strong phases difference between twist-2 and twist-3 spectator terms, our predictions are in good agreement with the experimental data.

In conclusion, what we have shown in this Letter, is that by adjusting the parameters for the spectator hard scattering contributions, as with the annihilation terms for charmless B decays, QCDF is able to produce appreciable non-factorizable contributions to $B^+ \to \chi_{c0,2}K^+$ decays close to experiments, in contrast with the $B^+ \to J/\psi K^+$ decay which needs a large factorizable contribution in addition to the small non-factorizable one obtained in QCDF [23].

Acknowledgement

G.Z. acknowledges the support from Alexander-von-Humboldt Stiftung.

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