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TeV-scale seesaw mechanism catalyzed by the electron mass

W. Grimus [a,*], L. Lavoura [b]

[a] University of Vienna, Faculty of Physics, Boltzmanngasse 5, A-1090 Vienna, Austria
[b] Technical University of Lisbon, Centre for Theoretical Particle Physics, 1049-001 Lisbon, Portugal

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ABSTRACT

We construct a model in which the neutrino Dirac mass terms are of order the electron mass and the seesaw mechanism proceeds via right-handed neutrinos with masses of order TeV. In our model the spectra of the three light and of the three heavy neutrinos are closely related. Since the mixing between light and heavy neutrinos is small, the model predicts no effects in pp and $p\bar{p}$ colliders. Possible signatures of the model are the lepton-number-violating process $e^-e^- \to H^-H^-$, where H^- is a charged scalar particle, lepton-flavour-violating decays like $\mu^- \to e^-e^-e^+$, or a sizable contribution to the anomalous magnetic dipole moment of the muon.

1. Introduction

The discovery of neutrino oscillations has established the existence of non-zero neutrino masses in the sub-eV range.[1] An appealing way of explaining such small neutrino masses is the seesaw mechanism [4], in which gauge-singlet right-handed neutrino fields ν_R with Majorana mass terms are added to the Standard Model (SM). Those mass terms are not generated by the Higgs mechanism and may, therefore, be much larger than the electroweak scale. Let ν_L be the neutral members of the left-handed leptonic SM gauge doublets and M_D and M_R the mass matrices of the fermionic bilinears $\bar{\nu}_R \nu_L$ and $\bar{\nu}_R C \bar{\nu}_R^T$, respectively. ($C$ is the charge-conjugation matrix in Dirac space.) We denote the mass scales of M_D and M_R by m_D and m_R, respectively. If m_R is much larger than m_D, then an effective Majorana mass matrix

$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D \tag{1}$$

is generated for the ν_L. According to (1), the scale m_ν of $\mathcal{M}_\nu$ is related to m_D and m_R through $m_\nu \sim m_D^2/m_R$. The mixing between the light and heavy neutrinos is of order m_D/m_R, hence very small.

We experimentally know that m_ν is in the range of 0.1 eV to 1 eV, if m_ν indicates the order of magnitude of the largest of the light-neutrino masses, but, in the framework of the seesaw mechanism, m_D and m_R remain a mystery. Since M_D is the neutrino counterpart of the charged-fermion mass matrices, m_D may vary in between 100 GeV (which is both the electroweak scale and the top-quark mass scale) and 1 MeV (the scale of the electron mass and of the up- and down-quark masses), and this spans a range of five orders of magnitude. As $m_R \sim m_D^2/m_\nu$, to these five orders of magnitude in m_D correspond ten orders of magnitude in m_R. If $m_D \sim 100$ GeV then $m_R \sim 10^{13}$ GeV; this is definitely below the typical GUT scale 10^{16} GeV — identifying m_R with the GUT scale would make the neutrino masses too small. If instead $m_D \sim m_\tau$, the mass of the tau lepton, then $m_R \sim 10^9$ GeV. If one wants to incorporate leptogenesis [5] into the seesaw mechanism, then the appropriate m_R would rather be 10^{11} GeV, which lies in between the two previous estimates. Finally, we may as well use $m_D \sim m_e$, the electron mass, and then $m_R \sim 1$ TeV — this was noticed, for instance, in [6].[2] Such a low m_R has the advantages that it coincides with the expected onset of physics beyond the SM and that it might produce testable effects of the seesaw mechanism at either present or future colliders.

The possibility that $m_D \sim m_e$ and $m_R \sim 1$ TeV is the starting point of this Letter. We construct a seesaw model in which the vacuum expectation value (VEV) responsible for the mass matrix M_D is of order m_e. In our model, which is inspired by [9] and [10],

* Corresponding author.
E-mail addresses: walter.grimus@univie.ac.at (W. Grimus), balio@cftp.ist.utl.pt (L. Lavoura).

[1] Cosmological arguments [1] and the negative result of the direct search for neutrino mass in tritium β decay [2] are also crucial in eliminating the possibility that the neutrino masses may be higher than about one eV. For a review on neutrino masses see [3].

[2] Of course, the simplest way to obtain $m_D \sim m_e$ is to *assume* tiny Yukawa couplings, as was done for instance in [7]; this is the opposite of what we do in this Letter — we discuss a scenario with Yukawa couplings of order 0.01–1. Another avenue which has been pursued are technicolor models with a suppressed m_D and $m_R \lesssim 10^3$ TeV [8]; however, in those models m_D is not claimed to be as low as m_e.

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each charged lepton α ($\alpha = e, \mu, \tau$) has its own Higgs doublet ϕ_α, whose VEV generates the mass m_α. On the other hand, there is only one Higgs doublet ϕ_0 which has Yukawa couplings to the ν_R and is, therefore, responsible for M_D. We furthermore make use of a mechanism, first put forward in [11] and later extended in [12], for suppressing the VEV of ϕ_0: this doublet has a *positive* mass-squared μ_0 (in the scalar-potential term $\mu_0 \phi_0^\dagger \phi_0$) and its VEV is triggered by a term $\phi_e^\dagger \phi_0$ in the scalar potential. Since ϕ_e is the Higgs doublet which gives mass to the electron, it must have a very small VEV, and this explains the smallness of the VEV of ϕ_0.

2. The model

Multiplets. The gauge-$SU(2)$ multiplets of our model are the following:

- Left-handed lepton doublets $D_{L\alpha} = (\nu_{L\alpha}, \alpha_L)^T$ and right-handed singlets $\nu_{R\alpha}$, α_R ($\alpha = e, \mu, \tau$).
- Four Higgs doublets ϕ_0 and ϕ_α.

We shall use indices k, l running over the range $0, e, \mu, \tau$. The VEV of the neutral component of ϕ_k is denoted v_k.

Symmetries. The family symmetries of our model are the following:

- Z_3 symmetry $e \to \mu \to \tau \to e$.[3]
- Three Z_2 symmetries [9]

$$\mathbf{z}_\alpha: \quad D_{L\alpha} \to -D_{L\alpha},\ \alpha_R \to -\alpha_R,\ \nu_{R\alpha} \to -\nu_{R\alpha}. \tag{2}$$

  Notice that ϕ_α does *not* change sign under $\mathbf{z}_\alpha$. The symmetries $\mathbf{z}_\alpha$ may be interpreted as discrete lepton numbers.
- Three Z_2 symmetries [9]

$$\mathbf{z}_\alpha^h: \quad \alpha_R \to -\alpha_R,\ \phi_\alpha \to -\phi_\alpha. \tag{3}$$

  The $\mathbf{z}_\alpha^h$ are meant to ensure that α_R has no Yukawa couplings to the ϕ_β ($\beta \neq \alpha$).

Yukawa Lagrangian. The multiplets and symmetries of the theory lead to the Yukawa Lagrangian

$$\mathcal{L}_Y = -y_1 \sum_\alpha \bar{D}_{L\alpha} \alpha_R \phi_\alpha - y_2 \sum_\alpha \bar{D}_{L\alpha} \nu_{R\alpha} \left(i\tau_2 \phi_0^* \right) + \text{H.c.} \tag{4}$$

This has, remarkably, only two Yukawa coupling constants. The Lagrangian (4) enjoys the accidental symmetry

$$\begin{aligned} \mathbf{z}_\nu: \quad & \phi_0 \to -\phi_0,\ \nu_{Re} \to -\nu_{Re}, \\ & \nu_{R\mu} \to -\nu_{R\mu},\ \nu_{R\tau} \to -\nu_{R\tau}. \end{aligned} \tag{5}$$

Charged-lepton masses. The mass of the charged lepton α is $m_\alpha = |y_1 v_\alpha|$. Therefore $m_e : m_\mu : m_\tau = |v_e| : |v_\mu| : |v_\tau|$. In our model one cannot obtain a small m_e by just tuning some Yukawa couplings — one really needs a small VEV v_e. For instance, even if there were no further Higgs doublets in the theory and v_τ by itself alone saturated the relation

$$v^2 \equiv \sum_k |v_k|^2 \leqslant (174\ \text{GeV})^2, \tag{6}$$

one would still need $|y_1| \sim 0.01$ because $m_\tau \approx 1.78$ GeV. We would then have $|v_e| \sim 50$ MeV. On the other hand, we may assume that there are in the full theory some extra Higgs doublets other than the four ϕ_k, which give mass to the quarks — in particular, for the top-quark mass a doublet with a large VEV is mandatory. Then $|v_\tau|$ may be significantly smaller than 174 GeV and, accordingly, $|v_e|$ will be significantly smaller than 50 MeV too; in particular, $|v_e| \sim m_e$ is possible.

Soft symmetry breaking. We assume that the Z_3 symmetry and the three $\mathbf{z}_\alpha$ symmetries are softly broken in the dimension-three neutrino Majorana mass terms[4]

$$\mathcal{L}_M = \frac{1}{2} \sum_{\alpha,\beta} \bar{\nu}_{R\alpha} (M_R)_{\alpha\beta} C \bar{\nu}_{R\beta}^T + \text{H.c.} \tag{7}$$

We furthermore assume that the symmetries $\mathbf{z}_\alpha^h$ are also softly broken, now by the dimension-two terms $\phi_k^\dagger \phi_l$ ($k \neq l$) in the scalar potential. However, we shall assume that *the combined symmetry*

$$\begin{aligned} \mathbf{z}_e^h \circ \mathbf{z}_\nu: \quad & \phi_0 \to -\phi_0,\ \phi_e \to -\phi_e,\ e_R \to -e_R, \\ & \nu_{Re} \to -\nu_{Re},\ \nu_{R\mu} \to -\nu_{R\mu},\ \nu_{R\tau} \to -\nu_{R\tau} \end{aligned} \tag{8}$$

is broken only spontaneously. Then the scalar potential is

$$\begin{aligned} V = {} & \sum_k \left[\mu_k \phi_k^\dagger \phi_k + \lambda_k \left(\phi_k^\dagger \phi_k \right)^2 \right] \\ & + \sum_{k \neq l} \left[\lambda_{kl} \phi_k^\dagger \phi_k \phi_l^\dagger \phi_l + \lambda'_{kl} \phi_k^\dagger \phi_l \phi_l^\dagger \phi_k + \lambda''_{kl} \left(\phi_k^\dagger \phi_l \right)^2 \right] \\ & + \left(\mu_{0e} \phi_0^\dagger \phi_e + \mu_{\mu\tau} \phi_\mu^\dagger \phi_\tau + \text{H.c.} \right). \end{aligned} \tag{9}$$

The quartic couplings in V, those with coefficients generically represented by the letter λ, obey all the family symmetries of the model. Because of the couplings $\lambda''_{kl} (\phi_k^\dagger \phi_l)^2$, the only $U(1)$ symmetry of V is the one associated with weak hypercharge; therefore, there are no Goldstone bosons in the model.

Suppression of $\boldsymbol{v_0}$. If μ_0 *is positive*, then v_0 will induced by v_e and by the first term in the last line of (9):

$$v_0 \approx -v_e \frac{\mu_{0e}}{\mu_0}. \tag{10}$$

We envisage the possibility that $|y_2|$ and, possibly, also $|y_1|$ are of order 1, because that would enhance scalar effects and the experimental testability of our model, cf. Sections 3 and 4 below. Still, it is possible, as we mentioned earlier, that $|y_1| \sim 0.01$ and $|v_e| \sim 50$ MeV. However, even in that case $|v_0|$ could easily be much smaller than $|v_e|$, as we pondered in [12]. Indeed, $|\mu_{0e}|$ could *naturally* [14] be small, and there is no reason why μ_0 should not be rather large, maybe even of order TeV. Then $|v_0|$ might be much smaller than $|v_e|$ — this is the mechanism that we envisaged in the introduction. Thus:

- If there are in the theory some Higgs doublets beyond the four ϕ_k, the Yukawa coupling constant y_1 in (4) may be of order one and then $|v_e| \sim m_e$. In that case, $|\mu_{0e}|$ and μ_0 may be allowed to be of the same order of magnitude and $|v_0| \sim |v_e| \sim m_e$.

[3] If we go beyond our main aim of achieving a light seesaw scale and furthermore want to obtain the predictions of maximal atmospheric-neutrino mixing and a vanishing reactor-neutrino mixing angle, then one may extend this Z_3 symmetry to the full S_3 permutation symmetry of e, μ and τ [13].

[4] If we want to predict maximal atmospheric mixing, then we must assume an S_3 instead of a Z_3 family symmetry and, furthermore, assume that the subgroup of S_3, the μ–τ interchange symmetry, is preserved in the soft breaking [9,10,13].

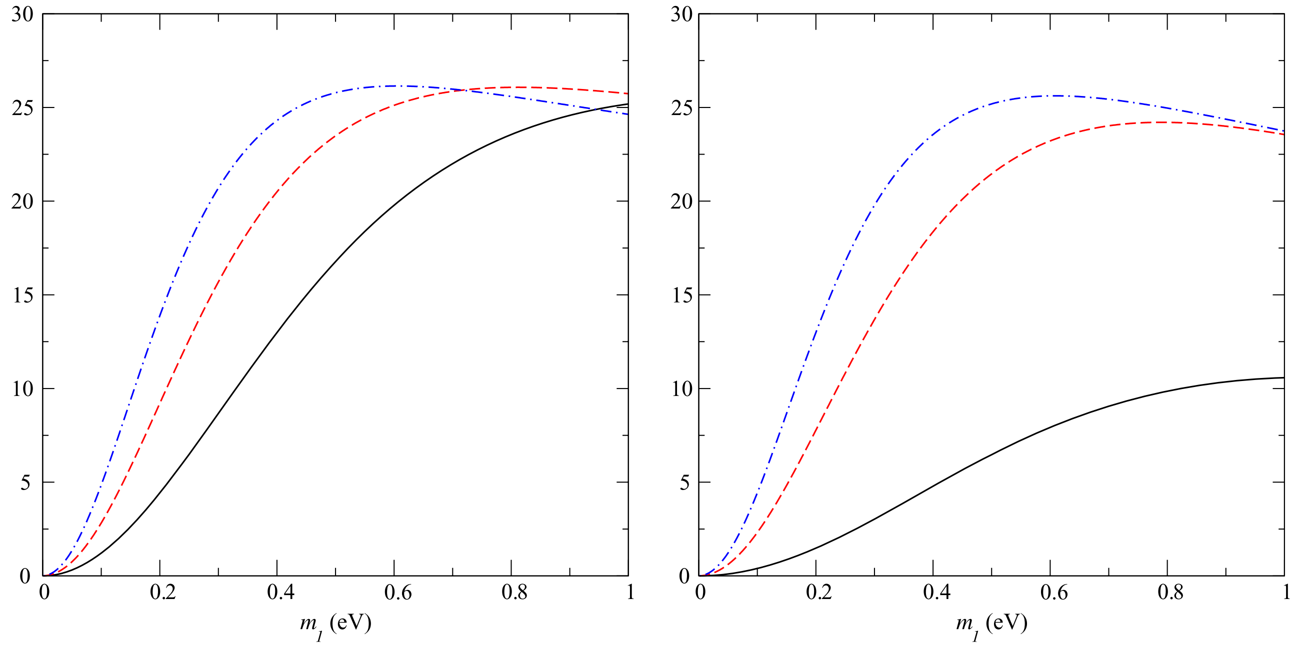

Fig. 1. $\sigma(e^-e^- \to H^-H^-)/\sigma_{\text{QED}}$ as a function of m_1 when $\sqrt{s} = 500$ GeV (full curves), 750 GeV (dashed curves) and 1 TeV (dashed-dotted curves). The mass of H^- is $m_H = 120$ GeV in the left figure, $m_H = 240$ GeV in the right one. We use Eqs. (14), (16) and (17), $|y_2| = 1$, $|v_0| = 511$ keV, $U_{e1}^2 = 0.7$, $U_{e2}^2 = 0.3$ and $m_2^2 = m_1^2 + 8 \times 10^{-5}$ eV^2.

- If there are only the four ϕ_k,[5] then y_1 is much smaller than one and $|v_e| \sim 100\, m_e$. In that case, we may naturally assume $|\mu_{0e}| \ll \mu_0$ because the theory acquires the extra symmetry $\mathbf{z}_\nu$ when $\mu_{0e} = 0$. We might then still obtain $|v_0| \sim m_e$.

Lepton mixing. The neutrino Dirac mass matrix is in our model proportional to the unit matrix:

$$M_D = y_2^* v_0 \mathbb{1}. \tag{11}$$

Therefore, the lepton mixing matrix U, which diagonalizes $\mathcal{M}_\nu$, also diagonalizes M_R:

$$U^T \mathcal{M}_\nu U = \text{diag}(m_1, m_2, m_3), \tag{12}$$

$$U^\dagger M_R U^* = -\exp\left[2i \arg\left(y_2^* v_0\right)\right] \text{diag}(M_1, M_2, M_3), \tag{13}$$

the m_j and M_j $(j = 1, 2, 3)$ being, respectively, the light- and heavy-neutrino masses. Consequently, there is a close relationship between the spectra of the light and heavy neutrinos:

$$M_j = \frac{|y_2 v_0|^2}{m_j}. \tag{14}$$

Following our rationale, we shall assume $|y_2 v_0| \sim m_e$.

3. Possible collider effects

In our model we assume $m_D \sim m_e \sim 1$ MeV while $m_R \sim 1$ TeV. Otherwise our model is a normal seesaw model, therefore in it the mixing between the light and the heavy neutrinos is of order $m_D/m_R \sim 10^{-6}$. This small mixing suppresses most possible signatures of heavy right-handed neutrinos that have been considered in the literature [15]. For instance, the Drell–Yan production of a virtual W boson and its subsequent decay $W^{\pm *} \to \ell^\pm N_j$, where $\ell^\pm$ is a charged lepton and N_j a heavy Majorana neutrino, is negligible because it is suppressed by the mixing between the light and heavy neutrinos.

Suppose that the Higgs doublet ϕ_q (which may be one of our four doublets or an additional one) couples to the quarks and, in particular, generates the top-quark mass. Then we may envisage the Drell–Yan production of a virtual $\phi_q^\pm$ at the LHC, followed by the transition $\phi_q^\pm \to \phi_0^\pm$ and the decay $\phi_0^\pm \to \ell^\pm N_j$, finally leading to heavy-neutrino production. However, since the VEV of ϕ_q^0 is necessarily large and the VEV of ϕ_0^0 is very small, the mixing $\phi_q^\pm$–$\phi_0^\pm$ will in general be small, unless we invoke finetuning in the scalar potential.

In contrast to what happens at hadron colliders, in an electron–electron collider the interesting lepton-number-violating process $e^-e^- \to H^-H^-$ might occur [6]. This process is due to the Majorana nature of the heavy neutrinos and is one of the processes, other than neutrinoless $\beta\beta$ decay, via which it might be possible to probe lepton-number violation [16]. Let us suppose for simplification that $H^- \equiv \phi_0^-$, the charged component of the scalar doublet ϕ_0. Then, the relevant Yukawa Lagrangian for $e^-e^- \to H^-H^-$ is given by

$$\mathcal{L}\left(e^-e^- \to H^-H^-\right) = y_2^* H^+ \sum_{j=1}^{3} U_{ej}^* \bar{N}_j P_L e + \text{H.c.}, \tag{15}$$

where P_L is the negative-chirality projection matrix. This leads to the total cross section (see [6] for a special case)

$$\sigma\left(e^-e^- \to H^-H^-\right) = \frac{|y_2|^4}{16\pi s^2 \beta} \sum_{j,k=1}^{3} \left(U_{ej} U_{ek}^*\right)^2 M_j M_k f\left(\frac{w_j}{\beta}, \frac{w_k}{\beta}\right), \tag{16}$$

where s is the square of the energy of the e^-e^- system in its centre-of-momentum reference frame, $\beta = (1 - 4m_H^2/s)^{1/2}$ (m_H is the mass of H^-) and $w_j = 1 - 2m_H^2/s + 2M_j^2/s$. The function f is given by

[5] In that case we might envisage a scenario in which the VEVs of ϕ_τ and ϕ_μ would be responsible, respectively, for the masses of the up-type and down-type quarks. Other possibilities may of course also be considered.

$$f(a,b) = \begin{cases} \frac{1}{b^2-a^2}\left(b\ln\left|\frac{a+1}{a-1}\right| - a\ln\left|\frac{b+1}{b-1}\right|\right) \Leftarrow b \neq a, \\ \frac{1}{a^2-1} + \frac{1}{2a}\ln\left|\frac{a+1}{a-1}\right| \Leftarrow b = a. \end{cases} \tag{17}$$

Notice that the cross section (16) depends on the Majorana phases of the products $(U_{ej}U_{ek}^*)^2$ ($j \neq k$). In Fig. 1 we have plotted $\sigma(e^-e^- \to H^-H^-)$ as a function of the light-neutrino mass m_1 in a number of cases. Following our rationale, we have taken $|y_2 v_0| = m_e$ in Eq. (14). On the other hand, in Eq. (16) we have taken $|y_2| = 1$, bearing in mind that $\sigma(e^-e^- \to H^-H^-)$ depends very strongly on this Yukawa coupling. As for the U_{ej} matrix elements, we have used the values $|U_{e1}|^2 = 0.7$, $|U_{e2}|^2 = 0.3$ and $U_{e3} = 0$, which almost coincide with the present best fit [17]; this has the advantage that then the third-neutrino mass m_3 and the type of neutrino mass spectrum become irrelevant — we only have to take into account the experimental value of $m_2^2 - m_1^2$, which we fixed at 8×10^{-5} eV^2. We have moreover assumed that the phase of $(U_{e1}U_{e2}^*)^2$ is zero — this choice maximizes $\sigma(e^-e^- \to H^-H^-)$. In Fig. 1 this cross section is given in units of $\sigma_{\mathrm{QED}} = 4\pi\alpha^2/(3s)$, with $\alpha = 1/128$.

In the limit $M_j^2 \gg s, m_H^2$ for all $j = 1, 2, 3$, one obtains

$$f\left(\frac{w_j}{\beta}, \frac{w_k}{\beta}\right) \approx \frac{2\beta^2}{w_j w_k} \approx \frac{s^2\beta^2}{2M_j^2 M_k^2}. \tag{18}$$

Therefore, in that limit

$$\sigma\left(e^-e^- \to H^-H^-\right) \approx \frac{\beta m_{\beta\beta}^2}{32\pi|v_0|^4}, \tag{19}$$

where $m_{\beta\beta} = |\sum_j m_j (U_{ej})^2|$ is the effective mass measured in neutrinoless $\beta\beta$ decay. The approximation (19) indicates a close relationship and, indeed, an approximate proportionality between $\sigma(e^-e^- \to H^-H^-)$ and $m_{\beta\beta}^2$. Eq. (19) overestimates the cross section: $\sigma(e^-e^- \to H^-H^-)$ is much smaller than indicated by that approximation whenever $m_1 \gtrsim 0.5$ eV. On the other hand, for $m_1 \lesssim 0.1$ eV the approximation (19) becomes quite good, but at the same time the cross section becomes small. With the values of U used in Fig. 1, for $m_1 = 0.1$ eV, $\sqrt{s} = 10^3$ GeV and $m_H = 120$ GeV, Eq. (19) gives a cross section about 14% too large; for smaller s or larger m_H the discrepancy is smaller. Note that a cross section of about $10^{-2} \times \sigma_{\mathrm{QED}}$ is still considered reasonable for detection of $e^-e^- \to H^-H^-$ at an e^-e^- collider [6].

4. Possible non-collider effects

We next investigate whether large Yukawa couplings y_1 and y_2 in Eq. (4) might induce measurable effects in non-collider physics.

4.1. The magnetic dipole moment of the muon

A promising observable is a_μ, the anomalous magnetic moment of the muon. There is a puzzling 3σ discrepancy [18]

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 255(63)(49) \times 10^{-11} \tag{20}$$

between experiment and the SM prediction for that observable. In our model there are contributions to a_μ from one-loop diagrams involving either charged or neutral scalars. We consider the latter firstly. The fields ϕ_k^0 are written in terms of the physical neutral scalars S_b^0 ($b = 2, \ldots, 8$) as

$$\phi_k^0 = v_k + \sum_{b=1}^{8} \frac{\mathcal{V}_{kb} S_b^0}{\sqrt{2}}. \tag{21}$$

The scalar S_1^0 corresponds to the Goldstone boson eaten by the gauge boson Z^0 and is, therefore, unphysical. The complex 4×8 matrix $\mathcal{V}$ is such that

$$\begin{pmatrix} \mathrm{Re}\,\mathcal{V} \\ \mathrm{Im}\,\mathcal{V} \end{pmatrix} \tag{22}$$

is *orthogonal*[6]; therefore, $|\mathcal{V}_{kb}|^2 \leqslant 1$. For $b \geqslant 2$, let m_b be the mass of S_b^0 and $\delta_b = m_\mu^2/m_b^2 \ll 1$. The contribution of the physical neutral scalars to a_μ is given by

$$a_\mu(\phi^0) = \sum_{b=2}^{8} \frac{\delta_b}{16\pi^2} \int_0^1 \frac{\mathrm{d}x}{\delta_b x^2 - x + 1}\left[x^2\,\mathrm{Re}\,A_b + \left(x^2 - x^3\right)|A_b|\right] \tag{23}$$

$$= \sum_{b=2}^{8} \left\{ \frac{\delta_b}{16\pi^2}\left[\frac{|A_b|}{3} - \left(\frac{3}{2} + \ln\delta_b\right)\mathrm{Re}\,A_b\right] + \mathrm{O}\left(\delta_b^2\right)\right\}, \tag{24}$$

where $A_b = y_1^2 \mathcal{V}_{\mu b}^2$. One sees that there is a term $\delta_b \ln\delta_b/(16\pi^2)$ which is $\sim 10^{-7}$ when $m_b \sim 100$ GeV. Thus, if one assumes $|A_b| \sim 1$ then $|a_\mu(\phi^0)|$ might well be very large.

We proceed to analyze the diagrams involving charged scalars in the loop. The three left-handed neutrinos $\nu_{L\alpha}$ and the three right-handed neutrinos $\nu_{R\alpha}$ are written in terms of the six physical Majorana neutrinos χ_i ($i = 1, \ldots, 6$) as

$$\nu_{L\alpha} = \sum_{i=1}^{6} R_{\alpha i} P_L \chi_i, \qquad \nu_{R\alpha} = \sum_{i=1}^{6} S_{\alpha i}^* P_R \chi_i, \tag{25}$$

where $P_{L,R}$ are the chirality projectors in Dirac space and R, S are 3×6 matrices. The matrix

$$\begin{pmatrix} R \\ S \end{pmatrix} \tag{26}$$

is 6×6 unitary [20]. The fields ϕ_k^+ are written in terms of the physical charged scalars S_a^+ ($a = 2, 3, 4$) as

$$\phi_k^+ = \sum_{a=1}^{4} \mathcal{U}_{ka} S_a^+. \tag{27}$$

The scalar S_1^+ corresponds to the Goldstone boson eaten by the gauge boson W^+ and is, therefore, unphysical. The complex 4×4 matrix $\mathcal{U}$ is unitary [19].

Let m_i be the mass of the physical neutrino χ_i and, for $a \geqslant 2$, let m_a be the mass of S_a^+. The contribution of the diagrams with charged scalars to a_μ is given by

$$a_\mu(\phi^+) = \frac{1}{16\pi^2} \sum_{a=2}^{4} \sum_{i=1}^{6} \int_0^1 \mathrm{d}x\, \frac{m_\mu^2}{m_\mu^2 x^2 + (m_a^2 - m_\mu^2 - m_i^2)x + m_i^2} \times \left[\left(|y_1 R_{\mu i} \mathcal{U}_{\mu a}|^2 + |y_2 S_{\mu i} \mathcal{U}_{0a}|^2\right)\left(x^3 - x^2\right) + 2\frac{m_i}{m_\mu}\mathrm{Re}\left(y_1^* y_2^* R_{\mu i} S_{\mu i} \mathcal{U}_{\mu a}^* \mathcal{U}_{0a}\right)\left(x - x^2\right)\right]. \tag{28}$$

The three light neutrinos $\chi_{1,2,3}$ have masses much smaller than m_μ, hence these masses are negligible. For those neutrinos the matrix elements $S_{\mu i} \sim 10^{-6}$ are also negligible. One then has

[6] For more details on this notation see [19].

$$a_\mu(\phi^+)_{\text{light neutrinos}} \approx -\frac{|y_1|^2}{96\pi^2}\sum_{a=2}^{4}\delta_a|\mathcal{U}_{\mu a}|^2, \tag{29}$$

where $\delta_a = m_\mu^2/m_a^2$. For $m_a \sim 100$ GeV one thus has $a_\mu(\phi^+)_{\text{light neutrinos}} \sim -10^{-9}|y_1|^2$, which is not very significant. The three heavy neutrinos $\chi_{4,5,6}$ have masses comparable to those of the charged scalars. For those neutrinos the $R_{\mu i} \sim 10^{-6}$. One then has

$$\begin{aligned} a_\mu(\phi^+)_{\text{heavy neutrinos}} \approx \sum_{a=2}^{4}\sum_{i=4}^{6}\Bigg\{ & \frac{|y_2 S_{\mu i}\mathcal{U}_{0a}|^2}{16\pi^2}\frac{m_\mu^2}{m_i^2} f\left(\frac{m_a^2}{m_i^2}\right) \\ & + \frac{\mathrm{Re}(y_1^* y_2^* R_{\mu i} S_{\mu i}\mathcal{U}_{\mu a}^*\mathcal{U}_{0a})}{8\pi^2}\frac{m_\mu}{m_i} \\ & \times\left[\frac{1}{6}+\left(\frac{m_a^2}{m_i^2}-1\right)f\left(\frac{m_a^2}{m_i^2}\right)\right]\Bigg\}, \end{aligned} \tag{30}$$

where

$$f(x) = \frac{1}{3(x-1)} - \frac{x}{2(x-1)^2} + \frac{x}{(x-1)^3} - \frac{x\ln x}{(x-1)^4} \tag{31}$$

$$= -\frac{1}{3} + \left(-\frac{11}{6} - \ln x\right)x + O(x^2). \tag{32}$$

For Yukawa couplings of order 1, and for heavy neutrinos with masses of order 1 TeV, one obtains $a_\mu(\phi^+)_{\text{heavy neutrinos}} \sim 10^{-10}$, which is smaller than the experimental error in Eq. (20) and hence irrelevant.

So one concludes that the largest non-standard contribution to a_μ in our model is in principle $a_\mu(\phi^0)$, which is proportional to $|y_1|^2$ and may be as large as 10^{-7} if $|y_1|^2 \sim 1$. One must invoke either a small Yukawa coupling y_1 or cancellations between the contributions of the various neutral scalars for our model not to give too large a contribution to a_μ. One may also view (24) as a possible way to explain the discrepancy (20).

4.2. The decay $\mu^- \to e^-e^+e^-$

In our model, there are also lepton-flavour-changing processes due to the soft breaking of the family lepton numbers [20]. The process most likely to be observable is in general [20] $\mu^- \to e^-e^+e^-$,[7] which is mediated by $\mu^- \to e^- S_b^{0*}$ if the Yukawa couplings are large. Using the formulas in [20] we obtain the approximate upper bound

$$\mathrm{BR}(\mu^- \to e^-e^+e^-) \lesssim 10^{-5}|y_1 y_2|^4\left[\sum_b |\mathcal{V}_{eb}|^2\frac{(100\ \mathrm{GeV})^2}{m_b^2}\right]^2|F|^2, \tag{33}$$

where

$$F = \sum_{i=1}^{3} U_{ei}U_{\mu i}^* \ln\frac{M_i^2}{\bar\mu^2}, \tag{34}$$

$\bar\mu$ being an arbitrary mass parameter on which F finally does not depend. Clearly, if y_1, y_2 and F are all of order one, then $\mathrm{BR}(\mu^- \to e^-e^+e^-)$ will be much too large in our model, since experimentally that branching ratio cannot be larger than 1.0×10^{-12} at 90% confidence level [18]. One possibility to avoid this problem is by making $|y_1| \sim 0.01$, which is possible as we have seen in Section 2, and would have the further advantage of also suppressing $a_\mu(\phi^0)$, while keeping y_2 at order one in order not to suppress $\sigma(e^-e^- \to H^-H^-)$. Another possibility is to assume that the mass spectrum of the light neutrinos is almost degenerate, which in turn renders the mass spectrum of the heavy neutrinos almost degenerate too. With this condition and assuming U_{e3} to be exactly zero, we estimate

$$|F| \simeq |U_{e2}U_{\mu 2}|\frac{\Delta m_\odot^2}{m_1^2}. \tag{35}$$

Taking, for instance, $m_1 = 0.3$ eV, one obtains $|F|^2 \sim 10^{-7}$, which is sufficient to make Eq. (33) compatible with the experimental bound.

Therefore, our model may be compatible with the present experimental bound on $\mathrm{BR}(\mu^- \to e^-e^+e^-)$, if the mass spectrum of the light neutrinos is almost degenerate or the Yukawa coupling y_1 is of order 0.01. On the other hand, the Yukawa coupling y_2 may perfectly well be of order one.

5. Conclusions

We have started in this Letter with the trivial observations that, in the seesaw mechanism, a mass scale of 1 MeV in the neutrino Dirac mass matrix corresponds to right-handed neutrinos in the TeV range, and that the scale 1 MeV might be provided by the electron mass. We have then constructed a simple model, which extends the SM with three right-handed neutrino singlets and four Higgs doublets, in which that observation is realized. Our model has the following properties:

1. Each charged lepton α has a corresponding Higgs doublet ϕ_α which, through its VEV v_α, generates the charged-lepton mass $m_\alpha = |y_1 v_\alpha|$.
2. The Dirac mass matrix M_D of the neutrinos is generated by another Higgs doublet, ϕ_0, whose VEV v_0 is induced by the VEV v_e such that $v_0 \sim v_e$ or smaller, hence $v_0 \propto m_e$.
3. In the appropriate weak basis, the mass matrix of the charged leptons is diagonal while M_D is proportional to the unit matrix; this yields the simple relation $M_j \propto m_e^2/m_j$ between the masses M_j of the heavy Majorana neutrinos and the masses m_j of the light Majorana neutrinos.
4. Moreover, the diagonalization matrix of the light-neutrino mass matrix — which is just the lepton mixing matrix — and the diagonalization matrix of the heavy-neutrino mass matrix are complex conjugate of each other.
5. The mixing between light and heavy neutrinos is small, of order 10^{-6}.

The last property prevents heavy-neutrino production in pp and $p\bar p$ colliders. However, at an e^-e^- collider one might test the mechanism of the model by searching for the process $e^-e^- \to H^-H^-$, where H^- is a charged scalar; this process is somehow a high-energy analogue of neutrinoless $\beta\beta$ decay. The charged scalar would mainly decay into a light neutrino and the electron, if the heavy neutrinos are heavier than the scalar. Thus the signal would amount to e^-e^- plus missing momentum. At an e^+e^- collider one should, of course, look for $e^+e^- \to H^+H^-$.

If the Yukawa coupling y_1 of our model is of order one, then the anomalous magnetic moment of the muon may be too large, and the branching ratio of the flavour-changing process $\mu^- \to e^-e^+e^-$ as well; in this case the suppression mechanisms discussed in Section 4 have to be invoked. The simplest way to make our model compatible with experimental constraints is by

[7] The process $\mu^- \to e^-\gamma$ usually is more suppressed than $\mu^- \to e^-e^+e^-$ in this type of models [20].

choosing $|y_1| \sim 0.01$. However, this does not impede the process $e^-e^- \to H^-H^-$, which proceeds through a different Yukawa coupling y_2.

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