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Annihilation rates of heavy 1^{--} S-wave quarkonia in Salpeter method

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ABSTRACT

Annihilation rates of vector 1^{--} charmonium and bottomonium $^3S_1$ states $V \to e^+e^-$ and $V \to 3\gamma$, $V \to \gamma gg$ and $V \to 3g$ are estimated in the relativistic Salpeter method. In our calculations, special attention is paid to the relativistic correction, which is important and cannot be ignored for excited $2S$, $3S$ and higher excited states. We obtain $\Gamma(J/\psi \to 3\gamma) = 6.8 \times 10^{-4}$ keV, $\Gamma(\psi(2S) \to 3\gamma) = 2.5 \times 10^{-4}$ keV, $\Gamma(\psi(3S) \to 3\gamma) = 1.7 \times 10^{-4}$ keV, $\Gamma(\Upsilon(1S) \to 3\gamma) = 1.5 \times 10^{-5}$ keV, $\Gamma(\Upsilon(2S) \to 3\gamma) = 5.7 \times 10^{-6}$ keV, $\Gamma(\Upsilon(3S) \to 3\gamma) = 3.5 \times 10^{-6}$ keV and $\Gamma(\Upsilon(4S) \to 3\gamma) = 2.6 \times 10^{-6}$ keV.

1. Introduction

Annihilation decay of 1^{--} S-wave heavy quarkonia has been extensively studied [1–7]. The interest in this study comes from several sources. First, the annihilation amplitudes are related to the behavior of wave function, enabling an understanding of the formalism of inter-quark interactions. Further more, it can be a sensitive test of potential models [8]. Finally, the ratio of the decay widths, e.g. $\Gamma(V \to e^+e^-)/\Gamma(V \to 3g)$, is sensitive to the running coupling constant $\alpha_s(\mu)$, where V is a heavy quarkonium vector state and μ is the scale ($\mu = m_c$ for J/ψ and $\mu = m_b$ for Υ), and may provide very useful information for α_s at the heavy quark mass scale [9,10].

In our previous Letters [11], two-photon and two-gluon annihilation rates of $J^{PC} = 0^{-+}$, 0^{++} and 2^{++} $c\bar{c}$ and $b\bar{b}$ states are computed with the relativistic Salpeter method. Good agreement of the predictions with other theoretical calculations and the available experimental data is found. In the calculations, we found the relativistic corrections are large and not negligible, especially for high excited states, such as, the $2S$ and $3S$ states, because there are node structures in wave functions of $2S$ and $3S$ states, these cause large relativistic corrections even for heavy quarkonium like bottomonium. So in the theoretical studies concerning the highly excited states, a relativistic model is required.

The annihilation decays of the vector 1^{--} states are different from the C even states. Basically there are two types of annihilation decay modes, which are $V \to \gamma^* \to l^+l^-$ and $V \to 3\gamma$, γgg, $3g$. These decay widths have been studied in non-relativistic limit and found to be proportional to the square of the wave function at the origin $|\psi(0)|^2$ [1,12]. However, the decay rates of many processes are subject to substantial relativistic corrections [10,13,14]. In this Letter, we will continue to study the annihilation decays of $J^{PC} = 1^{--}$ $c\bar{c}$ and $b\bar{b}$ states with the relativistic Salpeter method.

There are two sources of relativistic corrections [4,11], one is the correction in relativistic kinematics which appears in the decay amplitudes through a well-defined form of relativistic wave function (i.e., not merely through the wave function at origin); the other relativistic correction comes via the relativistic inter-quark dynamics, which requires a relativistic formalism to describe the interactions among quarks and relativistic formalism to consider the transition amplitude. To consider the relativistic corrections, we choose the Salpeter method [15], which is an instantaneous version of Bethe–Salpeter method [16]. For the equal-mass quarkonium, the non-instantaneous correction is very small [17]. For the annihilation amplitude, we choose Mandelstam formalism [18], which is well suited for the computation of relativistic transition amplitude with Bethe–Salpeter wave functions as input.

In Section 2, we give theoretical details for the annihilation amplitude in Mandelstam formalism and the corresponding wave function with a well-defined relativistic form. The decay width of $V \to \gamma^* \to e^+e^-$ and $V \to 3\gamma$, γgg, $3g$ are formulated in this

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section. We will show the numerical results and give discussions in Section 3.

2. Theoretical details

2.1. The $V \to e^+e^-$ decay

According to Mandelstam formalism [18], the transition amplitude of a quarkonium decaying into an electron and a positron (see Fig. 1) can be written as

$$T_{e^+e^-} = i\sqrt{3}e^2 e_q \int \frac{d^4q}{(2\pi)^4} \mathrm{tr}\big[\chi(q)\gamma_\mu\big] \frac{g^{\mu\nu}}{M^2} \bar{u}_{r_1}(\vec{k}_1)\gamma_\nu v_{r_2}(\vec{k}_2), \quad (1)$$

where $e_q = \frac{2}{3}$ for charm quark and $e_q = -\frac{1}{3}$ for bottom quark; $\vec{k}_1$ and $\vec{k}_2$ are the momenta of electron and positron respectively; M is the meson mass; $\chi(q)$ is the Bethe–Salpeter wave function with the total momentum P and relative momentum q, related by

$$p_1 = \alpha_1 P + q, \quad \alpha_1 \equiv \frac{m_1}{m_1 + m_2},$$
$$p_2 = \alpha_2 P - q, \quad \alpha_2 \equiv \frac{m_2}{m_1 + m_2},$$

where $m_1 = m_2$ is the constitute quark mass of charm or bottom.

After performing the integration over q^0, one reduce the expression, with the notation of Salpeter wave function $\Psi(\vec{q}) = i\int \frac{dq_0}{2\pi}\chi(q)$, to

$$T_{e^+e^-} = \sqrt{3}e^2 e_q \int \frac{d\vec{q}}{(2\pi)^3} \mathrm{tr}\big[\Psi(\vec{q})\gamma_\mu\big] \frac{g^{\mu\nu}}{M^2} \bar{u}_{r_1}(\vec{k}_1)\gamma_\nu v_{r_2}(\vec{k}_2). \quad (2)$$

We note that the form of the wave function is also important in the calculation, since the corrections of the relativistic kinetics come mainly through it. By analyzing the parity and charge conjugation, the general form of relativistic wave function of 1^- state (1^{--} for equal mass systems) can be written as [19]

$$\begin{aligned} \Psi^\lambda_{1^-}(q_\perp) &= q_\perp \cdot \epsilon^\lambda_\perp \left[f_1(q_\perp) + \frac{\not{P}}{M} f_2(q_\perp) + \frac{\not{q}_\perp}{M} f_3(q_\perp) + \frac{\not{P}\not{q}_\perp}{M^2} f_4(q_\perp) \right] \\ &+ M\not{\epsilon}^\lambda_\perp f_5(q_\perp) + \not{\epsilon}^\lambda_\perp \not{P} f_6(q_\perp) + \big(\not{q}_\perp \not{\epsilon}^\lambda_\perp - q_\perp \cdot \epsilon^\lambda_\perp\big) f_7(q_\perp) \\ &+ \frac{1}{M}\big(\not{P}\not{\epsilon}^\lambda_\perp \not{q}_\perp - \not{P} q_\perp \cdot \epsilon^\lambda_\perp\big) f_8(q_\perp), \end{aligned} \quad (3)$$

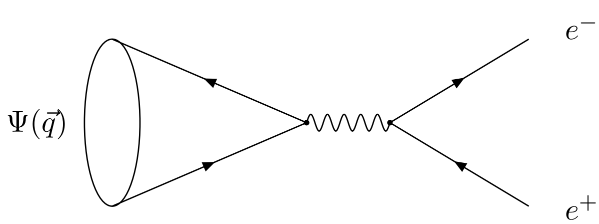

Fig. 1. Leptonic decay diagram of quarkonium.

where P and $\epsilon^\lambda_\perp$ are the momentum and polarization vector of the vector meson; $q_\perp = (0, \vec{q})$. The 8 wave functions f_i are not independent due to the equations $\varphi^{+-}_{1^-}(q_\perp) = \varphi^{-+}_{1^-}(q_\perp) = 0$. For quarkonium states we get the constraints on the components of the wave functions [19]:

$$f_1(q_\perp) = \frac{q^2_\perp f_3(q_\perp) + M^2 f_5(q_\perp)}{M m_1}, \qquad f_7(q_\perp) = 0,$$
$$f_8(q_\perp) = -\frac{M f_6(q_\perp)}{m_1}, \qquad f_2(q_\perp) = 0.$$

With these constraints, only four independent components f_3, f_4, f_5 and f_6 are left. Namely

$$\begin{aligned} \Psi^\lambda_{1^{--}}(q_\perp) &= q_\perp \cdot \epsilon^\lambda_\perp \left(\frac{q^2_\perp}{M m_1} + \frac{\not{q}_\perp}{M} \right) f_3(q_\perp) + q_\perp \cdot \epsilon^\lambda_\perp \frac{\not{P}\not{q}_\perp}{M^2} f_4(q_\perp) \\ &+ \left(M\not{\epsilon}^\lambda_\perp + q_\perp \cdot \epsilon^\lambda_\perp \frac{M}{m_1} \right) f_5(q_\perp) \\ &+ \left[\not{\epsilon}^\lambda_\perp \not{P} + \frac{\not{P}(q_\perp \cdot \epsilon^\lambda_\perp)}{m_1} - \frac{\not{P}\not{\epsilon}^\lambda_\perp \not{q}_\perp}{m_1} \right] f_6(q_\perp). \end{aligned} \quad (4)$$

These wave functions and the bound state mass M can be obtained by solving the full Salpeter equation with the constituent quark mass as input. We will not show the details of how to solve the full Salpeter equation, only give the final results. Interested readers can find the detail technique in Refs. [19,20].

Defining the decay constant f_V by

$$f_V M \epsilon^\lambda_\mu \equiv \langle 0|\bar{q}_1 \gamma_\mu q_2|V, \epsilon\rangle = \sqrt{3}\int \frac{d^3q}{(2\pi)^3} \mathrm{tr}\big[\varphi(\vec{q})\gamma_\mu\big], \quad (5)$$

and with Eq. (4) we can easily obtain

$$f_V = 4\sqrt{3}\int \frac{d\vec{q}}{(2\pi)^3} \left[f_5(\vec{q}) - \frac{\vec{q}^2}{3M^2} f_3(\vec{q}) \right]. \quad (6)$$

Summing over the polarizations of the final states and averaging over that of the initial state, neglecting the electron mass, it is easy to get the decay width

$$\Gamma_{e^+e^-} = \frac{4\pi}{3}\alpha^2 e^2_q f^2_V / M. \quad (7)$$

2.2. $V \to 3\gamma$, $V \to \gamma gg$ and $V \to 3g$ decays

With the notation and definition used in the previous subsection, the relativistic transition amplitude of a quarkonium decaying into three photons (see Fig. 2) can be written as

$$\begin{aligned} T_{3\gamma} &= \sqrt{3}(iee_q)^3 \int \frac{d^4q}{(2\pi)^4} \\ &\times \mathrm{tr}\left\{ \chi(q) \left[\not{\epsilon}_3 \frac{1}{\not{k}_3 - \not{p}_2 - m} \not{\epsilon}_2 \frac{1}{\not{p}_1 - \not{k}_1 - m} \not{\epsilon}_1 \right.\right. \\ &\left.\left. + \text{all other permutations of } 1, 2, 3 \right] \right\}, \end{aligned} \quad (8)$$

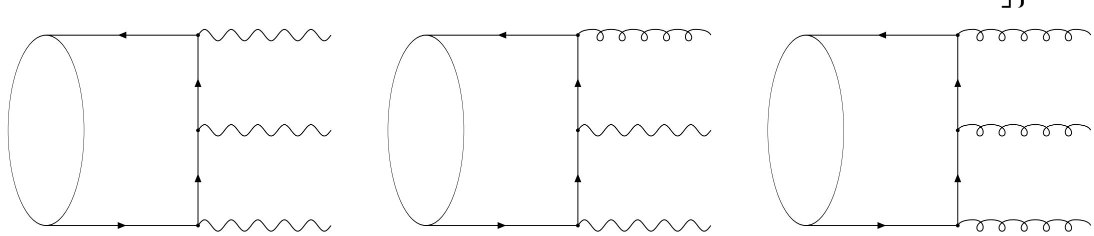

Fig. 2. 3γ, γgg, $3g$ annihilation diagrams of quarkonium. We do not show other diagrams with all the possible permutations of photons and gluons.

where k_1, k_2, k_3 and $\epsilon_1, \epsilon_2, \epsilon_3$ are the momenta and polarization vectors of three photons respectively.

Since $p_{10} + p_{20} = M$, we assume $p_{10} = p_{20} = M/2$ as did in Ref. [11]. Having this assumption, we can perform the integration over q_0 to reduce the expression to

$$T_{3\gamma} = \sqrt{3}(ee_q)^3 \int \frac{d^3\vec{q}}{(2\pi)^3} \times \mathrm{tr}\left\{\Psi(\vec{q})\left[\not{\epsilon}_3 \frac{1}{\not{k}_3 - \not{\tilde{p}}_2 - m}\not{\epsilon}_2 \frac{1}{\not{\tilde{p}}_1 - \not{k}_1 - m}\not{\epsilon}_1 + \text{all other permutations of } 1,2,3\right]\right\}, \quad (9)$$

where $\tilde{p}_1 = (\frac{M}{2}, \vec{q})$, $\tilde{p}_2 = (\frac{M}{2}, -\vec{q})$.

The decay width is given by

$$\Gamma_{3\gamma} = \frac{1}{3!}\frac{1}{8M(2\pi)^3}\int_0^{\frac{M}{2}} dk_1^0 \int_{\frac{M}{2}-k_1^0}^{\frac{M}{2}} dk_2^0 \frac{1}{3}\sum_{\text{pol}} |T_{3\gamma}|^2. \quad (10)$$

The width of $V \to \gamma gg$ and $V \to 3g$ are related to the three photon decay width by

$$\Gamma_{\gamma gg} = \frac{2}{3}\frac{\alpha_s^2}{\alpha^2 e_q^4}\Gamma_{3\gamma}, \quad (11)$$

$$\Gamma_{3g} = \frac{5}{54}\frac{\alpha_s^3}{\alpha^3 e_q^6}\Gamma_{3\gamma}. \quad (12)$$

For gluonic decay $V \to 3g$, the trace of color generators gives $\mathrm{tr}[T_a T_b T_c] = \frac{1}{4}(d_{abc} + i f_{abc})$, so the expression of decay width contains two parts, one of which is proportional to the square of symmetric constants and the other is proportional to the square of antisymmetric constants. The existence of antisymmetric term breaks the relation Eq. (12). However, our calculation shows that the antisymmetric term is sufficiently small compared to the symmetric term, so we can ignore it safely. In non-relativistic limit the antisymmetric term vanishes exactly.

Table 1
Mass spectra of the $c\bar{c}$ and $b\bar{b}$ $1^{--}(^3S_1)$ states ($^3D_1$ states are not presented) in unit of MeV. The experimental data are taken from PDG [22].

$\psi(nS)$	Th($c\bar{c}$)	Ex($c\bar{c}$)	$\Upsilon(nS)$	Th($b\bar{b}$)	Ex($b\bar{b}$)
J/ψ	3096.9	3096.916	$\Upsilon(1S)$	9460.3	9460.30
$\psi(2S)$	3688.4	3686.09	$\Upsilon(2S)$	10024	10023.26
$\psi(3S)$	4056.0	4039	$\Upsilon(3S)$	10371	10355.2
$\psi(4S)$	4327.7	4421	$\Upsilon(4S)$	10635	10579.4
$\psi(5S)$	4543.3		$\Upsilon(5S)$	10853	10865

3. Numerical results and discussions

To solve the full Salpeter equation, we choose a phenomenological Cornell potential. There are some parameters in this potential including the constituent quark mass and one loop running coupling constant. The following best-fit values of input parameters were obtained by fitting the mass spectra for heavy quarkonium 1^{--} states [21]:

$$m_c = 1.62\ \text{GeV}, \qquad m_b = 4.96\ \text{GeV}. \quad (13)$$

For $c\bar{c}$ system, we set $\Lambda_{QCD} = 0.27$ GeV. With this parameter set, we solve the full Salpeter equation and obtain the mass spectra shown in Table 1. To give numerical results, we need to fix the value of the renormalization scale μ in $\alpha_s(\mu)$. In the case of charmonium, we choose the charm quark mass m_c as the energy scale and obtain the coupling constant $\alpha_s(m_c) = 0.38$ [11].

For $b\bar{b}$ system we set $\Lambda_{QCD} = 0.20$ GeV. With this parameter set, the coupling constant at the scale of bottom quark mass is $\alpha_s(m_b) = 0.23$ [11]. The mass spectra are also shown in Table 1.

With the obtained wave function, Eq. (6) and Eq. (7), we calculate the decay width of $V \to e^+e^-$ for $c\bar{c}$ system. The results, with other theoretical predictions and experimental data from Particle Data Group, are shown in Table 2. Our results are larger than experimental data and consistent with the Beyer's [23] model version b results and Li's results [29]. The discrepancy between ours and experiment's may be due to the QCD corrections. We only have the leading order QCD correction $1 - \frac{16}{3}\frac{\alpha_s}{\pi}$ [3] in hand, while the large factor $\frac{16}{3}$ implies that high order QCD corrections can be still large and quite essential [30,31], so we only show the results without QCD corrections.

Decay widths of $\Upsilon(nS) \to e^+e^-$ are shown in Table 3. All the results, with or without QCD corrections, are consistent with each other, with only small discrepancies. Since the small value of α_s at the energy scale of bottom quark, corresponds to much smaller QCD corrections in bottomonium states than those in charmonium states, less discrepancies exit among the results of $\Upsilon(nS)$ decays than of $\psi(nS)$ decays.

The ratios of the high excited-state widths to the ground-state width $\Gamma(nS)/\Gamma(1S)$ are free from the QCD corrections and sensitive to wave functions. We show the ratios of leptonic decay widths in Table 4. Our theoretical values are comparable to the PDG data, except those in $\psi(3S)$ and $\psi(4S)$ states. It is can be seen from the table that the ratios, so do the decay widths, fall very slowly with successive radial excitations, which indicates that the relativistic corrections are large for high excited states.

Decay widths $\Gamma_{3\gamma}$, $\Gamma_{\gamma gg}$ and Γ_{3g} of charmonia and bottomonia are calculated with Eqs. (9)–(12). The results and other theoretical estimates as well as experimental data are shown in Tables 5 and 6. The decay widths quoted from Ref. [9] and Ref. [30] are estimated based on experimental data. As in the e^+e^- decays case, we only have the leading order QCD correction, e.g. $1 - 12.6\frac{\alpha_s}{\pi}$ [3],

Table 2
Decay width $\Gamma(\psi(nS) \to e^+e^-)$ in unit of keV. The results marked by † do not cover the contributions of QCD corrections.

$\psi(nS)$	J/ψ	$\psi(2S)$	$\psi(3S)$	$\psi(4S)$	$\psi(5S)$
Ours†	10.9	5.22	3.49	2.61	2.07
Beyer Ver.a† [23]	5.33	2.31	1.59	1.14	
Beyer Ver.b† [23]	11.2	4.06	2.74	2.06	
EQ† [24]	8.00	3.67			
VPBK† [25]	5.469	2.140	0.796	0.288	
GVGV [26]	2.94	1.22	0.76	0.43	0.27
IS [27]	6.72 ± 0.49	2.66 ± 0.19			
SYEF [28]	3.93	1.78	1.11		
LC† [29]	11.8	4.29	2.53	1.73	1.25
PDG [22]	$5.55 \pm 0.14 \pm 0.02$	2.38 ± 0.04	0.86 ± 0.07	0.58 ± 0.07	

Table 3
Decay width $\Gamma(\Upsilon(nS) \to e^+e^-)$ in unit of keV. The results marked by † do not cover the contributions of QCD corrections.

$\Upsilon(nS)$	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$\Upsilon(4S)$	$\Upsilon(5S)$
Ours†	1.47	0.736	0.530	0.425	0.359
Beyer Ver.a† [23]	1.24	0.51	0.35	0.28	
Beyer Ver.b† [23]	1.41	0.56	0.36	0.30	
EQ† [24]	1.71	0.76	0.55		
VPBK† [25]	1.320	0.628	0.263	0.104	0.0404
GVGV [26]	0.98	0.41	0.27	0.20	0.16
IS [27]	1.45 ± 0.07	0.52 ± 0.02	0.35 ± 0.02		
González [32]	1.7	0.61	0.39	0.27	0.21
PDG [22]	1.340 ± 0.018	0.612 ± 0.011	0.443 ± 0.008	0.272 ± 0.029	0.31 ± 0.07

Table 4
The ratios of the high excited-state widths to the ground-state width $\Gamma(nS)/\Gamma(1S)$ for decay $\Gamma(\psi(nS) \to e^+e^-)$ and $\Gamma(\Upsilon(nS) \to e^+e^-)$.

$\Gamma(\psi(nS))/\Gamma(\psi(1S))$	$\Gamma(2S)/\Gamma(1S)$	$\Gamma(3S)/\Gamma(1S)$	$\Gamma(4S)/\Gamma(1S)$	$\Gamma(5S)/\Gamma(1S)$
Ours	0.48	0.32	0.24	0.19
PDG [22]	0.43	0.15	0.10	
$\Gamma(\Upsilon(nS))/\Gamma(\Upsilon(1S))$	$\Gamma(2S)/\Gamma(1S)$	$\Gamma(3S)/\Gamma(1S)$	$\Gamma(4S)/\Gamma(1S)$	$\Gamma(5S)/\Gamma(1S)$
Ours	0.50	0.36	0.29	0.24
PDG [22]	0.46	0.33	0.20	0.23

Table 5
Decay widths of $\psi(nS) \to 3\gamma,\ \gamma gg,\ 3g$ in unit of keV. The data marked by $*$ is quoted from Ref. [35]. The results marked by † do not cover the contributions of QCD corrections.

Decay	Ours†	GI† [8]	ML [33]	PCP† [34]	SYEF [28]	Voloshin [30]
$\Gamma(J/\psi \to 3g)$	101	176	80 ± 40	63.72		61.5 ± 3.1
$\Gamma(\psi(2S) \to 3g)$	36.6	78.4		20.49		45.3 ± 9.3
$\Gamma(\psi(3S) \to 3g)$	24.7			11.92		
$\Gamma(\psi(4S) \to 3g)$	19.8			8.08		
$\Gamma(\psi(5S) \to 3g)$	16.7					
$\Gamma(J/\psi \to \gamma gg)$	6.18		7.5 ± 3			7.46 ± 2.80
$\Gamma(\psi(2S) \to \gamma gg)$	2.25					3.04
$\Gamma(\psi(3S) \to \gamma gg)$	1.52					
$\Gamma(\psi(4S) \to \gamma gg)$	1.22					
$\Gamma(\psi(5S) \to \gamma gg)$	1.03					
$\Gamma(J/\psi \to 3\gamma)$	0.68×10^{-3}				0.56×10^{-3}	$^{*}(1.12 \pm 0.47) \times 10^{-3}$
$\Gamma(\psi(2S) \to 3\gamma)$	0.25×10^{-3}					
$\Gamma(\psi(3S) \to 3\gamma)$	0.17×10^{-3}					
$\Gamma(\psi(4S) \to 3\gamma)$	0.13×10^{-3}					
$\Gamma(\psi(5S) \to 3\gamma)$	0.11×10^{-3}					

Table 6
Decay widths of $\Upsilon(nS) \to 3\gamma,\ \gamma gg,\ 3g$ in unit of keV. The results marked by † do not cover the contributions of QCD corrections.

Decay	Ours†	GI† [8]	ML [33]	KMRR [9]
$\Gamma(\Upsilon(1S) \to 3g)$	32.5	44.1	28 ± 6	42.9 ± 1.2
$\Gamma(\Upsilon(2S) \to 3g)$	12.0	22.5		
$\Gamma(\Upsilon(3S) \to 3g)$	7.47	16.9		
$\Gamma(\Upsilon(4S) \to 3g)$	5.52	12.1		
$\Gamma(\Upsilon(5S) \to 3g)$	4.41			
$\Gamma(\Upsilon(1S) \to \gamma gg)$	0.826		0.9 ± 0.2	1.20
$\Gamma(\Upsilon(2S) \to \gamma gg)$	0.304			
$\Gamma(\Upsilon(3S) \to \gamma gg)$	0.190			
$\Gamma(\Upsilon(4S) \to \gamma gg)$	0.140			
$\Gamma(\Upsilon(5S) \to \gamma gg)$	0.112			
$\Gamma(\Upsilon(1S) \to 3\gamma)$	0.15×10^{-4}			
$\Gamma(\Upsilon(2S) \to 3\gamma)$	0.57×10^{-5}			
$\Gamma(\Upsilon(3S) \to 3\gamma)$	0.35×10^{-5}			
$\Gamma(\Upsilon(4S) \to 3\gamma)$	0.26×10^{-5}			
$\Gamma(\Upsilon(5S) \to 3\gamma)$	0.21×10^{-5}			

in hand, and the large factor 12.6 implies that if we consider the QCD corrections, we need include high order QCD corrections not only the leading one. Besides, current leading order QCD factor is too sensitive to the value of α_s, which make other contributions, such as relativistic corrections, unclear, so we only show the results without QCD corrections. For the same reason in the leptonic decays case, we show the ratios $\Gamma_{3g}(nS)/\Gamma_{3g}(1S)$ in Table 7.

A typical non-relativistic calculation gives $\mathcal{B}(J/\psi \to 3\gamma) \sim 3 \times 10^{-5}$ [28,30], while our relativistic result $\mathcal{B}(J/\psi \to 3\gamma) \sim 0.73 \times 10^{-5}$ (the total width of J/ψ is 93.2 ± 2.1 keV [22]) is much smaller than the non-relativistic one, but within the experimental error bar. This indicates that the relativistic corrections for charmonia 3γ decays, so do the γgg, $3g$ decays, are large. This conclusion is also obtained by other authors, see Refs. [10,13,14]. Besides, our calculations show that the relativistic corrections for higher excited states are even lager than those for the ground state and lower excited states.

One can see from Tables 5–7, that the decay widths, which are sensitive to the wave functions of corresponding states, fall very slowly from $1S$ to $5S$. We obtained the similar results as the cases of e^+e^- decays. This behavior is different from the non-relativistic models, where the values fall quickly from $1S$ to $5S$. It shows that the relativistic corrections are large and important, especially for the higher excited states. It is believed that the relativistic corrections are small for bottomonium, however, we point that, this is true for ground state, but not exactly true for the excited states, especially for high excited states.

Table 7
The ratios of the high excited-state widths to the ground-state width $\Gamma(nS)/\Gamma(1S)$ for decay $\Gamma(\psi(nS) \to 3g)$ and $\Gamma(\Upsilon(nS) \to 3g)$.

$\Gamma(\psi(nS))/\Gamma(\psi(1S))$	$\Gamma(2S)/\Gamma(1S)$	$\Gamma(3S)/\Gamma(1S)$	$\Gamma(4S)/\Gamma(1S)$	$\Gamma(5S)/\Gamma(1S)$
Ours	0.36	0.24	0.20	0.17
GI [8]	0.45			
PCP [34]	0.32	0.19	0.13	
Voloshin [30]	0.74			
$\Gamma(\Upsilon(nS))/\Gamma(\Upsilon(1S))$	$\Gamma(2S)/\Gamma(1S)$	$\Gamma(3S)/\Gamma(1S)$	$\Gamma(4S)/\Gamma(1S)$	$\Gamma(5S)/\Gamma(1S)$
Ours	0.37	0.23	0.17	0.14
GI [8]	0.51	0.38	0.27	

Table 8
Relative deviations of decay width $\Gamma_{3\gamma}$ with assumptions $p_{10} = 0.9 \times M/2$, $p_{20} = 1.1 \times M/2$ from that with $p_{10} = p_{20} = M/2$.

nS	$1S$	$2S$	$3S$	$4S$	$5S$
$c\bar{c}$	−2.0%	−2.9%	−4.7%	−7.0%	−8.2%
$b\bar{b}$	−1.8%	−2.0%	−2.2%	−2.4%	−2.6%

In calculating the decay widths $\Gamma_{3\gamma}$, we assume $p_{10} = p_{20} = M/2$. To show the effect of relaxing this assumption, we take $p_{10} = 0.9 \times M/2,\ p_{20} = 1.1 \times M/2$ and estimate the relative deviations of decay widths $(\Gamma - \Gamma_0)/\Gamma_0$, which are shown in Table 8. We interchange the values of p_{10} and p_{20}, say, take $p_{10} = 1.1 \times M/2,\ p_{20} = 0.9 \times M/2$, and find that the results are exactly the same as the unchanged case as expected.

In summary, by solving the relativistic full Salpeter equation with a well-defined form of wave function, we estimate annihilation decay rates of heavy quarkonium $1^{--}(^3S_1)$ states including $V \to e^+e^-$, $V \to 3\gamma$, $V \to \gamma gg$ and $V \to 3g$. We conclude that the relativistic correction and QCD correction in these annihilation decays play important roles, and high order QCD corrections are expected.

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