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Procedia Engineering 15 (2011) 5284 – 5289

**Procedia  
Engineering**[www.elsevier.com/locate/procedia](http://www.elsevier.com/locate/procedia)

## Advanced in Control Engineering and Information Science

# The Vehicle Routing Problem with Simultaneous Pickup and Delivery Based on Customer Satisfaction

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### Abstract

The vehicle routing problem with simultaneous pickup and delivery considering customer satisfaction is based on a time window at each customer location. In such a problem, the transportation requests have to be performed by vehicles, each request having to be met as early as possible. The customer satisfaction is inversely proportional to the waiting time for the vehicle from the lower bound of the time window. The goal is to minimize the total length of vehicles' paths to reduce cost, and to maximize the sum of all customer satisfactions to improve service quality. Initial solution obtained by the cheapest insertion method can be improved by tabu search algorithm. Finally computational results are reported on test instances.

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*Keywords:* Vehicle Routing Problem; Cheapest Insertion Method; Tabu Search

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### 1. Introduction

The Vehicle Routing Problem (VRP) was put forward by Dantzig and Ramser<sup>[1]</sup> for the first time in 1959. In the classic vehicle routing problem, vehicles have to provide services for customers like retailers, subjected to the capacity of each vehicle, at the minimum total cost. The Vehicle Routing Problem with Time Window (VRPTW) is one of variants of VRP. In 1995, R A Russell<sup>[2]</sup> addressed the effective heuristics including both tour construction and local search tour improvement heuristics. And A Landrieu et al.<sup>[3]</sup> presented tabu search and probabilistic tabu search for the single vehicle, and G Pankratz<sup>[4]</sup> proposed the grouping genetic algorithm in which each gene provided a group of requests. In 2011, A

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Bettinelli et al.<sup>[5]</sup> presented a branch-and-cut-and-price algorithm. The Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRPSPD) is another extension of VRP. It was brought forward by Min<sup>[6]</sup> for the first time in 1989, who solved small scale of practical problems in which vehicles have the same capacities. [7-9] also developed different algorithms to solve different situations, respectively.

Furthermore, The Vehicle Routing Problem with Simultaneous Pickup and Delivery with Time Windows (VRPTWSPD) is the combination with VRPTW and VRPSPD. In [10], improved genetic algorithm is proposed to avoid effectively the common defects of early convergence and the diversity of population in traditional genetic algorithm. Chang et al.<sup>[11]</sup> studied the real-time situation.

This work focuses on The Vehicle Routing Problem with Simultaneous Pickup and Delivery Based on Customer Satisfaction (VRPSPDCS), which is the variant of VRPTWSPD. In today's competitive environment, it is obvious that logistics companies should make strategic and operational decisions in order to optimize the processes in their supply chain more efficiently. On the one hand, the strategic decisions concerns the design of routing networks since it offers great potential to reduce costs. On the other hand, the operational decisions must take into account the customers' time windows, since service quality evaluated by the customers is closely effected by the situation whether they are served as early as possible or not. In fact, because the number of vehicle is finite, not all customers' time windows requirement can be always met. In this paper the service quality is reflected by the customer satisfaction which is inversely proportional to the waiting time for the vehicle from the lower bound of the time window, that is to say, the shorter the waiting time the more the satisfaction. The overall distribution network is to minimize the total length of vehicles' paths, and to maximize the sum of all customer satisfactions. At present, there is little literature about VRPSPDCS. W Bin et al.<sup>[12]</sup> studied the opening situation in which vehicles are permitted not to return the company after finishing the task.

The rest of this paper is organized as follows. Problem definition and mathematical formulation are given in Section 2. Section 3 describes the cheapest insertion method to obtain the initial solution, and the tabu search algorithm is explained to improve the solution. Section 4 reports computational results.

## 2. Problem definition and mathematical formulation

The Vehicle Routing Problem with Simultaneous Pickup and Delivery Based on Customer Satisfaction (VRPSPDCS) can be defined as follows: let  $G=(V,A)$  be a complete directed network where  $V=\{O\} \cup T$ , in which  $O$  represent the company, and  $T=\{1,2,\dots,n\}$  represent customers, and  $A=\{(i,j)|i,j \in V, i \neq j\}$  is the set of arcs. Each arc  $(i,j) \in N$  has a nonnegative cost (distance)  $c_{ij}$  and triangular inequality holds (i.e.,  $c_{ij} + c_{jk} \leq c_{ik}$ ). There are  $K$  vehicles with capacity  $Q$  and a speed of  $v$ . Each customer  $j$  ( $j \in A$ ) has demands including pickup  $p_j$  and delivery  $d_j$  ( $0 < d_j, p_j \leq Q$ ). Customer  $j$ 's time window is  $[E_j, L_j]$  ( $j \in T$ ). Let  $S_j$  denote customer  $j$ 's satisfaction,  $a_j$  is the inversely proportional coefficient to customer  $j$ 's waiting time, denoted by  $w_j$ , from  $E_j$  ( $j \in T$ ). Note that the time used for pickup and delivery is trivial to be ignored. The problem is to minimize the total length of vehicles' paths to reduce cost, and to maximize the sum of all customer satisfactions to increase service quality.

To formulate the problem VRPSPDCS, the following variables are used:

$t_{ij}^k$ : if vehicle  $k$  travels directly from node  $i$  to node  $j$ , the starting time of pickup and delivery at node  $j$ ;

$x_{ij}^k$ : if vehicle  $k$  travels directly from node  $i$  to node  $j$ , the value is 1; otherwise, 0;

$y_{ij}^k$ : if vehicle  $k$  travels from node  $i$  to node  $j$ , demand to be delivered to customers;

$z_{ij}^k$ : if vehicle  $k$  travels from node  $i$  to node  $j$ , demand to be pickup to customers;

VRPEPDCS is given as follows

$$\min \sum_{i=0}^n \sum_{j=0}^n \sum_{k=1}^K c_{ij} x_{ij}^k$$

(1)

$$\max \sum_{j=1}^n S_j = \sum_{j=1}^n \sum_{i=0}^n \sum_{k=1}^K \frac{a_j}{1 + w_i(t_{ij}^k)}$$

(2)

$$s.t. \sum_{i=0}^n \sum_{k=1}^K x_{ij}^k = 1 \quad j \in T$$

(3)

$$\sum_{j=0}^n \sum_{k=1}^K x_{ij}^k = \sum_{j=0}^n \sum_{k=1}^K x_{ji}^k \quad i \in T$$

(4)

$$\sum_{j=0}^n \sum_{k=1}^K x_{ji}^k y_{ji}^k - \sum_{j=0}^n \sum_{k=1}^K x_{ij}^k y_{ij}^k = d_i \quad i \in T$$

(5)

$$\sum_{j=0}^n \sum_{k=1}^K x_{ij}^k z_{ij}^k - \sum_{j=0}^n \sum_{k=1}^K x_{ji}^k z_{ji}^k = p_i \quad i \in T$$

(6)

$$\sum_{j=0}^n x_{ij}^k (y_{ij}^k + z_{ij}^k) \leq Q \quad i \in V, k = 1, \dots, K$$

(7)

$$t_{0j}^k = \sum_{j=1}^n x_{0j}^k \frac{c_{0j}}{v} \quad j \in T, k = 1, \dots, K$$

(8)

$$w_j(t_{ij}^k) = \begin{cases} \max \{0, \sum_{l=0}^n x_{il}^k t_{il}^k + \frac{1}{v} c_{ij} - E_j\} & x_{ij}^k = 1 \\ M & x_{ij}^k = 0 \end{cases} \quad i \in T, j \in V, k = 1, \dots, K$$

(9)

$$x_{ij}^k \in \{0, 1\} \quad i, j \in V, i \neq j, k = 1, \dots, K$$

(10)

$$y_{ij}^k, z_{ij}^k, t_{ij}^k \geq 0 \quad i, j \in V, i \neq j, k = 1, \dots, K$$

(11)

where  $M$  is a large enough positive number .

In this formulation, objective function (1) minimizes the total transportation cost, and objective function (2) maximizes the overall customers' satisfaction. Constraints (3) and (4) are known as degree constraints. Constraints (5) and (6) are flow conservation constraints for delivery and pickup demands, respectively. Constraint (7) implies that total load on any arc must not exceed the vehicle capacity. While

constraints (8)–(9) provide how to calculate the waiting times of customers and ensure that the waiting times are nonnegative and reasonable. Finally, constraints (10)–(11) are known as integrality constraints which define the nature of the decision variables.

### 3. The problem-solving methodology

Combine the objective (1) and (2) to form the following formulation (12).

$$\min \lambda_1 \sum_{i=0}^n \sum_{j=0}^n \sum_{k=1}^K c_{ij} x_{ij}^k - \lambda_2 \sum_{i=1}^n w_i(t_i) \quad (12)$$

where  $\lambda_1 + \lambda_2 = 1$ .

#### 3.1. Initial Solution by Cheapest Insertion Method

Select the unvisited customer with the smallest index number to insert into two visited customers at the lowest cost. Under the capacity constraints of vehicles, the modified saving cost is defined as follows.

$$Sav_1(i, u, j) = (c_{iu} + c_{uj} - c_{ij}) / (2 \times c_{max})$$

$$Sav_2(i, u, j) = (t_{ju} - t_j) / w_{max}$$

$$Sav_3(i, u, j) = (W_u - W) / (m \times w_{max})$$

$$Sav(i, u, j) = \gamma_1 Sav_1(i, u, j) + \gamma_2 Sav_2(i, u, j) + \gamma_3 Sav_3(i, u, j)$$

Where  $u$  represents the customer waiting to insert,  $i$  and  $j$  represent two neighbor customers in the present routing paths.  $t_{ju}$  represent service time for customer  $j$  after inserting  $u$  before  $j$ .  $W$ ,  $W_u$  respectively represent the total waiting time before and after inserting  $u$ .  $Sav_1$  represents the additional length of paths after inserting  $u$ .  $Sav_2$  represents the delaying time after inserting  $u$ .  $Sav_3$  represents the additional waiting time in the routing after inserting  $u$ .  $m$  is the amount of customers in the routing network at present. Where  $\gamma_1, \gamma_2, \gamma_3$  are weight coefficient, satisfying  $\gamma_1 + \gamma_2 + \gamma_3 = 1$ .

### 3.2. Improvement Solution by Tabu Search

The framework of the tabu search is standard, and can be generalization as follows<sup>[10]</sup>:

*Step 1 Let the current solution be the one constructed according to Section 3.1*

*Step 2 While the stopping criterion is not satisfied, take the following steps:*

*2.1 Generate the neighborhood of the current solution by applying swap and insertion operations*

*2.2 Evaluate the set of moves generated and define the best non-tabu move to construct the new current solution*

*2.3 Execute the corresponding best move and update the data structures*

*2.4 If the new current solution is better than the best encountered, record it*

*Step 3 Return the best solution*

Now the main components have to be explained.

- (a) Neighborhood structure. We adopt the swap operation and the insertion operation to create the neighborhood of the solution, and the swap operation is the classical 2-opt.
- (b) Tabu list structure. Two arrays tabu-swap and tabu-insert are used to represent tabu moves. The evaluation of the tabu state of a move is obtained after comparing the functions of the move used.
- (c) Tabu list size. The value of the size is in the interval  $[n/4, n/2]$  to balance the different process.
- (d) Aspiration criterion. The global aspiration is used.
- (e) Stopping criterion. A certain number of iterations limit the process.

### 4. Computational Results

Several group problems are designed. Each problem size is from 10 to 50 customers, and is concerned with two different time windows about the same customer location. Results is reported in Table 1 with  $a_j=c_{0j}/2$ ,  $\lambda_1=\lambda_2=1/2$ ,  $\gamma_1=\gamma_2=\gamma_3=1/3$ . For each instance, the first feasible solution found during 10 execution is listed with respect to average objective function value, average number of iteration and average CPU time (in seconds). Each solution is compared with the best solution of objective formulation (12) and (1), respectively, encountered during the corresponding execution.

Table 1. Results of instances

Instances	Best solution of (1)	Feasible solution(average)			Best solution of (12)		
		Value	No. Iteration	CPU	Value	No. Iteration	CPU
A10	609.15	302.56	1052	17.21	302.56	1052	17.21
B10	651.33	314.74	3143	50.36	314.74	3143	50.36
A20	1652.52	785.40	958	85.12	751.23	11062	95.22
B20	2001.45	874.51	842	79.20	874.51	16215	121.45
A30	5324.12	1957.92	5647	342.58	1954.85	8253	542.06
B30	5120.88	2304.12	6214	601.81	2015.93	9547	853.10

### 5. Conclusion

VRPSPDCS is modeled to solve the combination of the minimization of the total length of vehicles' paths and the maximization of all customer satisfactions. Initial solution obtained by the nearest neighbor

method can be improved by tabu search algorithm. Finally computational results are reported on six group test instances which are different on problem size and time windows.

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