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Views, Factor Models and Optimal Asset Allocation

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Abstract

Making investment decisions in general is a decision-making problem under uncertainty. How well an actual investment portfolio performs depends on the future evolution of economic and financial variables such as interest rates, asset returns and inflation rates. The future evolution of these risk drivers is traditionally modelled using time series models, and it is assumed that historical data are relevant for assessing future risk and return. However, opinions vary about the extent to which all forward-looking information can be derived from historical data. Consequently, a framework for combining views and model-based (density) forecasts is indispensable. We present a concrete example of how views can consistently be combined with model-based (density) forecasts and how this affects investment decisions.

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Keywords: Imposing Views; Portfolio optimisation; Factor models.

1. Combining views and models

Making investment decisions is a decision-making problem under uncertainty and is, especially for institutional investors, part of the risk management process. In general, the risk management process is characterised by three phases. In the first phase, there is an assessment of the risk and return trade-off, taking account of the stakeholders' objectives, constraints and the assumptions on various asset classes and risk drivers; this phase then leads to a strategic asset allocation. In the second phase, called portfolio construction, the strategic asset allocation is translated

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into an actual investment portfolio. In the third phase, monitoring takes place to ensure that the assumptions contributing to the strategic asset allocation remain valid and that the implementation indeed conforms to the strategic asset allocation.

How well an actual investment portfolio will perform in terms of the objectives and constraints of the stakeholders will depend on the future evolution of economic and financial risk drivers such as interest rates, asset returns and inflation rates. The uncertainty about the future evolution of these risk drivers is traditionally modelled using time series models; see Campbell and Viceira (2002) for an example. A fundamental assumption underlying these time series modelling approaches is that there is relevance in historical data for assessing future risk and return. Although there are many arguments to support this claim, it is debatable whether all relevant forward-looking information can actually be derived from historical data. An obvious topical example in today's world is the impact of unprecedented central bank interventions on interest rate levels, and the expected speed of normalisation. Consequently, in practical applications, a framework for combining views and model-based (density) forecasts is indispensable.

Consistently combining expert views with model-based (density) forecasts is not at all straightforward. The difficulty is threefold: first, views are typically not formulated for financial variables, but rather in terms of macroeconomic variables such as economic growth and inflation; second, views are usually not formulated for the full investment horizon; and third, even when views are formulated in terms of certain financial variables – say, equity returns – on the full investment horizon, it is unclear how these views should impact other financial variables, e.g., bond returns. Especially when the number of views and number of assets in the investment portfolio is large, it is all the more important to apply a modelling approach to resolve these difficulties.

In this paper, we show how to consistently combine expert views with model-based (density) forecasts and outline how this can impact investment decisions. First, we present a model that is both realistic and simple enough to serve as an example and work out the methodology for this model. Then, we estimate the model on data and give a concrete example of how the views influence the forecast. Finally, we discuss the impact of the views on investment decisions using portfolio optimization.

2. Dynamic factor models

A realistic modelling application supporting the investment decision process typically involves forecasting the joint behaviour of a large number of financial and economic variables. Not all models are suited for this task, as estimating high dimensional models is usually difficult. The widely used Dynamic Factor Model (DFM) does not have this drawback and is very efficient in forecasting the joint behaviour of a large number of time series. The underlying assumption that validates the use of a DFM is that the simultaneous behaviour can be described by only a small number of (usually unobservable) factors. See Sargent and Sims (1977), Geweke (1977) and Stock and Watson (2002) for more details on DFMs.

A well-known example is the factor model presented in Stock and Watson (2002, 2011). In this type of DFM, the historical time series of the factors are estimated from a dataset containing 108 economic and financial time series for the United States by a principal component analysis (PCA). A common choice is to use the first four to 10 principal components. These principal components, and consequently the factors, explain a large part of the variance and correlation structure of the whole dataset.

The specific DFM used in this paper is based on the factor model presented in Stock and Watson (2012b) and has the following form:

$$F_t = C_F + AF_{t-1} + \varepsilon_t^F, \tag{1a}$$
$$R_t = C_R + BF_t + \varepsilon_t^R \quad , \tag{1b}$$

where $t = 1, \dots T$, F_t is a vector containing the factors, R_t is a vector of economic and financial variables regressed on the factors, ε_t^F is a vector with correlated normally distributed error terms, ε_t^R is a vector with uncorrelated normally distributed error terms, A and B are matrices with regression coefficients, and C_F and C_R are vectors with regression constants. In this particular example, the vector R_t of variables regressed on the factors consists of US equity returns (SP500), US 90-day treasury bill returns (TBILL), US government bond returns (BGOV), commodity returns in USD (COMM), US economic growth (GDP) and US price inflation (CPI). Although this factor model has its limitations, it is nonetheless realistic and simple enough to serve as an example of how factor model-based (density) forecasts can be combined consistently with views on the economy. Moreover, the methodology presented here works equally well with more complex factor models.

3. Views

Equation (1) gives us a method that, based entirely on historical data and model assumptions, produces a density forecast for the future evolution of the modelled asset classes and macroeconomic variables. As noted, in practical applications, it is necessary to combine (density) forecasts based on historical data with views on the economy that are assumed to contain forward-looking information that cannot be derived from historical data. A traditional approach to do this, although not in a time-dependent setting, is the method of Black and Litterman (1992). The Black-Litterman method uses a Bayesian framework in which views are specified by parameterising the density forecast to be combined with views and assuming distributions for these parameters. The classical Black-Litterman method assumes a multivariate normal distribution parameterised by a mean and covariance matrix, and assumes a normal distribution for the parameter representing the mean. Since the Black-Litterman method does not make use of any underlying model assumptions that are used to produce these forecasts, we believe that improvement is possible if the model structure at hand is used.

Given the importance of the topic from a practical point of view, it is striking that the number of publications that improve or build upon the Black-Litterman method is so modest. Among those worth noting are the following: an extension of the Black-Litterman method to non-normally distributed markets is presented in Meucci (2006); an extension of the Black-Litterman method that allows for the formulation of views in terms of underlying risk factors is presented in Meucci (2009); and entropy-based methods, which extend the Black-Litterman method to non-normal distributions and more general views, are presented in Pezier (2007) and Meucci (2008).

A less general approach not based on the Black-Litterman method is presented in Nogueira (2010). Here, a PCA is used to impose views on yield curve forecasts.

The method presented in this paper is particularly targeted at combining DFMs with views in a time-dependent setting. Technically, the methodology can be applied to any DFM. However, since the method presented uses a least squares-based method (see also Step 2 below), we assume, for consistency, that the DFM is estimated using ordinary least squares (OLS) as well. This assumption is realistic, being, for high dimensional models, the most simple and robust way to estimate these models.

The method for combining views with model-based (density) forecasts presented here consists of four steps: definition of the views; translation of the views to adjusted factors; extension of the views to the entire investment horizon (if necessary); and translation of adjusted factors to all non-view variables. These steps are discussed in detail below.

3.1. Step 1: Define the views

The first step is to define views in terms of expected values for a selected number of so-called view variables, denoted by R_t^{view}. It is sufficient to define the views for the first part of the horizon, i.e., up to some $0 < T' \leq T$.

Note that if the factor model includes stochastic volatility, it is also possible to formulate views on volatilities. In our example, US economic growth and US inflation will be the view variables and the views will be denoted by

$$V_t := \begin{bmatrix} V_t^{\text{GDP}} \\ V_t^{\text{CPI}} \end{bmatrix}, t = 1 \dots T' \leq T.$$

The difference between the views and the model-based expected values will be referred to as the set of required adjustments and will be denoted by

$$\Delta R_t^{\text{view}} = \begin{bmatrix} V_t^{\text{GDP}} \\ V_t^{\text{CPI}} \end{bmatrix} - \begin{bmatrix} R_t^{\text{GDP}} \\ R_t^{\text{CPI}} \end{bmatrix}, t = 1 \dots T' \leq T.$$

3.2. Step 2: Translate the views into adjusted factors

Next, we translate the set of required adjustments into corresponding adjustments of the expected value of the factors. The underlying idea is that equation (1b) implicitly defines a relation between the variables R_t regressed on the factors because all of these variables depend on the same set of factors. Therefore, if we adjust the (expected value of the density) forecast of the factors such that this yields the correct expected value of the view variables, the view automatically translates to all variables regressed on the factors.

The main difficulty is that the number of factors can be different from the number of view variables. When there are more factors than view variables, there are multiple solutions. When there are fewer factors than view variables, it is in general not possible to replicate the desired view exactly by changing the forecast of the factors. As a solution, we adjust the expected value of the factors such that the following conditions are satisfied:

1. *the required view is attained or approximated as well as possible for the view variables;*
2. *the expected value of the factors is adjusted as little as possible if there are multiple solutions.*

These conditions result in a procedure that seeks to keep historical relations between the view variables and the factors intact and that is in line with basic intuition on how the methodology should work. For example, if a view is formulated on interest rates and interest rates are mainly driven by the first factor, then, as expected, mainly the expected value of the first factor will be adjusted.

Additionally, we want these conditions to be in line with the method used to estimate the models at hand. Therefore, when there is a single solution, Condition 1 translates, at each time period for which a view is defined, into the following minimisation problem:

$$\min_{\mathrm{E}F_t^{\text{view}}} ||V_t - \mathrm{E}R_t^{\text{view}}|| \tag{2a}$$
$$\mathrm{E}R_t^{\text{view}} = C_R^{\text{view}} + B^{\text{view}}\mathrm{E}F_t^{\text{view}} \tag{2b}$$

Here, $\mathrm{t} = 1 \dots \mathrm{T}' \leq \mathrm{T}$, $||.||^2$ denotes the squared sum over the component of a vector, and equation (2b) corresponds to the part of equation (1b) that corresponds to the view variables. When there are multiple solutions, i.e., when Condition 2 applies, we obtain the following constrained minimisation problem for $t = 1 \dots T' \leq T$:

$$\min_{\mathrm{E}F_t^{\text{view}}} ||\mathrm{E}F_t^{\text{view}} - \mathrm{E}F_t||, \tag{3a}$$
$$||V_t - \mathrm{E}R_t^{\text{view}}|| = 0, \tag{3b}$$
$$\mathrm{E}R_t^{\text{view}} = C_R^{\text{view}} + B^{\text{view}}\mathrm{E}F_t^{\text{view}}, \tag{3c}$$

where $t = 1 \dots T' \leq T$. In the case of the factor model at hand, these optimisation problems can be solved analytically with standard techniques. The solution is given by:

$$\mathrm{E}F_t^{\text{view}} = \mathrm{E}F_t + \Delta F_t^{\text{view}}, \tag{4a}$$
$$\Delta F_t^{\text{view}} = P\Delta R_t^{\text{view}}, \tag{4b}$$

where P is the pseudo-inverse of B^{view}. More information on the pseudo inverse can be found in standard linear algebra textbooks. Moreover, algorithms to calculate the pseudo inverse of a matrix are standard in numerical software packages.

3.3. Step 3: Extend the views over the horizon

In the case where the view is formulated for only the first part of the investment or decision-making horizon, i.e., if $T' < T$, the adjusted factors are, currently, only determined for the first part of the horizon. Since the views only impact the expected value of the factors, we use factor model equation (1a) to extend the view to the entire horizon in a way that is consistent with the factor model, i.e., for $T' < t \leq T$ the adjusted factors satisfy:

$$\mathrm{E}F_t^{\text{view}} = \mathrm{C_F} + \mathrm{A}\mathrm{E}F_{t-1}^{\text{view}}. \tag{5}$$

The intuition here is that, if possible, we want to stay close to the model structure and thus let the expected value of the factors evolve as prescribed by equation (1a).

3.4. Step 4: Analyse the impact of the views

Because the factors are now adjusted and contain both the views and the historical information, all variables driven by these factors will be adjusted accordingly. The basic idea behind the methodology is that we adjust the factors (in expectation), while keeping the (factor) relation between the variables as defined by equation (1b) intact. The impact of the views on any economic or financial variable can now be analysed.

When the number of factors is smaller than the number of view variables, the view can generically not be replicated exactly. In such a case, it is also worthwhile checking to what extent the formulated views are replicated. The difference between the views and their replication can be seen as a measure of to what extent the views are consistent with the (historical) DFM structure.

In all these steps, the (estimated) historical relationship between variables as captured by the factor model plays a fundamental role. In this sense, the (historical) relationships between variables as contained in the DFM serve as the "dictionary" for translating the views for the view variables to other variables in the model.

3.5. Extensions

We list three extensions of this methodology that are worth noting at this point, not least because some of them are only left out for simplicity's sake.

The first extension is to not only adjust the factors (in expectation), but to also adjust the error terms ε_t^R (in expectation) to replicate the view. This especially makes sense if the view variables are poorly explained by the factors and the standard deviation of the error terms is relatively high. These error terms represent the idiosyncratic part of the volatility of a variable.

The second extension is the possibility to specify an uncertainty or level of confidence for a view. Although this extension might seem essential from an academic point of view, we believe that in practical situations the actual formulation of such an uncertainty is difficult since formulating views for "only" expected values is already difficult enough.

A third extension is the formulation of views on the volatility of the forecast, e.g., a view on the future evolution of the volatility of equity returns. This extension can be reached by including volatility state variables in the DFM and using these state variables to drive the (conditional) volatility of the corresponding asset returns. What is essential for the methodology, as currently presented, is how the view variables depend on the factors. There are no requirements on how other variables in the model depend on the view variables and factors. Hence, a view on volatility can be formulated as a view on the expected value of the volatility state variable.

3.6. Relation to Black-Litterman

The method presented here differs in several ways from the Black-Litterman method and its extensions. The most important difference is the time-dependent setting: the classical Black-Litterman combines a one-period density forecast with views and the method here combines a time-dependent density forecast with time-dependent views. Although it is possible to indicate some differences underlying the methodology, a real comparison cannot be made without an extension of the Black-Litterman method to a time-dependent setting, which is beyond the scope of this paper.

Step 2 of the approach presented is where the views and the model-based (density) forecast are combined with a least squares criterion that tries to keep the dynamics of the factor model intact. In the Black-Litterman and related approaches, the forecasted distribution, parameterised by a mean and covariance matrix, itself is used to combine the view with the forecast, i.e., a statistical criterion. As a result, the Black-Litterman method also updates the covariance matrix of the forecast combined with the views. In summary, the difference between the method here and the Black-Litterman method is twofold: a time-dependent setting versus a one-period setting, and a least squares criterion to stay close to the model structure versus a statistical criterion to stay close to the forecasted distribution.

4. Data description

Now that we have discussed the DFM and our approach for combining them with views, we are ready to work out an example and investigate the impact of views on optimal asset allocations. As a first step, we turn to the data. We estimate factor model (1a) on an extension of the Stock and Watson dataset presented in Stock and Watson (2012a, 2012b). The default Stock and Watson dataset consists of quarterly observations from 1959 till the end of 2008 of 108 US-related time series of the following major categories: economic growth, industrial production, employment-related time series, housing prices, interest rates, credit and money market-related time series, consumer prices and inflation, commodity prices, equity returns and exchange rates. For better forecasting accuracy of bond returns, the dataset has been extended to include not only (quarterly) interest rate changes, but also interest rate levels. Data processing and outlier corrections are performed as in Stock and Watson (2012a, 2012b).

The dataset of time series regressed on the principal components (see equation (1b)) consists of the log returns of the following indices also contained in the Stock Watson dataset: a US inflation index (CPI), a US economic growth index (GDP), a commodity price index (COMM) and the S&P 500 Index (SP500). Additionally, our dataset also contains a total return index for 90-day treasury bills (TBILL) and a total return index for government bonds (BGOV). These two bond indices are available on Global Financial Data (2013), which in turn based these indices on Homer and Sylla (2005) and data from the Federal Reserve.

5. Model estimation

Given the historical data and the model structure, we estimate the model using a three-step approach. In the first step, principal components are estimated from standardised time series of the Stock and Watson dataset. Based on the well-known elbow rule, which goes back to Thorndike (1953), we select the first seven principal components to serve as historical time series for the factors. The historical time series of the factors components are standardised to have zero mean and a standard deviation of one. In the second step, the model, as given by (1), is estimated on historical data using ordinary least squares.

Table 1. Imposed long-term expected annual nominal log returns based on long-term expected values in 2009; see CBO (2009), Dimson et. al. (2009) and Keating and O'Sullivan (2009).

Time series	Long-term expectation
GDP	2.2%
CPI	2.4%
COMM	2.4%
SP500	8.0%
TBILL	3.2%
BGOV	4.3%

In the final step, the expected value of the long-term unconditional distribution is adjusted to yield realistic (long-term) forward-looking values. This step is especially important for optimal asset allocation, as unrealistic long-term forward-looking values will lead to unrealistic portfolios for long-term investors. Since this adjustment merely impacts the long-term forecast, this should not be confused with the view methodology discussed earlier. By adjusting the constant C_R in equation (1b), the long-term expected returns on treasury bills, government bonds, equity, commodity prices, inflation and economic growth are as given in Table 1.

6. Results

Now that we have described the model, the data, the estimation procedure and the methodology for combining model-based (density) forecasts with views, we will look at an explicit application. We will first discuss the (unadjusted density) forecast of the model as estimated on the dataset described in Section 4 containing data till the end of 2008. Then, we will build the adjusted (density) forecast of the model by combining the model with a view using the methodology introduced in Section 3, and we will discuss the differences with the unadjusted forecast. Finally, we will show how the view impacts optimal asset allocations for various investment horizons. These optimal asset allocations are affected by the expected future returns (with and without views) as well as the volatility of these future returns and the future correlation between asset classes.

6.1. Density forecasts

Figure 1 shows the deciles of the unadjusted density forecast of model (1) in quarterly log returns, i.e., the forecast of the model as estimated on the dataset described in Section 4 containing data till the end of 2008. From 2009 onwards, the thin black lines are the deciles of the unadjusted density forecast (the outer black lines denote the 1st percentile and 99th percentile). Until the end of 2008, the thin black lines are the approximation of the original data by the first seven principal components. The thick black lines in Figure 1 correspond to historical data. Since the start date of the simulation is end-2008, the unadjusted density forecast can be compared with the realisation,

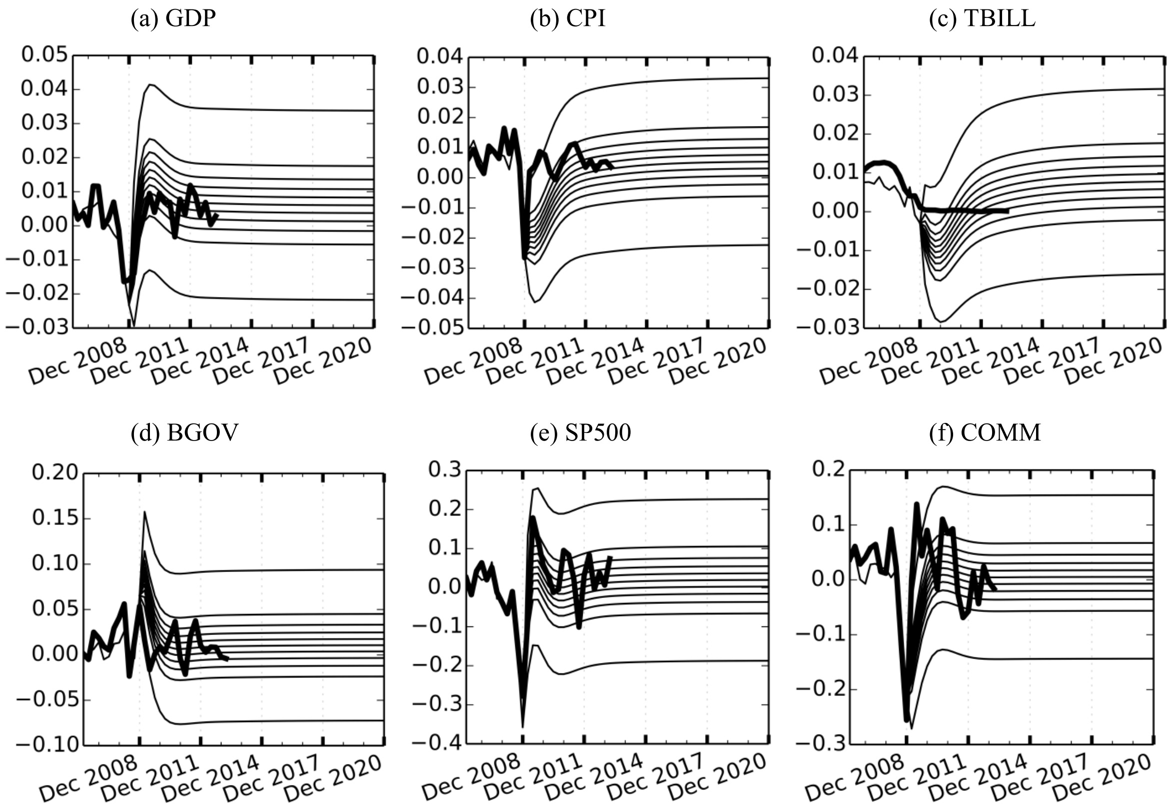

Fig. 1. Deciles (thin black lines) of the density forecast without view and realisation (thick black lines) in quarterly log returns.

i.e., an out of sample comparison with new data not used to estimate the model. As noted, the DFM used is both realistic and simple enough to serve as an example of how views and DFMs can be combined in a consistent framework, but the model has its limitations. This is also reflected in the forecast and its comparison with the realisation. Since, a priori, the probability that the realisation will end up between two subsequent deciles is 10%, the overall impression from Figure 1 is that the density forecast from model (1) performs quite well in terms of the accuracy of the marginal distributions except for treasury bills (TBILL). Although negative treasury bill yields are not impossible, the deciles indicate unrealistically large probabilities of negative yields. Since our focus is on combining factor model-based density forecasts with economic views and analysing the impact on optimal asset allocations, we take this limitation for granted.

6.2. Views

The next step is to formulate the views to combine with the (density) forecast of our DFM and to arrive at the adjusted (density) forecast. For this purpose, we selected the forecast for GDP and CPI from the IMF World Economic Outlook of April 2009; see IMF (2009). These views are readily available on the IMF website every quarter. Table 2 shows the IMF forecast (views) together with the unadjusted (expected value of the density) forecast from the DFM, as also shown in Figure 1.

Table 2. Unadjusted forecast from DFM and views based on IMF (2009) in annual log returns.

	GDP		CPI	
	IMF	Model	IMF	Model
2009	-2.8%	2.4%	-0.9%	-7.1%
2010	-0.1%	4.7%	-0.1%	-2.4%
2011	3.5%	3.1%	0.7%	0.2%
2012	3.6%	2.7%	1.7%	1.0%
2013	3.3%	2.6%	2.1%	1.3%
2014	2.5%	2.6%	2.2%	1.6%

Compared with the expected value of the forecast of the DFM, these views can be summarised as follows: lower economic growth in the years 2009-2011 and a higher inflation in the years 2009-2014. Note that these views are not formulated for the entire simulation horizon, but can be extended to the entire simulation horizon using equation (5).

Of course, one could also use other views or argue that IMF views might also be model-based, i.e., they might be produced by using a different DFM. Nevertheless, the purpose of the views here is to serve as an example. In practice, there are many types of reasoning that can lead to a formulation of a view, including: central bank interventions and the impact on expected interest rate levels; a macroeconomic department having an explicit view on certain countries or regions; and an investment committee with an explicit view on the risk premiums of certain asset classes.

A key characteristic of the view, however, should be that it can be explained by the factor structure that is based on historical relations. Note that, if this is not the case, e.g., when contradictory views are used for two variables that historically showed similar behaviour, it also does not make sense to use a historical data-based method to combine the views with the unadjusted forecast.

6.3. Impact of views on the (density) forecast

In Figure 2, the unadjusted (density) forecast is compared with the adjusted forecast (in expectation). Figures 2a and 2b show the unadjusted (expected value of the density) forecast and the IMF view for GDP and CPI extended to the whole simulation horizon using the DFM. Again, it can be seen that the IMF view can roughly be summarised as one of lower economic growth and higher inflation when compared with the DFM, which is mainly based on historical data. Figures 2c to 2f show the impact of the view on the (expected value of the density) forecast of the other variables. Each of these responses makes sense economically given the nature of the view for GDP and CPI.

First, Figure 2c shows that the view leads to higher expected TBILL returns, which is consistent with the higher expected inflation in the view. Second, Figure 2d shows that the view leads to lower expected bond returns (and, implicitly, higher bond yields) on short horizons, which is also consistent with the higher expected inflation included in the view. For longer horizons, the impact is smaller, probably due to the dampening effect of higher expected (implicit) bond yields. Third, Figure 2e shows that the view leads to lower equity returns, which is consistent with the lower expected growth in the view. Finally, Figure 4f shows that the impact of the view on expected commodity returns is limited. This can be due to the following offsetting effects: on the one hand, the lower expected growth in the view leads to lower expected commodity returns due to lower demand and, on the other hand, the higher expected inflation in the view leads to higher expected commodity returns because of their inflation sensitivity (some of the commodity prices may even contribute to the inflation itself).

(a) GDP (b) CPI (c) TBILL

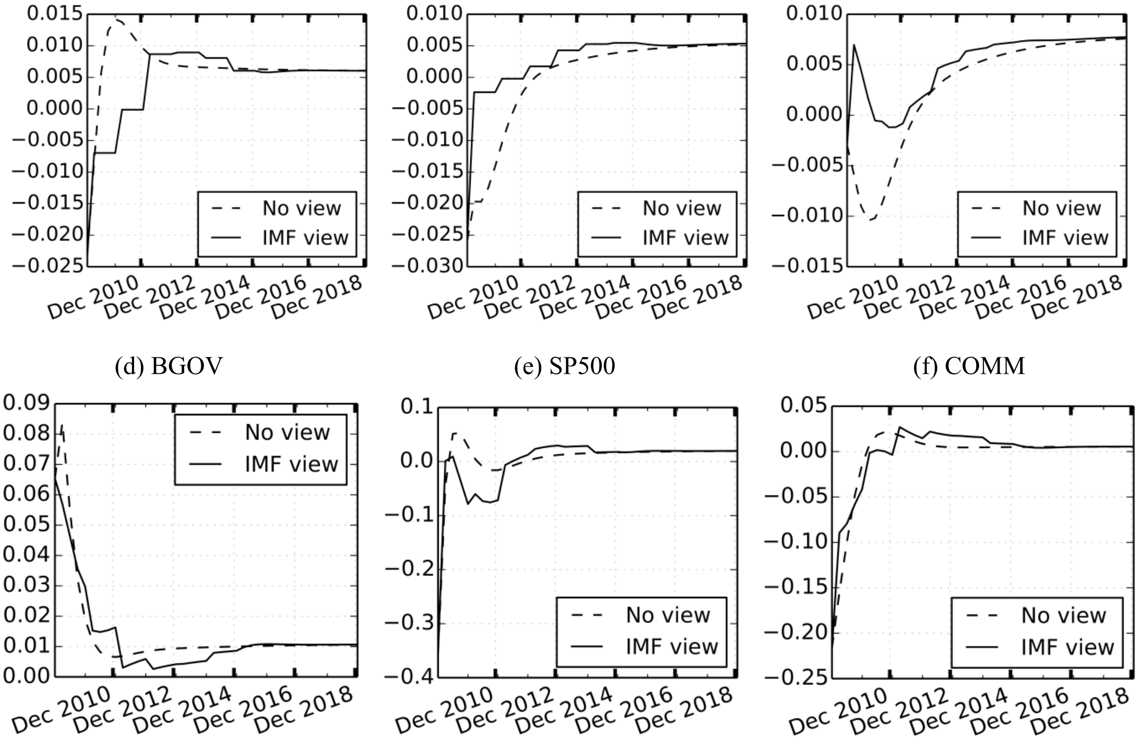

Fig. 2. Expected value of the (density) forecast with and without the IMF view in quarterly log returns.

6.4. Impact of views on the optimal asset allocation

So far, we have only discussed the expected impact on individual asset classes. Investors, however, are not only interested in individual asset classes, as their main interest lies in the entire portfolio. Therefore, we analyse the impact that views can have on investment decisions by comparing optimal portfolios constructed from the unadjusted and adjusted (density) forecast including the view. Optimal portfolios not only depend on expected returns, which are influenced by the view, but also on the correlations between the asset classes, which are not influenced by the view. Therefore, constructing optimal portfolios can also be seen as a way of analysing the impact on multivariate aspects of the forecast and not only on the marginal distributions.

To construct optimal portfolios from the (un)adjusted forecast several steps must be taken. First, the conditional distributions of the 1-period (quarterly) log returns of model (1) must be cumulated over the horizon to obtain the distribution of the cumulated log return over the horizon. By writing the cumulated log returns as a sum of independent random variables, i.e., assuming a random walk for the returns, they are annualised by dividing both the mean and the covariance matrix by the number of years in the horizon. Note that this does not mean that the 1-period returns as described by the DFM are independent. The random walk method is simply a way of annualising the distribution of the cumulative returns. Second, we want to perform the portfolio optimisation on the cumulated annualised returns rather than the cumulated annualised log returns. When the cumulated annualised log returns

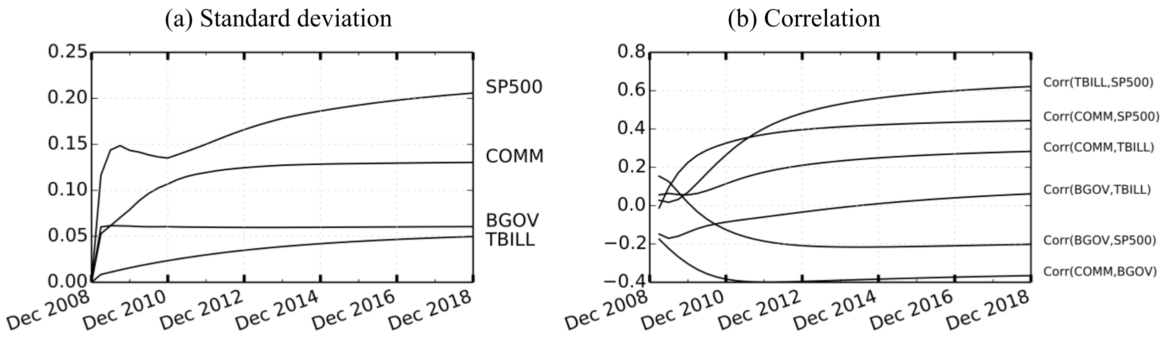

Fig. 3. Horizon dependence (term structure) of volatilities and correlations of cumulated annualised returns.

CLR have a normal distribution with mean μ_{CLR} and covariance matrix Σ_{CLR}, the cumulated annualised returns $\mathrm{CR} = e^{\mathrm{CLR}} - 1$ have a shifted log-normal distribution.

Figure 3 shows the volatilities and correlations of the cumulated annualised returns, as obtained after the first two steps. From Figure 3a, it can be seen that the annualised volatility of the cumulated returns of the asset class converge to plausible values and that they also have a logical ordering (equities more volatile than commodities etc.). Note that for portfolio optimisation, not only the expected value, but also the volatility and correlation structure plays a role. Figure 3b shows how the correlations between the cumulated annualised returns of the asset classes depend on the investment horizon. The combination of Figure 3 with the plot of the dependence of annualised return on the investment horizon is called the term structure of risk and return, i.e., of how risk and return can vary with the investment horizon.

The third step in our optimal portfolio construction is a simple mean-variance optimisation per horizon over four asset classes: US equities (SP500), US 90-day treasuries (TBILL), US government bonds (BGOV) and commodities (COMM). To obtain realistic portfolios, we impose the restriction that asset weights in the portfolio can only be positive. For each horizon, this results in an entire efficient frontier of portfolios.

In the fourth and final step, we show how the optimal portfolios depend on the investment horizon and how this dependence is influenced by the view. We select a specific portfolio on each of these efficient frontiers and plot this against the horizon. There are various choices to do this such as: a minimum variance portfolio, a maximum utility portfolio or a minimum value-at-risk portfolio. Here, we select the so-called tangent portfolio: the portfolio with the highest excess return (compared with the risk-free rate) per unit risk (here per unit of volatility). Figure 4 shows the term structure of the risk-free rate we use to select this tangent portfolio. This term structure is obtained as a least squares fit of a Nelson-Siegel-Svensson curve through the US treasury zero coupon yields of 31 December 2008, i.e., the start of the investment horizon in our case.

Figure 5 shows the portfolio weights of the tangent portfolio as a function of the horizon. Based on the IMF view (lower economic growth and higher inflation) and the adjusted expected values shown in Figure 2, we expect that when the IMF view is included, the optimal tangent portfolios will contain less equity and more treasury bills on short horizons. From Figure 5, we conclude that this is indeed the case. Additionally, commodities appear in the tangent portfolio on longer investment horizons. This can be understood from the fact that the expected returns on equities are more negatively affected by the view than the expected returns on commodities.

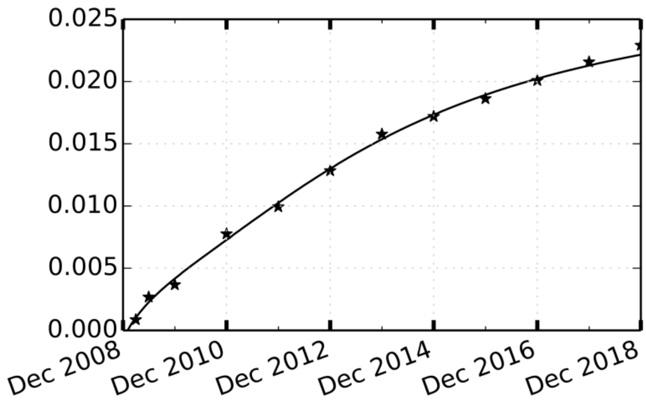

Fig. 4. Risk-free rate based on US treasury zero coupon yields of 31 December 2008.

7. Conclusion

We have described a methodology for consistently combining views on economies and financial markets with factor models. We have shown how this methodology can be applied in a relatively easy but realistic example: a factor model estimated on the Stock and Watson dataset combined with IMF views. This example illustrates the impact that views can have on optimal asset allocations across the investment horizon. Here, we have focused on translating views on macroeconomic variables such as GDP and CPI into the consequences for the expected returns on various asset classes. The example presented here and also in other experiments (see, e.g., van der Schans and Steehouwer (2012)) shows that working with views on a subset of asset classes has a substantial impact on other asset classes.

Given the impact of views on optimal portfolios, the results in this paper illustrate the importance of taking views into account in a consistent way in the investment decision process. In general, combining views with dynamic factor models can also have other applications such as in analysing, understanding and improving the dynamics of a specific factor model and constructing stress scenarios.

We emphasise again that a fundamental assumption underlying the presented methodology to translate views into other asset classes and economic variables is the validity of the estimated historical relation as captured by the factor model, i.e., the historical relationship should be able to describe the view appropriately. In this sense, the (historical) relationship between variables as contained in the factor model serves as the "dictionary" for translating the views from the view variables to other variables in the model.

There are several useful directions for extending the proposed methodology, such as including the idiosyncratic part of the volatility when translating the views, taking into account the uncertainty of the views and allowing for views on volatility. We leave these extensions for future research.

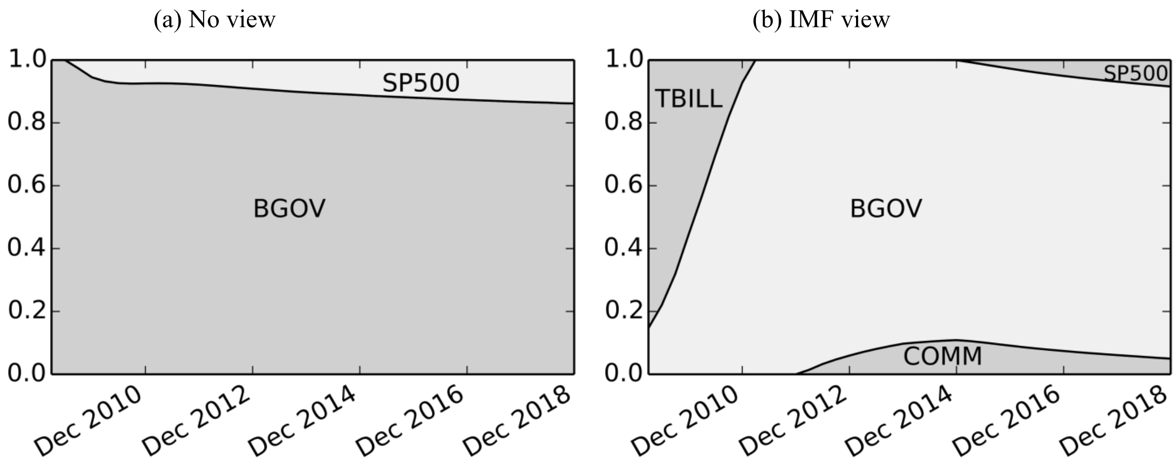

Fig. 5. Portfolio weights of the optimal tangent portfolios across the investment horizon, with and without the IMF view.

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