Comp. & Maths. with Appls.. Vol. 3, pp. 183–190. Pergamon Press 1977. Printed in Great Britain

COMPUTATION OF THE DATES OF THE HEBREW NEW YEAR AND PASSOVER

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Communicated by E. Y. Rodin

(*Received June*, 1976; *and in revised form October*, 1976)

Abstract—In 1802, Gauss published[1] a scheme for determining the Common Era (C.E.) date of Passover in a given year, but did not indicate how his results had been obtained. Almost a century later, M. Hamburger offered[2] a quite complicated proof of the Gaussian equations. The procedure is simplified by transforming the Gaussian Passover formulas into those for Rosh Hashonah (Hebrew New Year), from which the date of the *preceding* Passover follows at once.

I. FORMULAS*

(A) *The Date of Rosh Hashonah*

1. *The Julian date*

(a) *The 'undelayed' date of Tishri's Conjunction.*

(a1) The 'unadjusted' calendar day, given by

$$d'_R = i'_R + f'_R = 41.7972710 + 1.5542418n + 0.25r - 0.003177794H_R, \quad \text{or} \tag{1a}$$

$$= 29.8455877 + 1.5542418n + 0.25r - 0.003177794Y_R, \tag{1b}$$

where Y_R is the given calendar year in C.E.,

$H_R = Y_R + 3761$, is the same year in World Era (W.E.), for $Y_R > 0$,

$$n = (12H_R + 5)\text{mod}19, \quad \text{or} \tag{2a}$$

$$= (12(Y_R + 1))\text{mod}19, \tag{2b}$$

$$r = (H_R - 1)\text{mod}4, \quad \text{or} \tag{3a}$$

$$= (Y_R)\text{mod}4, \tag{3b}$$

i'_R and f'_R are the integral and fractional parts, respectively, of the sum (1a) or (1b).

(a2) The 'adjusted' calendar day, given by

$$d_R = i_R + f_R = (i'_R + 1) + (f'_R - 0.75), \quad \text{if} \quad f'_R \geqslant 0.75, \quad \text{or} \tag{4}$$

$$= i'_R + f'_R, \quad \text{if} \quad f'_R < 0.75.$$

(a3) The 'adjusted' day of the week is given by

$$w_R = (i_R + 1 + 5r + 3H_R)\text{mod}7, \quad \text{or} \tag{5a}$$

$$= (i_R + 5r + 3Y_R)\text{mod}7. \tag{5b}$$

With the exception of the delays, listed below, the Julian calendar day of the month August in Y_R, corresponding to the first day of the Hebrew month Tishri in H_R (which is also the first

*Definitions and derivations are given in Section II.

day of Rosh Hashonah), is given by

$I_R = i_R$, while the corresponding day of the week is
$W_R = w_R$.

(b) *The 'delayed' date.* In three situations,

$I_R = i_R + \delta$ and
$W_R = w_R + \delta$:

(b1) If $w_R = 1$ or 4 or 6; then $\delta = 1$.
(b2) If the following three conditions hold simultaneously:

$$w_R = 2 \quad \text{and} \quad n > 11 \quad \text{and} \quad f_R \geqslant 0.6477258; \quad \text{then} \quad \delta = 1. \tag{6}$$

(b3) If the following three conditions hold simultaneously:

$$w_R = 3 \quad \text{and} \quad n > 6 \quad \text{and} \quad f_R \geqslant 0.3828704; \quad \text{then} \quad \delta = 2. \tag{7}$$

2. *The Gregorian date*
The August Gregorian calendar day, corresponding to the Julian I_R, is

$$I_{GR} = I_R + \gamma, \tag{8}$$

while the day of the week remains, of course, unchanged: $W_{GR} = W_R$. At this writing, $\gamma = 13$.

(B) *The Date of the Preceding Passover*

The calendar day of Passover, in the year $H_P = H_R - 1 = Y_R + 3760 = Y_P + 3760$, fell in March on either $I_P = I_R - 10$, on the Julian calendar, or $I_{GP} = I_{GR} - 10$, on the Gregorian, while the day of the week was $W_P = W_R - 2$.

II. DERIVATION OF THE FORMULAS

(A) *Definitions*

1. *The Julian–Hebrew calendar date*
The Hebrew day starts at 6 p.m. of the preceding eve. It is divided into 24 hours, and the hour into 1080 parts (khalakim, kh). For purposes of computation, the Julian–Hebrew calendar day is given as

$$d = i + f,$$

where i is an integer, indicating the Julian calendar day of the month, and f is the fractional part of the Hebrew day (i.e. after 6 p.m.). Since C.E. lacks the year zero, the relation between H and a year in B.C.E. is not the same as that between H and a year in C.E. Thus,

H_P (at Passover)	$= Y_P + 3761,$	for	$Y_P < 0,$
	$= Y_P + 3760,$		$Y_P > 0,$
H_R (at Rosh Hashonah)	$= Y_R + 3762,$		$Y_R < 0,$
	$= Y_R + 3761,$		$Y_R > 0.$

2. *The Molad*
The mean conjunction, initiating a lunar month, is the computed moment, when the moon would have the same longitude as the sun, if both moved uniformly. It is called the Molad (birth) of the month.

3. *The lunation*

The mean interval between two successive Molads is called a lunation, or synodic month, and its length has been taken as the constant

$$m = 29\,\text{day},\ 12\,\text{h},\ 795\,\text{kh} = 29 + 13755/25920\,\text{day} = 29\ \text{day},\ 12\ \text{h},\ 44\,\text{min},\ 3\tfrac{1}{3}\,\text{s}.*$$

(B) *The Date of Rosh Hashonah*

1. *The Julian date*

(a) *The undelayed date.*

(a1) The unadjusted Molad of Tishri.

The Molad of Tishri, for a given Hebrew year H_R, is calculated from

$$d'_R = i'_R + f'_R = d_{1R} + \epsilon, \quad \text{where} \tag{9}$$

$$d_{1R} = i_{1R} + f_{1R},$$

is the Molad of Tishri in the year 1, W.E., and the epact

$$\epsilon = N_H - N_J, \tag{10}$$

where N_H and N_J are, respectively, the (non-integral) number of Hebrew days and the (integral) number of Julian days, accrued in the interval

$$V = H_R - 1.$$

(a1.1) The spurious Molad

The Molad d_{1R} is not to be confused with the biblical first day of Creation which—some Talmudists assert—took place almost half a year later, five days before the Molad of Aviv (later called Nisan). For this reason, d_{1R} is called Molad Tohu (imaginary) and is used merely as a convenient origin for computing the date of Rosh Hashonah. The custom of observing Tishri 1 as a High Holiday (originally called 'The Day of Remembrance'—of our transgressions, perhaps) and the names of the months were acquired during the Babylonian Exile.

The ancient astronomers found, by backward extrapolation, that Molad Tohu fell on Monday, Julian 7 October, at 5 h, 204 kh, in the year 3761 B.C.E. Therefore its mnemonic is Ba'Ha'Ra"D, signifying 2 (Monday)'5'200"4. For convenience in computation, this Molad is changed into its August equivalent, namely

$$d_{1R} = 68 + 467/2160\,\text{days}.$$

(a1.2) Computation of N_J

Let L be the number of leap years in V,.and set

$V = 4L + r,\ r < 4$, from which
$r = (H_R - 1)\text{mod}4$, explaining equation (3a). Also

$$L = \tfrac{1}{4}(V - r), \quad \text{an integer.} \tag{11}$$

Since every Julian leap year has one extra day,

$$N_J = 365V + L = 365\tfrac{1}{4}V - \tfrac{1}{4}r, \text{ an integer.} \tag{12}$$

**m* is not a constant; its last term is—at the present time—slightly above 2.8s. It is gradually diminishing at the rate of about 0.2s per millenium.

(a1.3) Computation of N_H

(a1.3.i) The 19-year cycle

The length of an ordinary Hebrew year,

$$h = 12m = 354\,\text{day},\ 8\,\text{h},\ 876\,\text{kh}, \tag{13}$$

is, obviously, far short of the tropical solar year, and the ancient astronomers were forced to introduce an intercalary month every two or three years. An embolismic year, containing an extra 13th month,

$$e = 13m = 383\,\text{day},\ 21\,\text{h},\ 589\,\text{kh}. \tag{14}$$

In the 5th century B.C.E., the Greek astronomer Meton conjectured that 235 lunations might be approximately equal to 19 tropical solar years. Some seven centuries later, the Hebrew astronomer, Rabbi Adda, announced his computation for the length of the tropical solar year to be

$$S = 365\,\text{day},\ 5\,\text{h},\ 55\,\text{min},\ 25 + 25/57\,\text{s}$$

and 'proved' (no doubt to everyone's amazement but his own) that a cycle of

$$12h + 7e = 235\ \text{lunations} = 19S\ \text{exactly!*}$$

The ordinal ('golden') number, g, ranging from 1 to 19, for each Hebrew Calendar year is, of course,

$$g = (H)\text{mod}19. \tag{15}$$

The set of g's for the seven e's in a cycle consists of 3, 6, 8, 11, 14, 17, 19. The fact, that the Gaussian equation for n indicates a linear relation between it and $(H)\text{mod}19$, led to the conjecture that the above set bears a linear relation to the integer n, $0 \leq n \leq 6$. Therefore, trial multiples of 19 were added to the above g's, and the sums rearranged, until the sequence exhibited a constant first difference, namely $17 + 0 \times 19$, $6 + 1 \times 19, \ldots, 8 + 3 \times 19$, differing from each other by 8. This determined the function

$$g = (8n + 17)\text{mod}19. \tag{16}$$

The equation was then tested for $7 \leq n \leq 18$, and, fortunately, the resulting g's were those of the h's.

To invert (16), it is necessary to isolate n – on the right side of the equation—by finding the smallest (odd) integer K_1, so that $19K_1 + 1 = 8K_2$, where K_2 is also an integer. These conditions are fulfilled by $K_1 = 5$, $K_2 = 12$. This indicates that both sides of (16) should be multiplied by 12, from which we obtain

$n = (12g + 5)\text{mod}19 = (12H + 5)\text{mod}19$, which explains equation (2a).

(a1.3.ii) The number of e's in V

If we define the function $E(x)$ as the number of e's in x consecutive Hebrew years, the relations between n, g, and $E(n)$ are shown in Table 1.

Table 1

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
g	17	6	14	3	11	19	8	16	5	13	2	10	18	7	15	4	12	1	9
$E(n)$	7	0	0	1	1	1	2	2	3	3	3	4	4	4	5	5	5	6	6

*The correct figures for the last two terms in S are 48 min, 46s. The solar year diminishes at the rate of about 0.6s per a hundred millenia.

From (15) and (16), it is seen that $g - 1 = H - 1 + 19k_1 = 8n + 16 + 19k_2$, where k_1 and k_2 are integers or zero. So

$$V = 19K + 8n + 16, \tag{17}$$

where K is an integer or zero. Hence

$E(V) = 7K + E(8n + 16)$. Set
$8n + 16 = 19u + v$, where $v < 19$. Then
$E(8n + 16) = 7u + E(v) = q$ (say). Thus
$E(V) = 7K + q$.

The value of $E(v)$ can be obtained from the last line in Table 1, while we are computing the value of q from Table 2.

Table 2

n	$8n + 16$	u	v	$E(v)$	$7u$	q
0	16	0	16	5	0	$5 = 5 + 3 \times 0$
1	24	1	5	1	7	$8 = 5 + 3 \times 1$
2	32	1	13	4	7	$11 = 5 + 3 \times 2$
18	160	8	8	3	56	$59 = 5 + 3 \times 18$

Evidently,

$E(8n + 16) = 5 + 3n$, so that
$E(V) = 7K + 5 + 3n$.

On the other hand, if we multiply both sides of equation (17) by 7, we obtain

$7V = 19 \times 7K + 19 \times 3n - n + 19 \times 5 + 17 = 19E(V) - n + 17$, from which
$E(V) = (7V + n - 17)/19$.

Since each e contains one extra month, the total number of lunations in V years is

$$T = 12V + E(V), \quad \text{and}$$

$$N_H = mT = 12mV + m(7V + n - 17)/19 = (235\, m/19)V + (m/19)(n - 17). \tag{18}$$

Consequently, in view of (10), (12) and (18), the epact $\epsilon = (235m/19)V + (m/19)(n - 17) - (365\frac{1}{4}V - \frac{1}{4}r)$; finally, in view of (9) $d'_R = i'_R + f'_R = 68 + 467/2160 + 365\frac{1}{4} - 252\, m/19 + (m/19)n + \frac{1}{4}r - (365\frac{1}{4} - 235m/19)H_R$, which yields equation (1a).

(a2) The adjusted Molad of Tishri.

According to one of the rules of the Sanhedrin (Hebrew Supreme Council), when a Molad is computed to occur on (or past) noon, it is considered 'zaken' (old). Eighteen, or more, hours past 6 p.m. corresponds to $f'_R \geq 0.75$ day, and therefore 0.25 day is added to the old Molad, which causes the following day to be accepted as the correct 'young' one. This explains the definition of d_R in equation (4).

(a3) The adjusted day of the week.

The weekday, corresponding to the Julian calendar day i_R, is obtained from

$w_R = w_{1R} + w_J$, where

w_{1R} = the weekday, on which the i_Rth of August fell in the year 1, W.E.,

w_J = the supplement, accrued to w_{1R} during the N_J days in V years.

Since the Molad of Tishri 1 in the year 1 occurred on Monday, 7 October,

$w_{1R} = (i_R + 2 - 68)\text{mod}7 = (i_R + 4)\text{mod}7$, while
$w_J = (365V + L)\text{mod}7 = (V + 8L)\text{mod}7 = (3V - 2r)\text{mod}7$, in view of (11).
$w_R = (i_R + 4 + 3(H_R - 1) + 5r)\text{mod}7$, explaining equation (5a).

(b) *The delayed date*

The adjusted Molad of Tishri is not necessarily set to the first of that month, because of the restrictions, imposed by the Sanhedrin, in the 4th century C.E., regarding its acceptability as the first day of Rosh Hashonah. It should be stated that the mathematicians, who are preparing the Julian–Hebrew, or Gregorian–Hebrew calendars, can vary the number of days, ρ, in any Hebrew calendar year, H, to fit the rules of the Delays. The two months, following Tishri, have been made flexible in length, and may have 29, or 30, days each, when it becomes necessary to either shorten or lengthen the 354-day 'regular' ordinary year, or the 384-day regular embolismic year. Thus ρ can vary either between 353 and 355, or else between 383 and 385. The Delays are:

(b1) ADU, mnemonic for 1/4/6.

The first Day of Rosh Hashonah (i.e. Tishri 1) may not fall on Sunday, Wednesday, or Friday. The reason given for the Wednesday and Friday prohibitions is that Yom Kippur (the Day of Atonement), which comes 9 days later, would fall on a Friday or Sunday, and in the warm climate of the Middle East, food would not stay fresh for two days of Sabbaths (a High Holiday is also termed Sabbath). The Sunday prohibition is attributed to the fact that the 7th Day of Tabernacles, which comes 20 days later, and on which ritual demands that twigs be beaten, would fall on Saturday, when all labor is prohibited. However, one cannot suppress the suspicion that the exclusion of Sunday on both High Holy Days might be a reaction to the decision—at the Nicean conference of 325 C.E.—to postpone Easter for a week, if the computed date of this lunar Holiday coincides with Passover. The prohibition ADU causes the Hebrew calendar-makers to postpone Tishri 1 till the next day.

(b2) B'TU'THaK"PaT, mnemonic for 2'15'500"89.

If the Molad of Tishri in an H_R, following an embolismic year, $H_R - 1$, occurs on Monday at (or after) 15 h, 589 kh, the New Year is put off to Tuesday. From Table 1, we note that, when $n > 11$, the g's exceed by 1 those of the seven e's. Moreover, the above time of day corresponds to $f_R \geqslant 0.6477238$. This explains the conditions in (6).

The reasons for this Delay are a bit complicated. The length of a Hebrew year, modulo 7 days, is called the Character, C, of the year. From equation (14), it follows that the Character of the embolismic year $H_R - 1$ is $C_e = 5$ day, 21 h, 589 kh. If we subtract C_e from the Molad of the current Tishri, we obtain the Tishri Molad of $H_R - 1$. This difference is 2 day, 15 h, 589 kh—5 day, 21 h, 589 kh, which is the same as 8 day, 39 h—5 day, 21 h = 3 day, 18 h (Tuesday at noon). The Molad of the previous year was, evidently, 'zaken' and had to be updated. But Wednesday is a prohibited day, by ADU, so Rosh Hashonah in the year $H_R - 1$ must have been celebrated on Thursday. But, if the current Molad were to be taken as Tishri 1, the ρ of that year would have been only 382—less than the statutory minimum—since $5 + (382)\text{mod}7 = 9$(Monday). The embolismic year is therefore lengthened by one day, by delaying the current Rosh Hashonah till Tuesday.

(b3) Ga'T'Ra"D, mnemonic for 3'9'200"4.

If in an ordinary calendar year, H_R, the Molad of Tishri is computed to be on Tuesday, at (or after) 9 h, 204 kh, the New Year is postponed till Thursday. From Table 1, we note that, when $n > 6$, the g's are those of h's. Moreover, the above time of day corresponds to $f_R \geqslant 0.3828704$ day. This explains the conditions in (7).

The reasons for this Delay are similar to the ones above.

From (13), the Character of H_R is $C_h = 4$ day 8 h 876 kh. If this be added to the current Molad, we obtain 7 day, 18 h, as the Molad of Tishri in the year $H_R + 1$, namely Saturday at (or after) noon, which is 'zaken' and must be updated. But Sunday is prohibited by ADU. Thus, Rosh Hashonah of the year $H_R + 1$ will be celebrated on Monday. But, if Tishri 1 of the current year were allowed to fall on Tuesday, ρ of this year would be 356—greater than the statutory maximum—since $3 + (356)\text{mod}7 = 9$(Monday). However, if the current ρ were reduced by only one day, Tishri 1 would fall on Wednesday, prohibited by ADU, so that ρ must be reduced to 354, setting Rosh Hashonah to Thursday.

2. *The Gregorian date*

The γ in equation (8) was added to the Julian calendar by Pope Gregory XIII. Starting as 10 days on Julian 5 October, 1582, γ has gradually increased, under the rule that every calendar year, divisible by 100—but not by 400—be made an ordinary—instead of a leap—year. The corresponding values are shown in Table 3.

Table 3

Julian dates		γ
From 5 October	1582	10
19 February	1700	11
18 February	1800	12
17 February	1900	13
16 February	2100	14
etc.		

(The newspapers of 27 May 1976, carried the claim by Mr. W. A. Radspinner that the Gregorian calendar is fast by 0.12037037 day per 400 years, and suggested a scheme for correcting the error.)

C. *The Date of the Preceding Passover*

The 15th of the Hebrew month Nisan—in which the first day of Passover is celebrated—in the year

$$H_P = H_R - 1 = Y_R + 3760 = Y_P + 3760,$$

precedes Rosh Hashonah—in the year $H_R = Y_R + 3761$—by 163 days. Therefore, if P be the calendar day in March (in either the Julian or Gregorian system), on which Passover falls, while R be the Rosh Hashonah day in August of the chosen system, then $31 - P + 122 + R = 163$ days, so that

$P = R - 10$, on either the Julian, or Gregorian calendar.

The corresponding day of the week is

$W_P = W_R - (163)\text{mod}7 = W_R - 2.$

From the last equation it follows that Passover may not start either on Monday, Wednesday, or Friday. It can occur on Sunday, and—as we mentioned earlier—if it coincides with the computed Easter Sunday, the latter is postponed a week. The Christian calendar-makers are, therefore, just as adept at calculating the day of Passover, as are the Jewish ones, which makes me wonder, whether the present paper should have been entitled "Not For Jews Only".

It will be noted, that the Jewish soli-lunar calendar is burdened with errors in both its lunation, and solar year, 'constants'. As a consequence, the Holy Days are beginning to stray—on occasion—from the statutory limits—21 March–19 April, Gregorian, for Passover, and 31 August–29 September for Rosh Hashonah—as is evident from some of the data in section III of this paper.

III. APPLICATION OF THE FORMULAS

(A) *Pertinent dates*

$Y < 0$: $H_R = Y_R + 3762$; $H_P = H_R - 1 = Y_R + 3761$.

EVENTS

Start of period *Y*	*H*	
—3761	1	Molad Tohu. Used as origin for computing Rosh Hashonah.
—586	3176	Bayblonian Exile. The seventh month, Tishri, celebrated, at its true Molad, as a High Holy Day, under various names.

—70	3691	Second temple destroyed. Phase calendar method, of observing the true Molad, in use. Passover is beginning to be celebrated in the present form.
		$Y > 0$: $H_R = Y_R + 3761$; $H_P = H_R - 1 = Y_R + 3760$.
359	4119	Fixed calendar method introduced, whereby Molads are calculated; delays initiated.
1583	5343	Gregorian Calendar in use.

(B) *Computed Dates of Rosh Hashonah and of Preceding Passover*

$Y_R = Y_P$ — Rosh Hashonah

	H_R	n	i_R	f_R	w_R	δ	I_R	W_R	Day	Julian	γ	Gregorian
—3761	1	17	68.21620		2		68	2	Mon.	Oct. 7		
—500	3262	9	45.66948		2		45	2	Mon.	Sep. 14		
—50	3712	13	51.20644		3		51	3	Tue.	Sep. 20		
1502	5263	5	33.34375		6	1	34	7	Sat.	Sep. 3		
1600	5361	3	29.42384		6	1	30	7	Sat.	Aug. 30	10	Sep. 9
1700	5461	6	34.01879		3	0	34	3	Tue.	Sep. 3	11	Sep. 14
1800	5561	9	38.11373		6	1	39	7	Sat.	Sep. 8	12	Sep. 20
1976	5737	12	42.21717		6	1	43	7	Sat.	Sep. 12	13	Sep. 25
1977	5738	5	31.58430		3	0	31	3	Tue.	Aug. 31	13	Sep. 13
1978	5739	17	50.48202		2	0	50	2	Mon.	Sep. 19	13	Oct. 2
1979	5740	10	40.09915		7	0	40	7	Sat.	Sep. 9	13	Sep. 22
1980	5741	3	28.21628		4	1	29	5	Thu.	Aug. 29	13	Sep. 11

Passover

H_P	I_P	W_P	Day	Julian	Gregorian
...	...	...	...	...	...
...	...	...	...	...	...
3711	41	1	Sun.	Mar. 10	...
5262	24	5	Thu.	Mar. 24	...
5360	20	5	Thu.	Mar. 20	Mar. 30
5460	24	1	Sun.	Mar. 24	Apr. 4
5560	29	5	Thu.	Mar. 29	Apr. 10
5736	33	5	Thu.	Apr. 2	Apr. 15
5737	21	1	Sun.	Mar. 21	Apr. 3
5738	40	7	Sat.	Apr. 9	Apr. 22
5739	30	5	Thu.	Mar. 30	Apr. 12
5740	19	3	Tue.	Mar. 19	Apr. 1

Summary

Y Formulas	δ	γ	Passover
do	–	–	old
not	–	–	rite
apply	–	–	new
do		–	rite
apply		10	
		11	
		12	
		13	

Acknowledgements—The author wishes to render grateful acknowledgment, for their gracious and valuable assistance, to Professor L. F. Meyers of Ohio State University; Rabbis A. Bogner and E. J. Lipman, both of Washington; and to Drs. L. E. Doggett and P. K. Seidelmann of the Naval Observatory.

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