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The Fading Amateur: William Lenhart and 19th-Century American Mathematics

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William Lenhart was an American mathematician who was active in the first part of the 19th century. His biographer, Samuel Tyler, described him as the greatest Diophantine algebraist who ever lived. Lenhart was introduced to Diophantine analysis by Robert Adrain, who wrote the first article on this topic that was published in the United States. Lenhart often worked on mathematical problems as a distraction from pain that was caused by an accident that occurred when he was a young man. He was an amateur mathematician in a country where mathematics was undergoing the process of professionalization. © 1990 Academic Press, Inc.

William Lenhart war ein amerikanischer Mathematiker, der in der ersten Hälfte des 19. Jahrhunderts aktiv war. Sein Biograph Samuel Tyler beschrieb ihn als den größten diophantischen Algebraiker, der jemals gelebt hat. Lenhart wurde von Robert Adrain in die diophantische Analysis eingeführt, der den ersten Aufsatz über dieses Thema verfaßte, der in den Vereinigten Staaten von Amerika veröffentlicht wurde. Lenhart arbeitete oft an mathematischen Problemen zur Ablenkung von Schmerzen, an denen er wegen eines Unfalles litt, der ihm als junger Mann widerfahren war. Er war ein Amateurmathematiker in einem Land, in dem sich die Mathematik in einem Professionalisierungsprozeß befand. © 1990 Academic Press, Inc.

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There were amateur mathematicians in the United States after William Lenhart (1787–1840); there still are today. But Lenhart, although strictly an amateur in the sense that he neither received nor sought to receive revenue from his mathematical work, was one of the leading American mathematicians of his generation. He is the last person, to my knowledge, who was both.

Lenhart was highly regarded by his countrymen. By the time of his death, he was heralded as a mathematician of extraordinary ability. Samuel Tyler, although not a scientist, was one of the foremost Baconians in America and Lenhart's nephew by marriage [King n.d., 3] [1]. He said of his uncle's contributions to the *Mathematical Miscellany:* "They have gained for him a reputation as the greatest Diophantine Algebraist that ever lived; and this is no mean renown, when it is considered that a Euler, a Lagrange, and a Gauss are his competitors" [Tyler 1841, 401]. Such words (see also [Mason 1841]) might be dismissed as exaggerated praise from family and friends, but others with far more mathematical knowledge gave Lenhart similar compliments. Probably the most impressive came from

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Charles Gill, a competent, if self-taught, mathematician. He wrote to Lenhart: "No one will now deny that you have done more with the Diophantine analysis than any man who ever lived" [2]. In addition, the mathematical astronomer Daniel Kirkwood described Lenhart as an "eminent mathematician" and cited Gill's evaluation, implying his agreement [Kirkwood 1875]. (On Kirkwood, see [Numbers 1977, Chap. 4; Marsden 1973].)

Lenhart was born in York, Pennsylvania, in 1787 to parents of Swiss and German ancestry. His father was a clock maker and silversmith. As a youth, Lenhart, like Newton, had an unusual ability to devise and construct mechanical toys, which he made in his father's shop. Lenhart was apparently educated entirely by his parents, except for a period of about 18 months, which he spent in the York Academy. Robert Adrain [3], one of the best American mathematicians of the time, was teaching there and considered young Lenhart to have exceptional mathematical ability. Lenhart probably received all his formal instruction in mathematics beyond arithmetic from Adrain during this period. Thus Adrain probably introduced Lenhart to Diophantine analysis—a branch of number theory that seeks rational solutions to indeterminate problems—in which teacher and pupil later distinguished themselves. Adrain wrote the first article on the topic to appear in an American publication [Adrain 1806–1807].

Following his education at the York Academy, Lenhart, then about 17, was urged by his father to obtain a position in business. He worked first in Baltimore and then in Philadelphia, where he soon proved to be an exceptional bookkeeper. But Lenhart was not totally enthusiastic about his career. Many years later he wrote "they made a merchant of me to gather ware and gaze upon empty space with a vacant stare, instead of a mathematician to gather Honour and Fame and become useful" (Lenhart to Gill, 7 Sept. 1837, CUL).

After being offered a partnership in a business firm in 1812, Lenhart went home on a holiday to see his family, friends, and fiancée, and met with a freak accident. He was taking a ride in a gig when he passed by a traveling menagerie. The roar of a lion in the caravan frightened his horse, and Lenhart, thrown from his gig, fractured a leg and seriously injured his back. He never recovered; for the rest of his life he suffered extreme pain, exacerbated by a second fracture of the same leg.

The accident forced Lenhart to withdraw from his business partnership and sever his engagement. By strict economy he was able to live comfortably on his savings and a stipend from his father. He sought relief from pain by traveling and by playing the flute and violin, for which he had a significant talent. He also sought relief by taking opium, which was then widely available without prescription, and by doing mathematical problems, often concurrently. He wrote to Gill, who was then editor of the country's only mathematical journal, the *Mathematical Miscellany:*

> My afflictions however do not prevent me from pursuing my thoughts on Diophantine subjects for it not infrequently happens that whilst I am in bed, in the midst of excruciating pain and under the influence of opium, which I am sometimes obliged to use freely, I discover

> views, calculate and obtain formulas, that it seems to me, I could not possibly do in any other way or at any other time. (2 Nov. 1836, CUL)

Lenhart's description of his mathematical work, although he was likely being falsely modest, was more accurate than those of his sympathetic contemporaries. He wrote: "It is certainly interesting and contains matter which, without subjecting myself to the charge of vanity, I think I may venture to say, is, if not ingenious, at least novel and exceedingly curious" (to Gill, 20 Dec. 1837, CUL).

Although certainly not comparable to that of Gauss, Lagrange, or Euler, Lenhart's work showed ingenuity and originality [4]. When about 19, he was awarded a prize medal for contributing the best solution to a prize problem in the *Mathematical Correspondent*. As modest as this accomplishment might appear, it was the highest honor in mathematics available in the United States at that time [5]. His work in number theory was also meritorious; L. E. Dickson cites him 11 times in his *History of the Theory of Numbers* [1919] [6]. But Lenhart's results are of minor importance. They generally consist of finding a particular solution or solutions to a specific problem such as to find "three squares such that the sum of every two shall make a square" [Lenhart 1839, 132], or "to find n cube numbers such that if from each of them a given number (a) be subtracted, the sum of the remainders shall be a square" [Lenhart 1837, 263].

Until the publication of Gauss's *Disquisitiones Arithmeticae* in 1801, work in number theory consisted largely of unrelated, if sometimes brilliant, results. Lenhart used none of the general methods that Gauss introduced and became aware of them only near the end of his life. He saw these methods referred to in two solutions to a problem in the *Miscellany* [7]. This prompted Lenhart to write to Gill to inquire about the works of both Gauss and Legendre (15 July 1839, CUL).

In addition to being unaware of current developments in number theory, Lenhart did not prove any general existence theorems, such as Benjamin Peirce's proof that no odd perfect number has fewer than four prime factors [Peirce 1832]. Nor did he devise any new general methods of attacking Diophantine problems, as did Gill in his *Applications of the Angular Analysis to the Indeterminate Problems of the Second Degree* [1848]. In this work Gill used trigonometric functions to solve, among other things, the problem of finding n squares, the sum of any $n - 1$ of which is a square. Gill thereby generalized a result of Euler, who had solved the problem for the case n equals 4. These and similar considerations, in my opinion, show that both Peirce and Gill were better number theorists than Lenhart.

Moreover, Lenhart's breadth of mathematical knowledge was narrow; he admitted reluctantly to Gill that he owned very few mathematical books (Lenhart to Gill, 2 Nov. 1836, CUL; see also [Kirkwood 1875, 181]), and on another occasion wrote:

> I very much like . . . all geometrical and other questions that are perfect in their results; but care very little (perhaps because I know so little) about the many abstruse questions in mechanics and on curves . . . which seem to me to be entirely speculative and in no manner practically useful. Do not I beg of you, suppose that I object to all questions in mechanics and

> on curves etc., I object only to such as I have described—such as contain impossibilities in their enunciation. Nor do not think the worse of me for expressing an opinion so unmathematical. (Lenhart to Gill, 7 Sept. 1837, CUL)

By expressing a distaste for mathematics other than synthetic geometry and Diophantine analysis, Lenhart was behaving like a typical amateur. He was simply complaining about the value of mathematics that he did not understand [8]. It is hard to think of mathematics that is more applicable than mechanics and less applicable than number theory.

Furthermore, that Lenhart worked in number theory is significant. This field of mathematics is probably the one best suited to the amateur. The problems are easily stated and understood by someone with little or no mathematical training, and often solutions can be devised that require ingenuity, but not extensive knowledge.

Although Lenhart considered himself an amateur in the sense that his interest in mathematics was purely recreational, he did not consider himself an amateur in the sense of achievement. To the contrary, he felt that he had reached the peak of mathematical accomplishment. This extract from a letter to Gill indicates how he valued his work:

> While I was engaged in the study which produced so many curious results, and which had occupied the attention of so many eminent mathematicians in ancient as well as in modern times, I really felt as though I had been (and perhaps really was) admitted into the great Temple of Numbers—even into the sanctum sanctorum—and permitted there to revel amongst and to select from the wonders contained therein whatever I might desire—a privilege not granted by the master of the Temple to anyone living before—and hence the vast variety of original matter on old subjects that I have already and may probably in the future produce on new, must be ascribed or credited to the Great Master of the Temple, and not to any peculiar talent or extraordinary exertion of your humble and severely afflicted friend. (n.d., CUL)

Lenhart's estimation of his abilities was indeed high. He was not just one of many men who had enjoyed access to the secrets of the Temple of Numbers; he had been the sole visitor. Part of the explanation for Lenhart's exaggerated self-esteem was that amateurs generally tend to overvalue their work. Lenhart simply did not know enough about mathematics to put his work in a reasonable perspective. Ignorance of mathematics may also explain the high praise Lenhart received from others. But Gill was well acquainted with contemporary European work in number theory, and even Tyler, who was not a mathematician, compared Lenhart's work with that of Gauss, Euler, and Lagrange, indicating that he had some knowledge of the principal contributors to number theory.

In contrast to the amateur Lenhart is Benjamin Peirce, the leading American mathematician of the mid-19th century. (On Peirce see Archibald et al. [1925], Kohlstedt [1976a, 143–50, 163–168], Lenzen [1968], and Pycior [1979].) No American scientist of the 19th century was more committed to professionalism than Peirce. The New York Mathematical Society, the first continuing professional mathematical association in the United States, as the forerunner of the American

Mathematical Society, was not founded until 1888, well after Peirce's death. However, more comprehensive scientific institutions were formed during Peirce's lifetime and Peirce was instrumental in the founding of many of them. He was a key figure in the transformation of the Association of American Geologists and Naturalists into the American Association for the Advancement of Science in 1848, and later served as the organization's president. In 1847 the American Academy of Arts and Sciences appointed him to a committee of five to devise a program for the organization of the Smithsonian Institution, and from 1855 to 1858 he worked with Alexander D. Bache and Joseph Henry to organize the Dudley Observatory at Albany, New York. Bache selected Peirce along with Louis Agassiz and Senator Henry Wilson of Massachusetts to draft a charter for the National Academy of Sciences, and Peirce was one of the original 50 members when the Academy was established in 1863.

To Peirce and many of his associates, the drive to achieve professionalism meant more than simply supplying an adequate number of jobs for scientists. It also meant making a sharp distinction between well-trained, knowledgeable scientists and dilettantes. Furthermore Peirce and his colleagues intended that the control of science should be firmly in the hands of the former. Although this process of professionalization caused friction between amateur scientists and the emerging body of professionals, most professionals attempted to maintain polite and cordial relations with amateurs. But Peirce had a particularly arrogant disdain for amateurs, which was apparent in many of his dealings with the American Association for the Advancement of Science. Perhaps the best-known incident was the Warner–Winslow affair, in which Peirce ignored charges by both John Warner and Charles Winslow that he had plagiarized their work [Kohlstedt 1976a, 81, 140–141, 143–150].

Despite their differences, such men as Peirce and Lenhart shared a profound reverence for science. To them, a scientist was a high-minded seeker of truth, void of prejudice and pettiness. Adrain expressed this attitude in a letter to a then-young Lenhart:

> I sincerely hope that you employ your leisure hours in making still further progress in science. The greatest glory which this world can afford confessedly belongs to him who stands foremost in the ranks of science, and you may certainly fill that station for some years, if God in his mercy spare your life and inspire you with the noble resolution of dedicating to grateful posterity the excellent talents for mathematics with which you are endowed. (quoted by [Mason 1841] and [Tyler 1841, 398])

Lenhart's complaint about the necessity of going into business was in accordance with Adrain's views, and its tone was repeated in many of Lenhart's other letters. Moreover, Adrain's sentiments were widely held by American scientists of the 19th century [Kohlstedt 1976a, 310] [9]. In particular, Benjamin Peirce's son, Charles Saunders Peirce, ascribed a similar attitude toward science to his father and his father's scientific associates. According to C. S. Peirce, these men did not regard science as " 'systemized knowledge' . . . nor anything set down in a book; but, on the contrary, a mode of life; not knowledge, but the devoted, well

considered life pursuit of knowledge; devotion to truth—not as one sees it, for that is no devotion to truth at all, but only to party—no, far from that, devotion to the truth that the man is not yet able to see but is striving to obtain'' [C. Peirce 1901, 693–694].

This attitude helps explain why Lenhart's countrymen so exaggerated his importance, and why amateurs were so irritating to men like Peirce. From the perspective of those who felt this way about science, scientific failure, either individually or nationally, was failure of enormous magnitude. In the first instance, Lenhart's failure to reach his potential, as predicted by Adrain, was not simply unfortunate, but tragic. Similarly, Lenhart's contemporaries had trouble dealing with the more general problem of America's failure to produce outstanding scientists; they remained dissatisfied with appeals to the country's scientific potential and accomplishments in other areas. For them, not only were scientific achievements valued above all others, but there was a moral and spiritual value associated with them. It followed that a country that lacked such achievements was morally deficient. In a new nation founded upon moral principles and assuming a certain moral superiority, this was intolerable.

Appreciation of the moral importance of science, however, also contributed to the 19th-century American scientists' disdain of amateurs. American professionals were attempting to gain recognition from European scientists and did not want American journals filled with embarrassing papers; also, professionals did not want amateurs to pester them at scientific meetings. But Peirce and others who viewed themselves as professional scientists had a higher purpose in discouraging amateurs; they felt a responsibility to protect science from those who were not suitable for nurturing her properly. They embraced the lofty responsibility of keeping science pure and holy.

A letter that Lenhart wrote to Peirce on December 30, 1839, serves as a focal point to discuss how the 19th-century scientist's attitude toward his work made it difficult to tolerate amateurs. The letter, which is remarkably similar to his other surviving letters, also gives a good picture of Lenhart:

> Two months and more have elapsed since I wrote to you . . . and as I thought at the time, a social and friendly letter soliciting a further development of your solution to my question no. 8 of the *Miscellany*, and I am yet without an answer, or even an acknowledgement of its receipt, an event I did not anticipate, and at which I feel somewhat pained. Had I supposed for one moment that my simple request would not have been attended to, or that a protracted silence, on your part would have been the result, you may be assured it had not been made. I wrote in confidence and thought of pleasing at the time, and under a full conviction of receiving from you an ample, and a spirited discussion in reply, and in due time; but in all this I have been mistaken. My doctrine is, and my practice in accordance ever has been promptly to notice, or if necessary, answer in full every polite and friendly letter of inquiry addressed to me, and believing your principles, on like occasion to be the same, I took it for granted that, in the course of a reasonable time, you would have favoured me with an answer to mine; but your long delay has proved me wrong, and also shown a want of courtesy towards me, that I certainly did not expect. I would briefly mention, in conclusion, that the Honored . . . Dr. Bowditch who was your inestimable friend, and efficient preceptor . . . did not refuse, but on the contrary was prompt in answering every inquiry addressed to him by me several years

ago; and had you done the same on the present occasion, my feelings . . . would have been spared: even this letter—a gentle rebuke—would have been avoided.

Be pleased to accept from me, the compliments of the season; and that you and all the inmates of old Harvard, may fully enjoy the feasts of these feasting times is the present wish of your respectful friend.

Wm Lenhart

P.S. I once, many years ago, saw Kersey's Algebra in Philadelphia. If you have it in the college Library, I should be greatly obliged to you, indeed, to get one of the students, or some other person in college to copy . . . the Diophantine problems relating to cubes, contained in the latter part of the work. . . . I am very anxious to possess them and therefore the favour, if granted shall be acknowledged by a thousand thanks, and reciprocated in any possible way. Permit me briefly to mention here that I am, and have been for many years, severely afflicted and in almost constant pain, occasioned by an injury to the spine and a consequent paralysis to the extremities; and that the Diophantine Analysis, requiring but few books and but little reading, as a recreation and a study in my confinement has been invaluable to me; and hence the above rather unreasonable request, and for the making of which I hope you will grant me pardon. [If any, or all of my papers in the *Miscellany*] have met your approbation, or afforded you pleasure, . . . it will be delightful to me to know it. [10]

Whether Peirce was guilty of the offenses with which Lenhart charged him, the disregard of amateurs implied in the letter was characteristic of him. Here we have a poor cripple with whom Peirce could not be bothered [11]. That Peirce's amateur mentor, Nathaniel Bowditch, corresponded with Lenhart implies that in passing from Bowditch's to Peirce's generation something had been lost, not only in courtesy and human compassion, but in the ideal of science as an unprejudiced search for the truth.

Clearly Peirce did not see things this way. It was necessary to draw sharp lines between those who were the keepers of science and those who were not. Such an important responsibility could not be left open to just anyone who cared to have a crack at it, meaning that an important distinction had to be maintained between professionals like Peirce and amateurs like Lenhart.

Yet the actual distinction was not so great as Peirce would undoubtedly have liked. Peirce did not have a German Ph.D., nor did his scientific accomplishments comfortably distinguish him from Lenhart. Peirce's knowledge of mathematics was far broader than Lenhart's and Peirce had graduated from Harvard, but the amount of mathematics taught there had been small, and Peirce had been fortunate to come under Bowditch's tutelage, just as Lenhart had been fortunate to come under Adrain's. Peirce was essentially self-taught; he was just better self-taught. His conception of science and himself made it impossible to treat Lenhart as a peer, as Bowditch had, or as an amateur who did interesting and creditable work.

Lenhart lived in a time when professionals were seeking to distance themselves from amateurs, but this was only one factor that contributed to the sorrow in his life. His limited education and mathematical knowledge, and his debilitating accident, all worked to deny him the success and happiness that he might have

expected. In addition to tragedy, however, there was triumph. Despite his many disadvantages, Lenhart rose to make real, if minor, contributions to the theory of numbers. He also won the personal and scientific respect of many of his contemporaries. Moreover, to paraphrase Lenhart, he was, if not a great mathematician, at least a novel and exceedingly curious one.

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NOTES

1. Tyler was highly respected by his contemporaries as an essayist, especially on the Baconian philosophy of science. See [Anon. 1847; Anon. 1859; Daniels 1967; Duyckinck & Duyckinck 1866; Lanman 1886; Lieber 1881]. There are two other biographical sketches of Lenhart [Kirkwood 1875; Mason 1841]. Both are by avowed friends of Lenhart, and, like the one by Tyler, are sympathetic. Except for a few biographical details, this essay is based on Lenhart's letters and his printed work.

2. Quoted by [Tyler 1841, 408]. As another example see Vernon to Gill, 12 June 1836, Charles Gill Papers, John M. Olin Library, Cornell University. Hereafter this collection will be denoted by "CUL." All citations are by permission of the Olin Library. On Gill see [Hogan 1985; McClintock 1913].

3. Robert Adrain and Nathaniel Bowditch, who will be mentioned below, were the leading American mathematicians of their generation. On Adrain see [Coolidge 1926; Hogan 1977]; on Bowditch see [Bowditch 1840; Pickering 1846].

4. Lenhart published work under his own name and also used the pseudonyms Mary Bond and Diophantus [Tyler 1841, 401].

5. Lenhart actually tied for the prize; two medals were given for this problem.

6. All the references to Lenhart are in Vol. 2. See pp. 206, 499, 506, 512, 561, 573, 582, 596, 604, 610, 726.

7. *Mathematical Miscellany* **1,** 362–364. Cajori [1890, 278] states that contributions by Peirce and Strong in the *Miscellany* were the only uses of Gaussian methods in number theory before the publication of the *American Journal of Mathematics* in 1878. For another use of Gaussian methods by Peirce, see the *Mathematical Miscellany* **2,** 91.

8. On amateurism see [Beach 1972; Daniels 1968; Kohlstedt 1976b; Reingold 1976].

9. A similar sentiment by another American mathematician of Lenhart's generation is found in a letter from Charles Gill to Theodore Strong, 18 Aug. 1838, CUL.

10. Benjamin Peirce Papers, Houghton Library, Harvard University. Quoted by permission of the Houghton Library.

11. It is highly likely that Peirce was aware of Lenhart's affliction even though Lenhart describes it in this letter. Lenhart made no secret of it; furthermore, Lenhart was one of the leading dozen mathematicians in the country at the time, and the mathematicians did correspond with one another about each other.

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