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Pomeron in the $\mathcal{N}=4$ supersymmetric gauge model at strong couplings

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Abstract

We find the BFKL Pomeron intercept at $\mathcal{N}=4$ supersymmetric gauge theory in the form of the inverse coupling expansion $j_0 = 2 - 2\lambda^{-1/2} - \lambda^{-1} + 1/4\lambda^{-3/2} + 2(1+3\zeta_3)\lambda^{-2} + O(\lambda^{-5/2})$ with the use of the AdS/CFT correspondence in terms of string energies calculated recently. The corresponding slope $\gamma'(2)$ of the anomalous dimension calculated directly up to the fifth order of perturbation theory turns out to be in an agreement with the closed expression obtained from the recent Basso results.

1. Introduction

Pomeron is the Regge singularity of the t-channel partial wave [1] responsible for the approximate equality of total cross-sections for high energy particle–particle and particle–antiparticle interactions valid in an accordance with the Pomeranchuk theorem [2]. In QCD the Pomeron is a colorless object, constructed from reggeized gluons [3].

The investigation of the high energy behavior of scattering amplitudes in the $\mathcal{N}=4$ Supersymmetric Yang–Mills (SYM) model [4–6] is important for our understanding of the Regge processes in QCD. Indeed, this conformal model can be considered as a simplified version of QCD, in which the next-to-leading order (NLO) corrections [7] to the Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation [3] are comparatively simple and numerically small. In the $\mathcal{N}=4$ SYM the

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equations for composite states of several reggeized gluons and for anomalous dimensions of quasi-partonic operators turn out to be integrable at the leading logarithmic approximation [8,9]. Further, the eigenvalue of the BFKL kernel for this model has the remarkable property of the maximal transcendentality [5]. This property gave a possibility to calculate the anomalous dimensions (AD) γ of the twist-two Wilson operators in one [10], two [5,11], three [12], four [13, 14] and five [15] loops using the QCD results [16] and the asymptotic Bethe ansatz [17] improved with wrapping corrections [14][1] in an agreement with the BFKL predictions [4,5].

On the other hand, due to the AdS/CFT correspondence [19–21], in $\mathcal{N}=4$ SYM some physical quantities can be also computed at large couplings. In particular, for AD of the large spin operators Beisert, Eden and Staudacher constructed the integral equation [22] with the use the asymptotic Bethe ansatz. This equation reproduced the known results at small coupling constants and is in a full agreement (see [23,24]) with large coupling predictions [25,26].

With the use of the BFKL equation in a diffusion approximation [3,4,6], strong coupling results for AD [25] and the Pomeron-graviton duality [27] the Pomeron intercept was calculated at the leading order in the inverse coupling constant (see the Erratum [28] to the paper [12]).[2] Similar results in the $\mathcal{N}=4$ SYM and QCD were obtained in Refs. [29] and [30]. The Pomeron-graviton duality in the $\mathcal{N}=4$ SYM gives a possibility to construct the Pomeron interaction model as a generally covariant effective theory for the reggeized gravitons [31].

Below we use recent calculations [32–35] of string energies to find the strong coupling corrections to the Pomeron intercept $j_0=2-\Delta$ in next orders. We discuss also the relation between the Pomeron intercept and the slope of the anomalous dimension at $j=2$.

2. BFKL equation at small coupling constant

The eigenvalue of the BFKL equation in $\mathcal{N}=4$ SYM model has the following perturbative expansion [4,5] (see also Ref. [6])

$$j-1=\omega=\frac{\lambda}{4\pi^2}\left[\chi(\gamma_{BFKL})+\delta(\gamma_{BFKL})\frac{\lambda}{16\pi^2}\right], \quad \lambda=g^2N_c, \tag{1}$$

where λ is the t'Hooft coupling constant. The quantities χ and δ are functions of the conformal weights m and $\widetilde{m}$ of the principal series of unitary Möbius group representations, but for the conformal spin $n=m-\widetilde{m}=0$ they depend only on the BFKL anomalous dimension

$$\gamma_{BFKL}=\frac{m+\widetilde{m}}{2}=\frac{1}{2}+i\nu \tag{2}$$

and are presented below [4,5]

$$\chi(\gamma)=2\Psi(1)-\Psi(\gamma)-\Psi(1-\gamma), \tag{3}$$

$$\delta(\gamma)=\Psi''(\gamma)+\Psi''(1-\gamma)+6\zeta_3-2\zeta_2\chi(\gamma)-2\Phi(\gamma)-2\Phi(1-\gamma). \tag{4}$$

Here $\Psi(z)$ and $\Psi'(z)$, $\Psi''(z)$ are the Euler Ψ-function and its derivatives. The function $\Phi(\gamma)$ is defined as follows

$$\Phi(\gamma)=2\sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{k+\gamma}\beta'(k+1), \tag{5}$$

[1] The anomalous dimensions up to four loops were calculated also with the use of the Baxter equation [18].

[2] The value of this intercept was estimated earlier in Ref. [11].

where

$$\beta'(z) = \frac{1}{4}\left[\Psi'\left(\frac{z+1}{2}\right) - \Psi'\left(\frac{z}{2}\right)\right]. \tag{6}$$

Due to the symmetry of ω to the substitution $\gamma_{BFKL} \to 1 - \gamma_{BFKL}$ expression (1) is an even function of ν

$$\omega = \omega_0 + \sum_{m=1}^{\infty} (-1)^m D_m \nu^{2m}, \tag{7}$$

where

$$\omega_0 = 4\ln 2 \frac{\lambda}{4\pi^2}\left[1 - \bar{c}_1 \frac{\lambda}{16\pi^2}\right] + O(\lambda^3), \tag{8}$$

$$D_m = 2(2^{2m+1} - 1)\zeta_{2m+1} \frac{\lambda}{4\pi^2} + \frac{\delta^{(2m)}(1/2)}{(2m)!} \frac{\lambda^2}{64\pi^4} + O(\lambda^3). \tag{9}$$

According to Ref. [5] we have

$$\bar{c}_1 = 2\zeta_2 + \frac{1}{2\ln 2}\left(11\zeta_3 - 32\,\mathrm{Ls}_3\left(\frac{\pi}{2}\right) - 14\pi\zeta_2\right) \approx 7.5812, \tag{10}$$

where (see [36])

$$\mathrm{Ls}_3(x) = -\int_0^x \ln^2\left|2\sin\left(\frac{y}{2}\right)\right| dy. \tag{11}$$

Thus, the rightmost Pomeron singularity of the partial wave $f_j(t)$ in the perturbation theory is situated at

$$j_0 = 1 + \omega_0 = 1 + 4\ln 2 \frac{\lambda}{4\pi^2}\left[1 - \bar{c}_1 \frac{\lambda}{16\pi^2}\right] + O(\lambda^3) \tag{12}$$

for small values of coupling λ.

In turn, the anomalous dimension γ also has the square-root singularity in this point, which means, that the convergency radius of the perturbation series in λ for the anomalous dimension $\gamma = \gamma(\omega, \lambda)$ at small ω is given by the expression

$$\lambda_{cr} = \frac{\pi^2\omega}{\ln 2}\left(1 + \bar{c}_1 \frac{\omega}{16\ln 2}\right) + O(\omega^3). \tag{13}$$

To clarify this statement let us write representation (7) for ω in the diffusion approximation for arbitrary λ

$$\omega = \omega_0(\lambda) - D_1(\lambda)\nu^2 + O(\nu^4). \tag{14}$$

From this expression we obtain, that the anomalous dimension has the square-root singularity

$$\lim_{\lambda \to \lambda_{cr}} \gamma = \sqrt{\frac{\omega_0'(\lambda_{cr})(\lambda_{cr} - \lambda)}{D_1(\lambda_{cr})}} + const, \tag{15}$$

where λ_{cr} is a function of ω satisfying the equation

$$\omega = \omega_0(\lambda_{cr}). \tag{16}$$

Therefore the perturbative series for the anomalous dimension γ

$$\gamma = \sum_{k=1}^{\infty} \lambda^k c_k(\omega), \tag{17}$$

has the finite radius of convergence $\lambda = \lambda_{cr}$ and its coefficients c_k behave at large k as follows

$$\lim_{k\to\infty} c_k = \lambda_{cr}^{-k} k^{-\frac{3}{2}} \frac{1}{2\sqrt{\pi}} \sqrt{\frac{\lambda_{cr}\omega_0'(\lambda_{cr})}{D_1(\lambda_{cr})}}. \tag{18}$$

It will be interesting to find higher order corrections to the BFKL intercept $\omega_0(\lambda)$ and the diffusion coefficient $D_1(\lambda)$ by comparing the above asymptotic expression for c_k with the analytic results at $k = 1$–5 obtained recently [5,12–15]. Note, that the BFKL singularity for positive ω is situated at positive $\lambda = \lambda_{cr}$. But it is expected, that with growing ω the nearest singularity, responsible for the perturbation theory divergency will be at negative λ. Positions of both singularities can be found from the perturbative expansion of γ with the possible use of appropriate resummation methods (cf. [12]).

Due to the Möbius invariance and hermicity of the BFKL hamiltonian in $\mathcal{N} = 4$ SUSY expansion (7) is valid also at large coupling constants. In the framework of the AdS/CFT correspondence the BFKL Pomeron is equivalent to the reggeized graviton [27]. In particular, in the strong coupling regime $\lambda \to \infty$

$$j_0 = 2 - \Delta, \tag{19}$$

where the leading contribution $\Delta = 2/\sqrt{\lambda}$ was calculated in Refs. [28–30]. Below we find next-to-leading terms in the strong coupling expansion of the Pomeron intercept. In the next section the simple approach to the intercept estimates discussed shortly in Ref. [28] will be reviewed.

3. AdS/CFT correspondence

Due to the energy–momentum conservation, the universal anomalous dimension of the stress tensor $T_{\mu\nu}$ should be zero, i.e.,

$$\gamma(j = 2) = 0. \tag{20}$$

It is important, that the anomalous dimension γ contributing to the DGLAP equation [37] does not coincide with γ_{BFKL} appearing in the BFKL equation. They are related as follows [7] (see also [38])

$$\gamma = \gamma_{BFKL} + \frac{\omega}{2} = \frac{j}{2} + i\nu, \tag{21}$$

where the additional contribution $\omega/2$ is responsible in particular for the cancelation of the singular terms $\sim 1/\gamma^3$ obtained from the NLO corrections (1) to the eigenvalue of the BFKL kernel [7].

Using above relations one obtains

$$\nu(j = 2) = i. \tag{22}$$

As a result, from Eq. (7) for the Pomeron intercept we derive the following representation for the correction Δ (19) to the graviton spin 2

$$\Delta = \sum_{m=1}^{\infty} D_m. \tag{23}$$

In the diffusion approximation, where $D_m = 0$ for $m \geqslant 2$, one obtains from (23) the relation between the diffusion coefficient D_1 and Δ (see [28])

$$D_1 \approx \Delta. \tag{24}$$

This relation was also obtained in Ref. [39].

According to (19) and (23), we have the following small-ν expansion for the eigenvalue of the BFKL kernel

$$j - 2 = \sum_{m=1}^{\infty} D_m\big(\big(-\nu^2\big)^m - 1\big), \tag{25}$$

where ν^2 is related to γ according to Eq. (21)

$$\nu^2 = -\left(\frac{j}{2} - \gamma\right)^2. \tag{26}$$

On the other hand, due to the AdS/CFT correspondence the string energies E in dimensionless units are related to the anomalous dimensions γ of the twist-two operators as follows [20,21][3]

$$E^2 = (j + \Gamma)^2 - 4, \quad \Gamma = -2\gamma \tag{27}$$

and therefore we can obtain from (26) the relation between the parameter ν for the principal series of unitary representations of the Möbius group and the string energy E

$$\nu^2 = -\left(\frac{E^2}{4} + 1\right). \tag{28}$$

This expression for ν^2 can be inserted in the r.h.s. of Eq. (25) leading to the following expression for the Regge trajectory of the graviton in the anti-de-Sitter space

$$j - 2 = \sum_{m=1}^{\infty} D_m\left[\left(\frac{E^2}{4} + 1\right)^m - 1\right]. \tag{29}$$

Note [28], that due to (28) expression (7) for the eigenvalue of the BFKL kernel in the diffusion approximation (24)

$$j = j_0 - \Delta\nu^2 = 2 - \Delta\big(\nu^2 + 1\big), \tag{30}$$

is equivalent to the linear graviton Regge trajectory

$$j = 2 + \frac{\alpha'}{2}t, \quad \alpha' t = \Delta\frac{E^2}{2}, \tag{31}$$

where its slope α' and the Mandelstam invariant t, defined in the 10-dimensional space, equal

$$\alpha' = \Delta\frac{R^2}{2}, \qquad t = \frac{E^2}{R^2} \tag{32}$$

and R is the radius of the anti-de-Sitter space.

[3] Note that our expression (27) for the string energy E differs from a definition, in which E is equal to the scaling dimension Δ_{sc}. But Eq. (27) is correct, because it can be presented as $E^2 = (\Delta_{sc} - 2)^2 - 4$ and coincides with Eqs. (45) and (3.44) from Refs. [20] and [21], respectively.

Now we return to Eq. (29) in general case. We assume below, that it is valid also at large j and large λ in the region

$$1 \ll j \ll \sqrt{\lambda}, \tag{33}$$

where the strong coupling calculations of energies were performed [32,35]. Comparing the l.h.s. and r.h.s. of (29) at large j values gives us the coefficients D_m and Δ (see Appendix A).[4]

4. Graviton Regge trajectory and Pomeron intercept

The coefficients D_1 and D_2 at large λ can be written as follows[5]

$$D_1 = \frac{2}{\sqrt{\lambda}}\left(1 - \frac{2a_{01}}{\sqrt{\lambda}}\right), \qquad D_2 = -\frac{8a_{10}}{\lambda^{3/2}}, \tag{34}$$

where a_{01} and a_{10} are calculated in Appendix A

$$a_{01} = -\frac{1}{4}, \qquad a_{10} = \frac{3}{8}. \tag{35}$$

As a result, we find eigenvalue (29) of the BFKL kernel at large λ in the form of the nonlinear Regge trajectory of the graviton in the anti-de-Sitter space

$$j - 2 = D_1 \frac{E^2}{4} + D_2\left[\left(\frac{E^2}{4}\right)^2 + \frac{E^2}{2}\right]. \tag{36}$$

Note, that the perturbation theory for the BFKL equation gives this trajectory at small $\omega = j - 1$ (see Eq. (1)) according to Eqs. (21) and (28). However the energy–momentum constraint (20), leading to $\omega = 1$ at $E = 0$, is not fulfilled in the perturbation theory, because at $\gamma \to 0$ the right-hand side of (1) contains the pole singularities which should be canceled after an appropriate resummation of all orders.

Neglecting the term $D_2 E^2/2 \sim E^2/\lambda^{3/2}$ at $\lambda \to \infty$ in comparison with a larger correction $a_{01} E^2/\lambda$, we obtain the graviton trajectory (36) in the form

$$j - 2 = \frac{2}{\sqrt{\lambda}}\left(1 - \frac{2a_{01}}{\sqrt{\lambda}}\right)\frac{E^2}{4} - \frac{8a_{10}}{\lambda^{3/2}}\left(\frac{E^2}{4}\right)^2. \tag{37}$$

Solving this quadratic equation, one can derive with the same accuracy (see [32,35])

$$\frac{2}{\sqrt{\lambda}}\frac{E^2}{4} = (j-2)\left(1 + 2\frac{a_{01} + a_{10}(j-2)}{\sqrt{\lambda}}\right). \tag{38}$$

On the other hand, due to (27) this relation can be written as follows

$$\frac{1}{2\sqrt{\lambda}}(j - 2\gamma)^2 = \frac{2}{\sqrt{\lambda}} + (j-2)\left(1 + 2\frac{a_{01} + a_{10}(j-2)}{\sqrt{\lambda}}\right) \tag{39}$$

and for $j - 2 \gg 1/\sqrt{\lambda}$ we have

$$j - 2\gamma = \sqrt{2(j-2)}\lambda^{1/4}\left[1 + \left(\frac{1}{j-2} + a_{01} + a_{10}(j-2)\right)\frac{1}{\sqrt{\lambda}}\right]. \tag{40}$$

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[4] When this paper was almost prepared for publication, we found the article [40] containing some of our results (see discussions in Appendix A).

[5] Here we consider only the calculation of the λ^{-1} correction to Pomeron intercept. More general results are presented in Appendix A.

In particular, for $j = 4$ one obtains the anomalous dimension for the Konishi operator $\gamma = \gamma_K$ [32] (see also Appendix B)

$$2 - \gamma_K = \lambda^{1/4}\left[1 + \left(\frac{1}{2} + a_{01} + 2a_{10}\right)\frac{1}{\sqrt{\lambda}}\right] = \lambda^{1/4}\left[1 + \frac{1}{\sqrt{\lambda}}\right]. \tag{41}$$

For the anomalous dimension at $j-2 \sim 1/\sqrt{\lambda}$ from (39) we obtain the square-root singularity similar to that appearing at small $j - 1 = \omega_0$ (8)

$$\gamma = -\frac{\lambda^{1/4}}{\sqrt{2}}\left(1 + \frac{a_{01}}{\sqrt{\lambda}}\right)\left(\sqrt{D_1 + j - 2} - \sqrt{D_1}\,\right), \tag{42}$$

where D_1 (34) is equal to the correction Δ to the graviton trajectory intercept with our accuracy

$$\Delta = D_1 \approx \frac{2}{\sqrt{\lambda}}\left(1 + \frac{1}{2\sqrt{\lambda}}\right).$$

Note, that in the region $j - 2 < -\Delta$, the anomalous dimension is complex similar to it in the perturbative regime at $j - 1 < \omega_0$ (8). Moreover, the position of the BFKL singularity of γ at large coupling constants can be found from the calculation of the radius of the divergency of the perturbation theory in $1/\sqrt{\lambda}$ at small $j - 2$.

5. Numerical analysis of the Pomeron intercept $j_0(\lambda)$

Let us obtain a unified expression for the position of the Pomeron singularity $j_0 = 1 + \omega_0$ for arbitrary values of λ, using an interpolation between weak and strong coupling regimes.

It is convenient to replace ω_0 with the new variable t as follows

$$t_0 = \frac{\omega_0}{1 - \omega_0}, \qquad \omega_0 = \frac{t_0}{1 + t_0}. \tag{43}$$

This variable has the asymptotic behavior $t_0 \sim \lambda$ at $\lambda \to 0$ and $t_0 \sim \sqrt{\lambda}/2$ at $\lambda \to \infty$ similar to the case of the cusp anomalous dimension (see, for example, [11]). So, following the method of Refs. [11,12,41], we shall write a simple algebraic equation for $t_0 = t_0(\lambda)$ whose solution will interpolate ω_0 for the full λ range.

We choose the equation of the form

$$k_0(\lambda) = k_1(\lambda)t_0 + k_2(\lambda)t_0^2, \tag{44}$$

where the following ansatz for the coefficients k_0, k_1 and k_2 is used:

$$k_0(\lambda) = \beta_0\lambda + \alpha_0\lambda^2, \qquad k_1(\lambda) = \beta_1 + \alpha_1\lambda, \qquad k_2(\lambda) = \gamma_2\lambda^{-1} + \beta_2 + \beta_2\lambda. \tag{45}$$

Here γ_2, α_i and β_i $(i = 0, 1, 2)$ are free parameters, which are fixed using the known asymptotics of ω_0 at $\lambda \to 0$ and $\lambda \to \infty$.

The solution of quadratic equation (44) is given below

$$t_0 = \frac{k_1}{2k_2}\left[\sqrt{1 + \frac{4k_0k_2}{k_1^2}} - 1\right]. \tag{46}$$

To fix the parameters γ_2, α_i and β_i $(i = 0, 1, 2)$, we use two known coefficients for the weak coupling expansion of ω_0:

$$\omega_0 = \tilde{e}_1\lambda + \tilde{e}_2\lambda^2 + \tilde{e}_3\lambda^3 + \cdots \quad (\text{at } \lambda \to 0) \tag{47}$$

with

$$\tilde{e}_1 = \frac{\ln 2}{\pi^2} \approx 0.07023, \qquad \tilde{e}_2 = -\tilde{e}_1 \frac{7.5812}{16\pi^2} \approx -0.00337 \tag{48}$$

and first four terms of its strong coupling expansion

$$\omega_0 = 1 - \Delta, \quad \Delta = \frac{2}{\sqrt{\lambda}}\left(1 + \frac{\tilde{t}_1}{\sqrt{\lambda}} + \frac{\tilde{t}_2}{\lambda} + \frac{\tilde{t}_3}{\lambda^{3/2}} + \frac{\tilde{t}_4}{\lambda^2} + \cdots\right) (\text{at } \lambda \to \infty) \tag{49}$$

with (see below Eq. (58))

$$\tilde{t}_1 = \frac{1}{2}, \qquad \tilde{t}_2 = -\frac{1}{8}, \qquad \tilde{t}_3 = -1 - 3\zeta_3, \qquad \tilde{t}_4 = 2a_{12} - \frac{145}{128} - \frac{9}{2}\zeta_3. \tag{50}$$

The coefficients $\tilde{e}_3$ and $\tilde{t}_4$ are unknown but we estimate them later from the interpolation.

Then, for the weak and strong coupling expansions of t one obtains

$$t_0 = e_1\lambda + e_2\lambda^2 + e_3\lambda^3 + \cdots \quad (\text{when } \lambda \to 0), \tag{51}$$

$$t_0 = \frac{\sqrt{\lambda}}{2}\left(1 - \frac{t_1}{\sqrt{\lambda}} - \frac{t_2}{\lambda} - \frac{t_3}{\lambda^{3/2}} - \frac{t_4}{\lambda^2}\right) + \cdots \quad (\text{when } \lambda \to \infty), \tag{52}$$

where

$$\begin{aligned}
&e_1 = \tilde{e}_1, \qquad e_2 = \tilde{e}_2 + \tilde{e}_1^2, \qquad e_3 = \tilde{e}_3 + \tilde{e}_1\tilde{e}_2 + \tilde{e}_1^3, \qquad t_1 = \tilde{t}_1 + 2 = \frac{5}{2},\\
&t_2 = \tilde{t}_2 - \tilde{t}_1^2 = -\frac{3}{8}, \qquad t_3 = \tilde{t}_3 - 2\tilde{t}_2\tilde{t}_1 + \tilde{t}_1^3 = -\frac{3}{4}(1 + 4\zeta_3),\\
&t_4 = \tilde{t}_4 - 2\tilde{t}_3\tilde{t}_1 - \tilde{t}_2^2 + 3\tilde{t}_2\tilde{t}_1^2 - \tilde{t}_1^4 = 2a_{12} - \frac{39}{128} - \frac{3}{2}\zeta_3.
\end{aligned} \tag{53}$$

Comparing the l.h.s. and the r.h.s. of Eq. (44) at $\lambda \to 0$ and $\lambda \to \infty$, respectively, we derive the following relations

$$\begin{aligned}
&\alpha_2 = 4\alpha_0, \qquad \alpha_2 = 10\alpha_0, \qquad \beta_1 = C_1\alpha_0, \qquad \beta_2 = C_2\alpha_0,\\
&\gamma_2 = C_3\alpha_0, \qquad \beta_0 = (C_2 - 22)\frac{\alpha_0}{4}
\end{aligned} \tag{54}$$

with the free parameter α_0 which disappears in the relationship k_1/k_2 and k_0/k_2 and, thus, in the results (46) for t_0.

Here

$$C_1 \approx 88.60, \qquad C_2 \approx 42.41, \qquad C_3 \approx -277.0, \tag{55}$$

which lead to the following predictions for the coefficients e_3 and t_4 in (51) and (52)

$$\begin{aligned}
&e_3 = -\frac{10e_2 + 2C_2e_1e_2 + 4e_1^2}{C_1 + 2C_3e_1} \approx -0.00079,\\
&t_4 = \frac{9 + 16(C_3 - 5C_1 + 7C_2)}{128} \approx -40.5774
\end{aligned} \tag{56}$$

and, respectively, for the corresponding terms in (47), (49) and (50)

$$\tilde{e}_3 \approx -0.00066, \qquad \tilde{t}_4 \approx -51.0117, \qquad a_{12} \approx -22.2348. \tag{57}$$

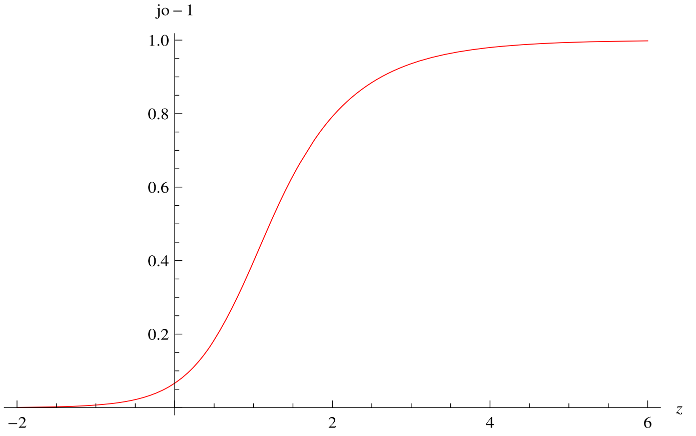

Fig. 1. (Color online.) The results for j_0 as a function of z ($\lambda = 10^z$).

Note that the results for the coefficients e_3, t_4, $\tilde{e}_3$, $\tilde{t}_4$ and a_{12} do not depend on the free parameter α_0.

On Fig. 1, we plot the Pomeron intercept j_0 as a function of the coupling constant λ. The behavior of the Pomeron intercept j_0 shown in Fig. 1 is similar to that found in QCD with some additional assumptions (see Ref. [30]).

6. Conclusion

We found the intercept of the BFKL Pomeron at weak and strong coupling regimes in the $\mathcal{N} = 4$ Supersymmetric Yang–Mills model.

At large couplings $\lambda \to \infty$, the correction Δ for the Pomeron intercept $j_0 = 2 - \Delta$ has the form (see Eq. (A.21))

$$\Delta = \frac{2}{\lambda^{1/2}}\left[1 + \frac{1}{2\lambda^{1/2}} - \frac{1}{8\lambda} - (1 + 3\zeta_3)\frac{1}{\lambda^{3/2}} + \left(2a_{12} - \frac{145}{128} - \frac{9}{2}\zeta_3\right)\frac{1}{\lambda^2} + O\left(\frac{1}{\lambda^{5/2}}\right)\right]. \tag{58}$$

The anomalous dimension has a square-root singularity at the value of the BFKL intercept both in the weak and strong coupling regimes. This value is related to the radius of convergency of perturbation theory in λ and $1/\sqrt{\lambda}$ near the points $j_0 = 1$ and $j_0 = 2$, respectively.

The fourth corrections in (58) contain unknown coefficient a_{12}, which will be obtained after the evaluation of spinning folded string on the two-loop level. Some estimations were given in Section 6.

The slope of the universal anomalous dimension at $j = 2$ known by the direct calculations [42] up to the fifth order of perturbation theory can be written as follows

$$\gamma'(2) = -\frac{\sqrt{\lambda}}{4}\frac{I_3(\sqrt{\lambda})}{I_2(\sqrt{\lambda})}, \tag{59}$$

according to the well-known Basso result [33] for local operators of an arbitrary twist.

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Appendix A

Here we discuss coefficients D_m and the Pomeron intercept $2-\Delta$ using expression (29) at comparatively large j in the region $j \ll \sqrt{\lambda}$.

A.1. String energy at $1 \ll j \ll \sqrt{\lambda}$

The recent results for the string energies [34] in the region restricted by inequalities (33) can be presented in the form[6]

$$\frac{E^2}{4} = \sqrt{\lambda}\frac{S}{2}\left[h_0(\lambda) + h_1(\lambda)\frac{S}{\sqrt{\lambda}} + h_2(\lambda)\frac{S^2}{\lambda}\right] + O\left(S^{7/2}\right), \tag{A.1}$$

where

$$h_i(\lambda) = a_{i0} + \frac{a_{i1}}{\sqrt{\lambda}} + \frac{a_{i2}}{\lambda} + \frac{a_{i3}}{\sqrt{\lambda^3}} + \frac{a_{i2}}{\lambda^2}. \tag{A.2}$$

The contribution $\sim \sqrt{S}$ can be extracted directly from the Basso result [33] taking $J_{an} = 2$ according to [34]:

$$h_0(\lambda) = \frac{I_3(\sqrt{\lambda}\,)}{I_2(\sqrt{\lambda}\,)} + \frac{2}{\sqrt{\lambda}} = \frac{I_1(\sqrt{\lambda}\,)}{I_2(\sqrt{\lambda}\,)} - \frac{2}{\sqrt{\lambda}}, \tag{A.3}$$

where $I_k(\sqrt{\lambda}\,)$ is the modified Bessel functions. It leads to the following values of coefficients a_{0i}

$$a_{00} = 1, \qquad a_{01} = -\frac{1}{2}, \qquad a_{02} = a_{03} = \frac{15}{8}, \qquad a_{04} = \frac{135}{128}. \tag{A.4}$$

The coefficients a_{10} and a_{20} come from considerations of the classical part of the folded spinning string corresponding to the twist-two operators[7] (see, for example, [35])

$$a_{10} = \frac{3}{4}, \qquad a_{20} = -\frac{3}{16}. \tag{A.5}$$

The one-loop coefficient a_{11} is found recently in the paper [34] (see also [43]), considering different asymptotical regimes with taking into account the Basso result [33]

$$a_{11} = \frac{3}{16}(1-\zeta_3), \tag{A.6}$$

where ζ_3 is the Euler ζ-function.

[6] Here we put $S = j-2$, which in particular is related to the use of the angular momentum $J_{an} = 2$ in calculations of Refs. [32,35].

[7] We are grateful to Arkady Tseytlin for explaining this point.

All calculations were performed for nonzero values of the angular momentum J_{an} (really, $J_{an}=2$ was used) and are applicable also to the finite S values.[8] Moreover, all these coefficients are in a full agreement with numerical Y-system predictions (see [45,32] and references therein).

A.2. Equations for coefficients D_m and the Pomeron intercept $2-\Delta$

Thus, from expression (A.1) we obtain the following expansions of even powers of E in the small parameter $j/\sqrt{\lambda}$

$$\left(\frac{E^2}{4}\right)^2 = \lambda\frac{S^2}{4}\left[h_0^2(\lambda)+2h_0h_1(\lambda)\frac{S}{\sqrt{\lambda}}\right], \qquad \left(\frac{E^2}{4}\right)^3 = \lambda^{3/2}\frac{S^3}{8}h_0^3(\lambda). \tag{A.7}$$

Comparing the coefficients in the front of S, S^2 and S^3 in the l.h.s. and r.h.s. of (29), we derive the equations

$$1 = \frac{\sqrt{\lambda}}{2}h_0\overline{D}_1, \quad \overline{D}_1 = (D_1+2D_2+3D_3), \tag{A.8}$$

$$0 = \frac{1}{2}h_1\overline{D}_1 + \frac{\lambda}{4}h_0^2\overline{D}_2, \quad \overline{D}_2 = (D_2+3D_3), \tag{A.9}$$

$$0 = \frac{1}{2\sqrt{\lambda}}h_2\overline{D}_1 + \frac{\sqrt{\lambda}}{4}h_0h_1\overline{D}_2 + \frac{\lambda^{3/2}}{8}h_0^3D_3. \tag{A.10}$$

Their perturbative solution leads is given below

$$\overline{D}_1 = \frac{2}{\sqrt{\lambda}}\frac{1}{h_0}, \qquad \overline{D}_2 = -\frac{2}{\lambda}\frac{h_1}{h_0^2}\overline{D}_1 = -\frac{4}{\lambda^{3/2}}\frac{h_1}{h_0^3}, \tag{A.11}$$

$$D_3 = \frac{4}{\lambda^2}\frac{2h_1^2-h_0h_2}{h_0^4}\overline{D}_1 = \frac{8}{\lambda^{5/2}}\frac{2h_1^2-h_2h_0}{h_0^5} \tag{A.12}$$

and, correspondingly,

$$D_2 = \overline{D}_2 - 3D_3, \qquad D_1 = \overline{D}_1 - 2\overline{D}_2 + 3D_3. \tag{A.13}$$

Finally, we obtain the correction Δ to the Pomeron intercept in the form

$$\begin{aligned}\Delta &= D_1+D_2+D_3 = \overline{D}_1 - \overline{D}_2 + D_3 \\ &= \frac{2}{\sqrt{\lambda}}\frac{1}{h_0^2} + \frac{4}{\lambda^{3/2}}\frac{h_1}{h_0^3} + \frac{8}{\lambda^{5/2}}\frac{2h_1^2-h_2h_0}{h_0^5},\end{aligned} \tag{A.14}$$

where the λ-dependence of parameters h_i is given in Eqs. (A.2) and (A.3).

A.3. Strong coupling expansions of D_m and Δ

Using expressions (A.4)–(A.6) we have

$$D_3 = \frac{8r_3}{\lambda^{5/2}} + O\left(\frac{1}{\lambda^{7/2}}\right), \qquad \overline{D}_2 = -\frac{4}{\lambda^{3/2}}\left[\bar{c}_2 + \frac{\bar{c}_3}{\lambda^{1/2}} + \frac{\bar{c}_4}{\lambda} + O\left(\frac{1}{\lambda^{3/2}}\right)\right], \tag{A.15}$$

$$\overline{D}_1 = \frac{2}{\lambda^{1/2}}\left[1 + \frac{\bar{d}_1}{\lambda^{1/2}} + \frac{\bar{d}_2}{\lambda} + \frac{\bar{d}_3}{\lambda^{3/2}} + \frac{\bar{d}_4}{\lambda^2} + O\left(\frac{1}{\lambda^{5/2}}\right)\right], \tag{A.16}$$

[8] The previous calculations [44] were done with the zero values of the angular momentum J_{an} and cannot be directly applied for the finite S values. We are grateful to Arkady Tseytlin for explaining this point to us.

where

$$\bar{c}_2 = a_{10} = \frac{3}{4}, \qquad \bar{c}_3 = a_{11} - 3a_{10}a_{01} = \frac{3}{16}(7 - 8\zeta_3), \qquad r_3 = 2a_{10}^2 - a_{20} = \frac{21}{16},$$
$$\bar{c}_4 = a_{12} + 3a_{10}\left(2a_{01}^2 - a_{02}\right) - 3a_{11}a_{01} = a_{12} - \frac{9}{16}(5 + 4\zeta_3) \tag{A.17}$$

and

$$\bar{d}_1 = -2a_{01} = \frac{1}{2}, \qquad \bar{d}_2 = 2a_{01}^2 - a_{02} = -\frac{13}{8}, \qquad \bar{d}_3 = 2a_{01}a_{02} - a_{01}^3 - a_{03} = -\frac{29}{8},$$
$$\bar{d}_4 = a_{01}^4 - 3a_{01}^2 a_{02} + 2a_{01}a_{03} + a_{02}^2 - a_{04} = -\frac{97}{128}. \tag{A.18}$$

Here a_{02}, a_{12}, a_{03} and a_{04} are parameters which should be calculated in future at two, three and four loops of the string perturbation theory. It is important, that the coefficients D_k tend to zero at large λ as $\lambda^{-n+1/2}$.

Analogously, we can obtain expressions for D_2, D_1 and Δ:

$$D_2 = -\frac{4}{\lambda^{3/2}}\left[c_2 + \frac{c_3}{\lambda^{1/2}} + \frac{c_4}{\lambda} + O\left(\frac{1}{\lambda^{3/2}}\right)\right], \tag{A.19}$$
$$D_1 = \frac{2}{\lambda^{1/2}}\left[1 + \frac{d_1}{\lambda^{1/2}} + \frac{d_2}{\lambda} + \frac{d_3}{\lambda^{3/2}} + \frac{d_4}{\lambda^2} + O\left(\frac{1}{\lambda^{5/2}}\right)\right], \tag{A.20}$$
$$\Delta = \frac{2}{\lambda^{1/2}}\left[1 + \frac{\hat{d}_1}{\lambda^{1/2}} + \frac{\hat{d}_2}{\lambda} + \frac{\hat{d}_3}{\lambda^{3/2}} + \frac{\hat{d}_4}{\lambda^2} + O\left(\frac{1}{\lambda^{5/2}}\right)\right], \tag{A.21}$$

where

$$c_2 = \bar{c}_2, \qquad c_3 = \bar{c}_3, \qquad c_4 = \bar{c}_4 + 6r_3, \qquad d_1 = \bar{d}_1 = \hat{d}_1, \tag{A.22}$$
$$d_2 = \bar{d}_2 + 4\bar{c}_2, \qquad d_3 = \bar{d}_3 + 4\bar{c}_3, \qquad d_4 = \bar{d}_4 + 4\bar{c}_4 + 12r_3, \tag{A.23}$$
$$\hat{d}_2 = \bar{d}_2 + 2\bar{c}_2, \qquad \hat{d}_3 = \bar{d}_3 + 2\bar{c}_3, \qquad \hat{d}_4 = \bar{d}_4 + 2\bar{c}_4 + 4r_3 \tag{A.24}$$

and all $\bar{c}_i$ and $\bar{d}_i$ are given above in Eqs. (A.17) and (A.18). So, we have

$$\hat{d}_1 = \frac{1}{2}, \qquad \hat{d}_2 = -\frac{1}{8}, \qquad \hat{d}_3 = -1 - 3\zeta_3, \qquad \hat{d}_4 = 2a_{12} - \frac{145}{128} - \frac{9}{2}\zeta_3. \tag{A.25}$$

Using a similar approach, the coefficients $\hat{d}_1$ and $\hat{d}_2$ were found recently in the paper [40]. The corresponding coefficients $c_{2,0}$ and $c_{3,0}$ in [40] coincide with our $\hat{d}_1$ and $\hat{d}_2$ but in the expression for the Pomeron intercept they contributed with an opposite sign. Further, in the talk of Miguel S. Costa "Conformal Regge theory" on IFT Workshop "Scattering amplitudes in the multi-Regge limit" (Universidad Autonoma de Madrid, 24–26 October 2012) (see http://www.ift.uam.es/en/node/3985) the sign of these contributions to the Pomeron intercept was correct but there is a misprint the definition of the parameter of expansion. Note, however, that we have the next term $\hat{d}_3$ in the strong coupling expansion.

A.4. Anomalous dimension near $j = 2$

At $j = 2$, the universal anomalous dimension is zero (20), but its derivative $\gamma'(2)$ (the slope of γ) has a nonzero value in the perturbative theory

$$\gamma'(2) = -\frac{\lambda}{24} + \frac{1}{2}\left(\frac{\lambda}{24}\right)^2 - \frac{2}{5}\left(\frac{\lambda}{24}\right)^3 + \frac{7}{20}\left(\frac{\lambda}{24}\right)^4 - \frac{11}{35}\left(\frac{\lambda}{24}\right)^5 + O\left(\lambda^6\right), \tag{A.26}$$

as it follows from exact three-loop calculations [12,28]. Two last terms were calculated by V.N. Velizhanin [42] from the explicit results for γ in five loops [15].

To find the slope $\gamma'(2)$ at large values of the coupling constant we calculate the derivatives of the l.h.s. and r.h.s. of Eq. (25) written in the form

$$j - 2 = \sum_{m=1} D_m\left[\left(\frac{j}{2} - \gamma\right)^{2m} - 1\right] \tag{A.27}$$

in the variable j for $j = 2$ using $\gamma(2) = 0$:

$$1 = \left(1 - 2\gamma'(2)\right)\sum_{m=1} m D_m \equiv \left(1 - 2\gamma'(2)\right)\overline{D}_1, \tag{A.28}$$

where $\overline{D}_1$ is found in (A.8). So we obtain explicitly

$$1 - 2\gamma'(2) = \frac{\sqrt{\lambda}}{2} h_0(\lambda). \tag{A.29}$$

Substituting (A.3) in (A.29), we have the closed form for the slope $\gamma'(2)$

$$\gamma'(2) = -\frac{\sqrt{\lambda}}{4}\frac{I_3(\sqrt{\lambda})}{I_2(\sqrt{\lambda})},$$

which is in full agreement with predictions (A.26) of perturbation theory.

Appendix B

We apply Eqs. (25) and (26) with $j = 4$ (and/or $S = 2$) and D_i ($i = 1, 2, 3$) obtained in Appendix A, to find the large λ asymptotics of the anomalous dimension of the Konishi operator. So, it obeys to the equation

$$2 = \sum_{m=1} D_m\left(x^m - 1\right), \quad x \equiv (2 - \gamma_k)^2. \tag{B.1}$$

1. It is convenient to consider firstly the particular case, when $\overline{D}_2 = \overline{D}_3 = 0$ and, thus, $D_1 = \overline{D}_1 = 2/\sqrt{\lambda}h_0$. So, we have

$$2 = \overline{D}_1(x - 1) \tag{B.2}$$

and

$$x = \frac{2}{\overline{D}_1} + 1 = \sqrt{\lambda}h_0 + 1, \tag{B.3}$$

where h_0 has the closed form (A.3). So, the anomalous dimension γ_K can be represented as

$$2 - \gamma_K = (\sqrt{\lambda}h_0 + 1)^{1/2} \approx \lambda^{1/4}\left(\sqrt{h_0} + \frac{1}{2\sqrt{\lambda}\sqrt{h_0}} - \frac{1}{8\lambda h_0^{3/2}} + O\left(\frac{1}{\lambda^2}\right)\right). \tag{B.4}$$

For the case of the classic string, where $h_0 = 1$, i.e. $a_{00} = 1$ and $a_{0i} = 0$ $(i \geqslant 1)$, we reconstruct well-known results[9]

$$2 - \gamma_K \approx \lambda^{1/4}\left(1 + \frac{1}{2\sqrt{\lambda}} - \frac{1}{8\lambda} + O\left(\frac{1}{\lambda^{3/2}}\right)\right). \tag{B.5}$$

For the exact values of h_0 done in Eqs. (A.2) and (A.4), we have

$$\begin{aligned} 2 - \gamma_K &\approx \lambda^{1/4}\left(1 + \frac{1 + a_{01}}{2\sqrt{\lambda}} + \frac{1}{2\lambda}\left[a_{02} - \frac{(1 + a_{01})^2}{4}\right] + O\left(\frac{1}{\lambda^{3/2}}\right)\right) \\ &= \lambda^{1/4}\left(1 + \frac{1}{4\sqrt{\lambda}} + \frac{29}{32\lambda} + O\left(\frac{1}{\lambda^{3/2}}\right)\right). \end{aligned} \tag{B.6}$$

2. In the case when all $\overline{D}_i$ $(i = 1, 2, 3)$ are nonzero, it is convenient to represent the solution of Eq. (B.1) in the following form

$$x = \sqrt{\lambda} h_0 + 1 + x_1 + \frac{x_2}{\sqrt{\lambda}}. \tag{B.7}$$

Expanding $\overline{D}_i$ in the inverse series of $\sqrt{\lambda}$ and compare the coefficients in the front of λ^0 and $1/\sqrt{\lambda}$, we have

$$x_1 = 2a_{10}, \qquad x_2 = 2a_{11} + 4a_{20}. \tag{B.8}$$

So, the solution of Eq. (B.7) with the coefficients (B.8) has the form

$$\begin{aligned} 2 - \gamma_K \approx \lambda^{1/4}\bigg(1 &+ \frac{a_{01} + 1 + 2a_{10}}{2\sqrt{\lambda}} + \frac{1}{2\lambda}\left[a_{02} + 2a_{11} + 4a_{20} - \frac{(1 + a_{01} + 2a_{10})^2}{4}\right] \\ &+ O\left(\frac{1}{\lambda^{3/2}}\right)\bigg). \end{aligned} \tag{B.9}$$

Using Eqs. (A.4)–(A.6) the exact values of a_{ij}, we have

$$2 - \gamma_K \approx \lambda^{1/4}\left(1 + \frac{1}{\sqrt{\lambda}} + \frac{1}{4\lambda}[1 - 6\zeta_3] + O\left(\frac{1}{\lambda^{3/2}}\right)\right). \tag{B.10}$$

We would like to note that our coefficient in the front of $\lambda^{-1/4}$ is equal to 1, which in an agreement with calculations performed in [45,32,35]. Further, the coefficient in front of $\lambda^{-3/4}$ agrees with the results of [34] (see also Refs. [43] and [46]).

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[9] We should remind that our anomalous dimension γ_K has the additional factor $-1/2$, i.e. $\gamma_K = -\gamma_K^{standard}/2$.

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