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Controllability of Sobolev-Type Semilinear Integrodifferential Systems in Banach Spaces

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Abstract—Sufficient conditions for controllability of Sobolev-type semilinear integrodifferential systems in a Banach space are established. The results are obtained by using the Schaefer fixed-point theorem. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords—Controllability, Integrodifferential systems, Sobolev-type systems, Fixed-point theorem.

1. INTRODUCTION

Controllability of nonlinear systems represented by ordinary differential equations in infinite-dimensional spaces has been extensively studied by several authors. Naito [1] discussed the controllability of nonlinear Volterra integrodifferential systems and in [2,3] he has studied the controllability of semilinear systems whereas Yamamoto and Park [4] investigated the same problem for parabolic equation with uniformly bounded nonlinear term. Chukwu and Lenhart [5] have studied the controllability of nonlinear systems in abstract spaces. Do [7] and Zhou [8] discussed the approximate controllability for a class of semilinear abstract equations. Kwun *et al.* [9] studied the approximate controllability for delay Volterra systems with bounded linear operators. Recently Balachandran *et al.* [9,10] studied the controllability and local null controllability of Sobolev-type integrodifferential systems and functional differential systems in Banach spaces by using Schauder's fixed-point theorem. The purpose of this paper is to study the controllability of Sobolev-type semilinear integrodifferential systems in Banach spaces by using the Schaefer fixed-point theorem. The Sobolev-type semilinear integrodifferential equation considered here serves as an abstract formulation of partial integrodifferential equation which arise in various applications such as in the flow of fluid through fissured rocks [11], thermodynamics [12], and shear in second order fluids [13,14].

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Consider the Sobolev-type semilinear integrodifferential system of the form

$$(Ex(t))' + Ax(t) = (Bu)(t) + f\left(t, x(t), \int_0^t g(t, s, x(s))\, ds\right), \qquad t \in J = [0, b], \tag{1}$$
$$x(0) = x_0,$$

where the state $x(.)$ takes values in the Banach space X and the control function $u(.)$ is given in $L^2(J, U)$, a Banach space of admissible control functions with U as a Banach space. B is a bounded linear operator from U into Y, a Banach space, $g : J \times J \times X \to X$ and $f : J \times X \times X \to Y$. The norm of X is denoted by $\|.\|$ and Y by $|.|$.

2. PRELIMINARIES

The operators $A : D(A) \subset X \to Y$ and $E : D(E) \subset X \to Y$ satisfy Hypothesis [C_i] for $i = 1, \dots, 4$.

[C_1] A and E are closed linear operators.

[C_2] $D(E) \subset D(A)$ and E is bijective.

[C_3] $E^{-1} : Y \to D(E)$ is continuous.

[C_4] For each $t \in [0, b]$ and for some $\lambda \in \rho(-AE^{-1})$, the resolvent set of $-AE^{-1}$, the resolvent $R(\lambda, -AE^{-1})$ is a compact operator.

Hypotheses [C_1] and [C_2] and the closed graph theorem imply the boundedness of the linear operator $AE^{-1} : Y \to Y$.

LEMMA. *(See [15].) Let $S(t)$ be a uniformly continuous semigroup and let A be its infinitesimal generator. If the resolvent set $R(\lambda : A)$ of A is compact for every $\lambda \in \rho(A)$, then $S(t)$ is a compact semigroup.*

From the above fact, $-AE^{-1}$ generates a compact semigroup $T(t), t \geq 0$ in Y. Thus, $\max_{t\in J}$ *$|T(t)|$ is finite and so denote $M_1 = \max_{t\in J}|T(t)|$.*

DEFINITION 1. *A solution $x \in C([0, b] : X)$ of the integral equation*

$$x(t) = E^{-1}T(t)Ex_0 + \int_0^t E^{-1}T(t-s)\left[(Bu)(s) + f\left(s, x(s), \int_0^s g(s, \tau, x(\tau))\, d\tau\right)\right] ds, \qquad t \in J,$$

is called a mild solution of problem (1).

DEFINITION 2. *In system (1), it is said to be controllable on the interval J if for every $x_0 \in D(E)$, $x_1 \in X$, there exists a control $u \in L^2(J, U)$ such that the mild solution $x(t)$ of (1) satisfies $x(b) = x_1$.*

[C_5] *The linear operator $W : L^2(J, U) \to X$, defined by*

$$Wu = \int_0^b E^{-1}T(b-s)Bu(s)\, ds,$$

has an invertible operator W^{-1} which takes values in $L^2(J, U) \setminus \ker W$ and there exist positive constants M_2, M_3 such that $\|B\| \leq M_2$ and $\|W^{-1}\| \leq M_3$.

[C_6] *For each $t, s \in J \times J$ the function $g(t, s, .) : X \to X$ is continuous and for each $x \in X$ the function $g(., ., x) : J \times J \to X$ is strongly measurable.*

[C_7] *For each $t \in J$ the function $f(t, ., .) : X \times X \to Y$ is continuous and for each $x, y \in X$ the function $f(., x, y) : J \to Y$ is strongly measurable.*

[C_8] *For every positive integer k there exists $h_k \in L^1(0, b)$ such that for a.e. $t \in J$*

$$\left[\sup_{\|x\| \leq k} \left\| f\left(t, x(t), \int_0^t g(t, s, x(s))\, ds\right)\right\| \leq h_k(t)\right].$$

[C_9] *There exists a continuous function* $m : J \times J \to [0, \infty)$ *such that*

$$\|g(t,s,x)\| \le m(t,s)\Omega(\|x\|), \qquad t,s \in J, \quad x \in X,$$

where $\Omega : [0,\infty) \to (0,\infty)$ *is a continuous nondecreasing function.*

[C_{10}] *There exists a continuous function* $p : J \to [0,\infty)$ *such that*

$$|f(t,x,y)| \le p(t)\Omega_0(\|x\| + \|y\|), \qquad t \in J, \quad x,y \in X,$$

where $\Omega_0 : [0,\infty) \to (0,\infty)$ *is a continuous nondecreasing function.*

[C_{11}]

$$\int_0^b \hat{m}(s)\,ds \le \int_c^\infty \frac{ds}{\Omega_0(s) + \Omega(s)},$$

where $c = \|E^{-1}\|M_1[|Ex_0| + Nb]$, $\hat{m}(t) = \max\{M_1\|E^{-1}\|p(t), m(t,t)\}$ and

$$N = M_2M_3\left[\|x_1\| + \|E^{-1}\|\, M_1|Ex_0| + M_1\,\|E^{-1}\|\int_0^b p(s)\Omega_0\left(\|x\| + \int_0^s m(s,\tau)\Omega(\|x\|)\,d\tau\right) ds\right].$$

We need the following fixed-point theorem due to [16].

Schaefer Theorem. *Let S be a convex subset of a normed linear space E and $0 \in S$. Let $F : S \to S$ be a completely continuous operator and let*

$$\zeta(F) = \{x \in S;\ x = \lambda Fx \text{ for some } 0 < \lambda < 1\}.$$

Then either $\zeta(F)$ is unbounded or F has a fixed point.

System (1) has a mild solution of the following form [15]

$$\begin{aligned} x(t) &= E^{-1}T(t)Ex_0 + \int_0^t E^{-1}T(t-s) \\ &\times \left[(Bu)(s) + f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right)\right] ds, \qquad t \in J, \end{aligned} \tag{2}$$

In order to study the controllability problem of (1) we introduce a parameter $\lambda \in (0,1)$ as in [17] and consider the following system

$$\begin{aligned} (Ex(t))' + \lambda Ax(t) &= \lambda(Bu)(t) + \lambda f\left(t, x(t), \int_0^t g(t,s,x(s))\,ds\right), \qquad t \in J, \\ x(0) &= x_0. \end{aligned} \tag{3}$$

Then the mild solution of (3) can be written as

$$x(t) = \lambda E^{-1}T(t)Ex_0 + \lambda\int_0^t E^{-1}T(t-s)\left[(Bu)(s) + f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right)\right] ds, \qquad t \in J.$$

3. MAIN RESULT

Theorem. *If Hypotheses [C_1]–[C_{11}] are satisfied, then the system (1) is controllable on J.*

Proof. Using Hypothesis [C_5] for an arbitrary function $x(.)$ define the control

$$u(t) = W^{-1}\left[x_1 - E^{-1}T(b)Ex_0 - \int_0^b E^{-1}T(b-s)f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right) ds\right](t).$$

We shall now show that when using this control the operator defined by

$$(Fx)(t) = E^{-1}T(t)Ex_0 + \int_0^t E^{-1}T(t-s)\left[(Bu)(s) + f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right)\right] ds,$$

has a fixed point. This fixed point is then a solution of equation (2).

Clearly, $(Fx)(b) = x_1$, which means that the control u steers the system from the initial state x_0 to x_1 in time b, provided we can obtain a fixed point of the nonlinear operator F.

First, we obtain *a priori* bounds for the following equation:

$$\begin{aligned} x(t) =& \lambda E^{-1}T(t)Ex_0 + \lambda \int_0^t E^{-1}T(t-\eta)BW^{-1}\bigg[x_1 - E^{-1}T(b)Ex_0 \\ &- \int_0^b E^{-1}T(b-s)f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right)ds\bigg](\eta)\,d\eta \\ &+ \lambda\int_0^t E^{-1}T(t-s)f\left(s,x(s),\int_0^s g(s,\tau,x(\tau))\,d\tau\right)ds. \end{aligned}$$

We have

$$\begin{aligned} \|x(t)\| \le& \left\|E^{-1}\right\| M_1|Ex_0| + \int_0^t \left\|E^{-1}\right\| M_1M_2M_3\bigg[\|x_1\| + \left\|E^{-1}\right\| M_1|Ex_0| \\ &+ \left\|E^{-1}\right\| M_1\int_0^b p(s)\Omega_0\left(\|x(s)\| + \int_0^s m(s,\tau)\Omega(\|x(\tau)\|)\,d\tau\right)ds\bigg]\,d\eta \\ &+ \left\|E^{-1}\right\| M_1\int_0^t p(s)\Omega_0\left(\|x(s)\| + \int_0^s m(s,\tau)\Omega(\|x(\tau)\|)\,d\tau\right)ds \\ \le& \left\|E^{-1}\right\| M_1|Ex_0| + \int_0^t \left\|E^{-1}\right\| M_1N\,ds \\ &+ \left\|E^{-1}\right\| M_1\int_0^t p(s)\Omega_0\left(\|x(s)\| + \int_0^s m(s,\tau)\Omega(\|x(\tau)\|)\,d\tau\right)ds \\ \le& \left\|E^{-1}\right\| M_1|Ex_0| + \left\|E^{-1}\right\| M_1Nb + \left\|E^{-1}\right\| M_1\int_0^t p(s)\Omega_0\bigg(\|x(s)\| \\ &+ \int_0^s m(s,\tau)\Omega(\|x(\tau)\|)\,d\tau\bigg)\,ds. \end{aligned}$$

Denoting by $v(t)$ the right-hand side of the above inequality we have $c = v(0) = \|E^{-1}\|M_1[|Ex_0| + Nb]$, $\|x(t)\| \le v(t)$, and

$$\begin{aligned} v'(t) &= M_1\left\|E^{-1}\right\| p(t)\Omega_0\left(\|x(t)\| + \int_0^t m(t,\tau)\,\Omega\left(\|x(\tau)\|\right)d\tau\right) \\ &\le M_1\left\|E^{-1}\right\| p(t)\Omega_0\left(v(t) + \int_0^t m(t,\tau)\Omega(v(\tau))\,d\tau\right). \end{aligned}$$

Let

$$w(t) = v(t) + \int_0^t m(t,\tau)\Omega(v(\tau))\,d\tau$$

Then $w(0) = v(0) = c,\ v(t) \le w(t)$, and

$$\begin{aligned} w'(t) &= v'(t) + m(t,t)\Omega(v(t)) \\ &\le M_1\left\|E^{-1}\right\| p(t)\Omega_0(w(t)) + m(t,t)\Omega(w(t)) \\ &\le \hat{m}(t)\left[\Omega_0(w(t)) + \Omega(w(t))\right]. \end{aligned}$$

This implies

$$\int_{w(0)}^{w(t)} \frac{ds}{\Omega_0(s)+\Omega(s)} \le \int_0^b \hat{m}(s)\,ds < \int_c^\infty \frac{ds}{\Omega_0(s)+\Omega(s)}, \qquad t \in J.$$

This inequality implies that there is a constant K such that $w(t) \le K$, $t \in J$, and hence, $\|x(t)\| \le K$, $t \in J$, where K depends only on b and on the functions $\hat{m}, \Omega_0$ and Ω.

Second, we must prove that the operator $F : C = C(J, X) \to C$ defined by

$$(Fx)(t) = E^{-1}T(t)Ex_0 + \int_0^t E^{-1}T(t-\eta)BW^{-1}\Bigg[x_1 - E^{-1}T(b)Ex_0 - \int_0^b E^{-1}T(b-s)f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right) ds\Bigg](\eta)\,d\eta + \int_0^t E^{-1}T(t-s)f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right) ds$$

is a completely continuous operator.

Let $B_k = \{x \in C : \|x\| \le k\}$ for some $k \ge 1$. We first show that F maps B_k into an equicontinuous family. Let $x \in B_k$ and $t_1, t_2 \in J$. Then if $0 < t_1 < t_2 \le b$,

$$\begin{aligned}
\|(Fx)(t_1) - (Fx)(t_2)\| &\le |T(t_1) - T(t_2)|\,\|E^{-1}\|\,|Ex_0| \\
&+ \Bigg\|\int_0^{t_1} [T(t_1-\eta) - T(t_2-\eta)]\,E^{-1}BW^{-1}\Bigg[x_1 - E^{-1}T(b)Ex_0 \\
&- \int_0^b E^{-1}T(b-s)f(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau)\,ds\Bigg](\eta)\,d\eta\Bigg\| \\
&+ \Bigg\|\int_{t_1}^{t_2} T(t_2-\eta)E^{-1}BW^{-1}\Bigg[x_1 - E^{-1}T(b)Ex_0 \\
&- \int_0^b E^{-1}T(b-s)f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right) ds\Bigg](\eta)\,d\eta\Bigg\| \\
&+ \left\|\int_0^{t_1} [T(t_1-s) - T(t_2-s)]\,E^{-1}f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right) ds\right\| \\
&+ \left\|\int_{t_1}^{t_2} T(t_2-s)E^{-1}f\left(s, x(s), \int_0^s g(s,\tau,x(\tau)\right) d\tau)\,ds\right\| \\
&\le |T(t_1) - T(t_2)|\,\|E^{-1}\|\,|Ex_0| \\
&+ \int_0^{t_1} |T(t_1-\eta) - T(t_2-\eta)|\,\|E^{-1}\|\,M_2M_3\Bigg[\|x_1\| + \|E^{-1}\|\,M_1\,|Ex_0| \\
&+ \|E^{-1}\|\,M_1\int_0^b h_k(s)\,ds\Bigg]\,d\eta \\
&+ \int_{t_1}^{t_2} |T(t_2-\eta)|\,\|E^{-1}\|\,M_2M_3\Bigg[\|x_1\| + \|E^{-1}\|\,M_1|Ex_0| \\
&+ \|E^{-1}\|\,M_1\int_0^b h_k(s)\,ds\Bigg]\,d\eta \\
&+ \int_0^{t_1} |T(t_1-s) - T(t_2-s)|\,\|E^{-1}\|\,h_k(s)\,ds \\
&+ \int_{t_1}^{t_2} |T(t_2-s)|\,\|E^{-1}\|\,h_k(s)\,ds.
\end{aligned}$$

The right-hand side tends to zero as $t_2 - t_1 \to 0$, since the compactness of $T(t)$ for $t > 0$ implies the continuity in the uniform operator topology.

Thus, F maps B_k into an equicontinuous family of functions. It is easy to see that the family FB_k is uniformly bounded.

Next we show $\overline{FB_k}$ is compact. Since we have shown FB_k is an equicontinuous collection, it suffices by the Arzela-Ascoli Theorem to show $\{(Fx)(t) : x \in B_k\}$ is precompact in X for any $t \in [0, b]$. Let $0 < t \leq b$ be fixed and ϵ a real number satisfying $0 < \epsilon < t$. For $x \in B_k$ we define

$$
\begin{aligned}
(F_\epsilon x)(t) = E^{-1}T(t)Ex_0 + \int_0^{t-\epsilon} E^{-1}T(t-\eta)BW^{-1} \Bigg[x_1 - E^{-1}T(b)Ex_0 \\
- \int_0^b E^{-1}T(b-s)f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right) ds \Bigg] (\eta)\,d\eta \\
+ \int_0^{t-\epsilon} E^{-1}T(t-s)f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right) ds
\end{aligned}
$$

Since $T(t)$ is a compact operator, the set $Y_\epsilon(t) = \{(F_\epsilon x)(t) : x \in B_k\}$ is precompact in X for every ϵ, $0 < \epsilon < t$. Moreover, for every $x \in B_k$ we have

$$
\begin{aligned}
\|(Fx)(t) - (F_\epsilon x)(t)\| \leq \int_{t-\epsilon}^t \Bigg\| E^{-1}T(t-\eta)BW^{-1} \Bigg[x_1 - E^{-1}T(b)Ex_0 \\
- \int_0^b E^{-1}T(b-s)f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right) ds \Bigg] (\eta) \Bigg\| \,d\eta \\
+ \int_{t-\epsilon}^t \left\| E^{-1}T(t-s)f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right) \right\| ds \\
\leq \int_{t-\epsilon}^t \|E^{-1}\| M_1M_2M_3 \Bigg[\|x_1\| + \|E^{-1}\| M_1 |Ex_0| \\
+ M_1 \|E^{-1}\| \int_0^b h_k(s)\,ds \Bigg] d\eta + \int_{t-\epsilon}^t \|E^{-1}\| M_1 h_k(s)\,ds.
\end{aligned}
$$

Therefore, there are precompact sets arbitrarily close to the set $\{(Fx)(t) : x \in B_k\}$. Hence, the set $\{(Fx)(t) : x \in B_k\}$ is precompact in X.

It remains to show that $F : C \to C$ is continuous. Let $\{x_n\}_0^\infty \subseteq C$ with $x_n \to x$ in C. Then there is an integer r such that $\|x_n(t)\| \leq r$ for all n and $t \in J$, so $x_n \in B_r$ and $x \in B_r$. By [C_7],

$$
f\left(t, x_n(t), \int_0^t g(t,s,x_n(s))\,ds\right) \to f\left(t, x(t), \int_0^t g(t,s,x(s))\,ds\right) \text{ for each } t \in J,
$$

and since

$$
\left| f\left(t, x_n(t), \int_0^t g(t,s,x_n(s))\,ds\right) - f\left(t, x(t), \int_0^t g(t,s,x(s))\,ds\right) \right| \leq 2h_r(t),
$$

we have by dominated convergence theorem,

$$
\begin{aligned}
\|Fx_n - Fx\| = \sup_{t\in J} \left\| \int_0^t E^{-1}T(t-\eta)BW_{-1} \left[\int -0^b T(b-s) \right.\right. \\
\left[f\left(s, x_n(s), \int_0^s g(s,\tau,x_n(\tau))\,d\tau\right) - f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\,d\tau\right) \right] \Bigg] (\eta)\,d\eta \\
+ \int_0^t E^{-1T}(t-s) \left[f\left(s, x_n(s), \int_0^s g(s,\tau,x_n(\tau))\,d\tau\right) \right.
\end{aligned}
$$

$$-f\left(s,x(s),\int_0^s g(s,\tau,x(\tau))\,d\tau\right)\Big]\,ds\Big\|$$

$$\leq \int_0^b \|E^{-1}\|\, M_1M_2M_3\left[M_1\int_0^b \Big\| f\left(s,x_n(s),\int_0^s g(s,\tau,x_n(\tau))\,d\tau\right)\right.$$

$$\left.-f\left(s,x(s),\int_0^s g(s,\tau,x(\tau))\,d\tau\right)\Big\|\right] d\eta$$

$$+\int_0^b \|E^{-1}\|\, M_1 \Big\| f\left(s,x_n(s),\int_0^s g(s,\tau,x_n(\tau))\,d\tau\right)$$

$$-f\left(s,x(s),\int_0^s g(s,\tau,x(\tau))\,d\tau\right)\Big\|\,ds \to 0.$$

Thus, F is continuous. This completes the proof that F is completely continuous.

Finally the set $\zeta(F)=\{x\in C: x=\lambda Fx, \lambda\in(0,1)\}$ is bounded, as we proved in the first step. Consequently, by Schaefer's Theorem the operator F has a fixed point in C. This means that any fixed point of F is a mild solution of (1) on J satisfying $(Fx)(t)=x(t)$. Thus, the system (1) is controllable on J.

4. EXAMPLE

Consider the following partial integrodifferential equation of the form

$$\frac{\partial}{\partial t}\left(z(t,x)-z_{xx}(t,x)\right)-z_{xx}(t,x)=Bu(t)+\mu_1\left(t,z(t,x),\int_0^t \mu_2(t,s,z(s,x))\,ds\right)$$

with $z(t,0)=z(t,1)=0$, $z(0,x)=z_0(x)$, $0<x<1$, $t\in J$. Assume that the following condition hold with $X=Y=L^2(0,1)$.

[A$_1$] The operator $B: U\to Y$, with $U\subset J$, is a bounded linear operator.

[A$_2$] The linear operator $W: L^2(J,U)\to X$, defined by

$$Wu=\int_0^b E^{-1}T(b-s)Bu(s)\,ds$$

has an bounded invertible operator W^{-1} which takes values in $L^2(J,U)\setminus\ker W$.

[A$_3$] Further the functions

$$\mu_2: J\times J\times X\to X$$

$$\mu_1: J\times X\times X\to Y$$

are all continuous, bounded and strongly measurable.

[A$_4$] Let $g(t,s,w)(x)=\mu_2(t,s,w(x))$ and $f(t,w,\sigma)(x)=\mu_1(t,w(x),\sigma(x))$. Define the operators $A: D(A)\subset X\to Y$, $E: D(E)\subset X\to Y$ by

$$Aw=-w'',\qquad Ew=w-w'',$$

respectively, where each domain $D(A)$, $D(E)$ is given by

$$\{w\in X, w, w' \text{ are absolutely continuous, } w''\in X,\ w(0)=w(1)=0\}.$$

Then A and E can be written, respectively, as (see [8])

$$Aw=\sum_{n=1}^{\infty} n^2(w,w_n)w_n,\ w\in D(A),$$

$$Ew = \sum_{n=1}^{\infty} \left(1+n^2\right)(w, w_n)w_n, \; w \in D(E),$$

where $w_n(x) = \sqrt{2}\sin nx$, $n = 1, 2, 3, \dots$, is the orthogonal set of eigenvectors of A. Furthermore for $w \in X$ we have

$$E^{-1}w = \sum_{n=1}^{\infty} \frac{1}{1+n^2}(w, w_n)w_n,$$

$$-AE^{-1}w = \sum_{n=1}^{\infty} \frac{-n^2}{1+n^2}(w, w_n)w_n,$$

$$(t)w = \sum_{n=1}^{\infty} e^{\left(-n^2/1+n^2\right)t}(w, w_n)w_n.$$

It is easy to see that $-AE^{-1}$ generates a strongly continuous semigroup $T(t)$ on Y and $T(t)$ is compact such that $\|T(t)\| \le e^{-t}$ for each $t > 0$.

[A_5] The functions μ_1, and μ_2 satisfy the following conditions.

(i) There exists a continuous function $q : J \times J \to [0, \infty)$ such that

$$\|\mu_2(t, s, w)\| \le q(t, s)\Omega_2(|w|),$$

where $\Omega_2 : [0, \infty) \to (0, \infty)$ is continuous and nondecreasing.

(ii) There exists a continuous function $l : J\times \to [0, \infty)$ such that

$$|\mu_1(t, w, \mu_2)| \le l(t)\Omega_3(|w|),$$

where $\Omega_3 : [0, \infty) \to (0, \infty)$ is continuous and nondecreasing.

Also we have

$$\int_0^b \hat{n}(s)\, ds \le \int_c^{\infty} \frac{ds}{\Omega_2(s) + \Omega_3(s)},$$

where $c = \|E^{-1}\|e^{-t}[|Ez_0| + Nb]$, and $\hat{n}(t) = \max\{e^{-t}\|E^{-1}\|l(t), q(t,t)\}$. Here N depends on E, A, B, μ_1, and μ_2. Further all the conditions stated in the above theorem are satisfied. Hence, the system (4) is controllable on J.

REMARK. Examples in which the operator W in Hypothesis [C_5] has an invertible operator are discussed by Carmichel and Quinn [18].

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