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ORIGINAL ARTICLE

A new design method based on firefly algorithm for IIR system identification problem

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Abstract In this paper a population based evolutionary optimization methodology called firefly algorithm (FFA) is applied for the optimization of system coefficients of the infinite impulse response (IIR) system identification problem. FFA is inspired by the flash pattern and characteristics of fireflies. In FFA technique, behaviour of flashing firefly towards its competent mate is structured. In this algorithm attractiveness depends on brightness of light and a bright firefly feels more attraction for the brighter one. For this optimization problem, brightness varies inversely proportional to the error fitness value, so the position of the brightest firefly gives the optimum result corresponding to the least error fitness in multidimensional search space. Incorporation of different control parameters in basic movement equation results in balancing of exploration and exploitation of search space. The proposed FFA based system identification approach has alleviated from inherent drawbacks of premature convergence and stagnation, unlike genetic algorithm (GA), particle swarm optimization (PSO) and differential evolution (DE). The simulation results obtained for some well known benchmark examples justify the efficacy of the proposed system identification approach using FFA over GA, PSO and DE in terms of convergence speed, identifying plant coefficients and mean square error (MSE) fitness values produced for both same order and reduced order models of adaptive IIR filters.

1. Introduction

Adaptive filter finds a wide scope of applications where the characteristics of the modelled system vary with time. In the non-stationary fields adaptive filter is placed in different areas or application areas such as prediction, system identification and modelling, equalization, interference cancellation, etc. Adaptive filters are important in the disciplines of control systems, communication, signal processing, image processing and speech processing. Complexity of adaptive filter has increased many folds in the environment of multiple input/output,

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variant-time behaviours, usage of long and complex transfer function, but still so many features are required to cope up with the ever challenging environment.

An adaptive filter also behaves like a filter with the exception of iteration based coefficient values due to incorporation of an adaptive algorithm to cope up with the ever changing environmental condition and/or unknown system parameters. The adaptive algorithm varies the filter's characteristics by manipulating or varying the filter coefficient values according to the performance criterion of the system. Error signal is between the output response of the unknown plant and the output response of the designed filter for the same input and adaptive filter works toward the minimization of the error signal with the proper adjustment of the filter coefficients.

Finite impulse response (FIR) and infinite impulse response (IIR) filters are the two types of digital filters. For adaptive IIR digital filter, due to the recursive nature, present output not only depends on the present input but also on the previous inputs and outputs. An IIR filter requires lower order compared to FIR filter to meet the same specifications (Hussain et al., 2011). In the present work adaptive IIR filter is considered for identifying/modelling an unknown plant.

Previously, as a classical approach of adaptive filtering, least mean square (LMS) technique and its variants were used extensively as optimization tools for adaptive filter. The high acceptance of classical optimization technique is due to the low complexity and simplicity of implementation. But the main drawback of the LMS technique is its slow convergence speed to reach the optimal solution. Several measures have been reported to boost up the speed of convergence (Guan et al., 2009; Shengkui et al., 2007).

In adaptive IIR filtering applications, non-differentiability and multimodal nature of the error surface are major points of concern. Classical optimization methods such as the LMS technique and its variants are gradient based optimization methods and are incapable to determine the global optimal solution due to the following inherent deficiencies:

- Requirement of continuous and differentiable cost function;
- Usually converges to the local optimum solution or revisits the same sub-optimal solution;
- Incapable to search the large problem space;
- Requirement of the piecewise linear cost approximation (linear programming);
- Highly sensitive to starting points when the number of solution variables is increased and as a result the solution space is also increased.

Due to the above shortfalls of classical optimization methods, different meta-heuristic search algorithms have successfully been implemented in digital filter optimization problems. Few such evolutionary optimization techniques aptly used are as follows: genetic algorithm (GA) is inspired by the Darwin's "Survival of the Fittest" strategy (Ma and Cowan, 1996); human searching nature is mimicked in seeker optimization algorithm (SOA) (Dai et al., 2010); cat swarm optimization (CSO) is based upon the behaviour of cat's tracing and seeking of an object (Panda et al., 2011); bee colony algorithm (BCA) is based upon honey searching behaviour of the bee swarm (Karaboga and Cetinkaya, 2011; Karaboga, 2009); gravitational search algorithm (GSA) is motivated by the gravitational laws and laws of motion (Rashedi et al., 2011); food searching behaviour is mimicked in bacterial foraging algorithm (Majhi and Panda, 2007). Conventional PSO has mimicked the behaviour of bird flocking or fish schooling (Panda et al., 2007; Chen and Luk, 2010; Krusienski and Jenkins, 2003, 2004; Majhi et al., 2008; Durmus and Gun, 2011; Mandal et al., 2012a, b; Pan and Chang, 2011); In Quantum behaved PSO (QPSO) quantum behaviour of particles in a potential well is adopted in conventional PSO algorithm (Fang et al., 2006; Sun et al., 2010); In PSO with Quantum Infusion (PSO-QI), a hybridized version of PSO and QPSO in which the fast convergence property of PSO and the convergence to a lower average error property obtained by QPSO have been combined to enhance the performance (Luitel and Venayagamoorthy, 2010). In Adaptive Inertia Weight PSO (AIW-PSO), a modified Versoria function is introduced to alter the inertia weight of conventional PSO to improve the convergence speed and optimization efficiency of standard PSO (Yu et al., 2009). To increase the randomness by the process of mutation, a random vector is introduced in basic QPSO for the enhancement of global search ability (Fang et al., 2009). Biological evolutionary strategy is adopted in the development of differential evolution (DE) algorithm (Karaboga, 2005). Along with DE, wavelet mutation (WM) strategy is utilized to develop the DEWM algorithm (Mandal et al., 2012c). Differential cultural (DC) algorithm as a global stochastic search technique tries to find out the global optima of the problem more rapidly (Gao and Diao, 2010). In (Chen, 2000) a new batch recursive adaptive simulated annealing (ASA) algorithm is developed for obtaining a faster convergence rate. Different adaptive filtering algorithms are considered in (Netto et al., 1995) with proper parameter values to overcome the problems of convergence to biased or local minimum solutions and slow convergence speed.

In this paper, the capabilities of finding near global optimal result in multidimensional search space using real coded GA (RGA), PSO, DE and FFA are investigated thoroughly for the identification of the unknown IIR system with the help of optimally designed adaptive IIR filter of the same order and reduced order as well.

GA is a probabilistic heuristic search optimization technique developed by Holland (Holland, 1992). PSO is a swarm intelligence based algorithm developed by Eberhart et al. (Kennedy and Eberhart, 1995; Eberhart and Shi, 1998). Several attempts have been taken towards the system identification problem with basic PSO and its modified versions (Panda et al., 2007; Chen and Luk, 2010; Krusienski and Jenkins, 2003; Krusienski and Jenkins, 2004; Majhi et al., 2008; Durmus and Gun, 2011; Pan and Chang, 2011; Fang et al., 2006, 2009; Sun et al., 2010; Luitel and Venayagamoorthy, 2010; Yu et al., 2009; Schoeman and Engelbrecht, 2010; Berro et al., 2010). The key advantage of PSO is its simplicity in computation and a few numbers of steps are required in the algorithm. DE algorithm was first introduced by Storn and Price in 1995 (Storn and Price, 1995). Like GA, it is a randomized stochastic search technique enriched with the operations of crossover, mutation and selection for finding the optimal solution in multidimensional search space.

It has been realized that GA is incapable for local searching (Karaboga, 2005) in a multidimensional search space and GA, PSO and DE suffer from premature convergence and get easily trapped to suboptimal solution (Karaboga, 2009; Ling et al.,

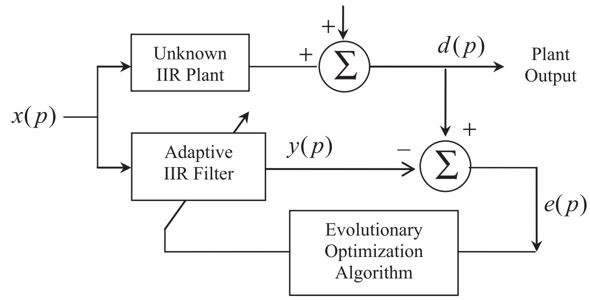

Figure 1 Adaptive IIR filter for system identification.

2008; Biswal et al., 2009). So, to enhance the performance of optimization algorithm in global search (exploration stage) as well as local search (exploitation stage), the authors propose an alternative superior technique called firefly algorithm (FFA) which is based on the characteristics and flashing behaviour of the firefly (Yang, 2009).

In this paper the performances of all the optimization algorithms are analysed with nine benchmark IIR plants and corresponding adaptive IIR filters. Simulation results obtained with the proposed FFA based technique are compared to those of real coded genetic algorithm (RGA), PSO and DE and algorithms of other reported literatures to demonstrate the effectiveness and superior performance of FFA for achieving the global optimal solution in terms of filter coefficients and the mean square error (MSE) in the adaptive system identification problem.

The rest of the paper is organized as follows: In section II, mathematical expression of an adaptive IIR filter and the objective function are formulated. In section III, FFA algorithm is briefly discussed for the adaptive IIR filter design problem. In section IV, comprehensive, demonstrative sets of results and illustrations are given to make a floor of comparative study among different algorithms. Finally, section V concludes the paper.

2. Design formulation

The main task of the system identification in this work is to vary the coefficients of the adaptive IIR filter iteratively using evolutionary algorithms unless and until the filter's output signal is matched to the output signal of unknown system when the same input signal is applied simultaneously to both the adaptive filter and unknown system under consideration. In other way, it can be said that in the system identification, the optimization algorithm searches for the adaptive IIR filter coefficients iteratively such that the filter's input/output relationship matches closely to that of the unknown system. Basic block diagram for system identification using adaptive IIR filter is shown in Fig. 1.

This section discusses the design strategy of adaptive IIR filter. The input–output relation is governed by the following difference Eq. (1):

$$y(p) + \sum_{k=1}^{n} a_k y(p-k) = \sum_{k=0}^{m} b_k x(p-k) \tag{1}$$

where $x(p)$ and $y(p)$ are the filter's input and output, respectively; and $n(\geqslant m)$ is the filter's order. With the assumption of coefficient $a_0 = 1$, the transfer function of the adaptive IIR filter is expressed as given in (2).

$$H(z) = \frac{\sum_{k=0}^{m} b_k z^{-k}}{1 + \sum_{k=1}^{n} a_k z^{-k}} \tag{2}$$

In this design approach the unknown plant of transfer function $H_s(z)$ is to be identified with the adaptive IIR filter $H_{af}(z)$ in such a way so that the outputs from both the systems match closely for the same given input. In this work, the input has been chosen as white noise.

In the system identification problem mean square error (MSE) of time samples is considered as the objective function, also known as error fitness function and is expressed in (3).

$$MSE = \frac{1}{N_s} \sum_{p=1}^{N} e^2(p) \tag{3}$$

In dB the mean square is expressed as MSE(dB)

$$= 10\log_{10}(J) \tag{4}$$

where the error signal is $e(p) = d(p) - y(p)$; $d(p)$ is the response of the unknown plant; $y(p)$ is the response of the adaptive IIR filter and N is the total number of samples. The main objective of any evolutionary algorithm considered in this work is to iteratively minimize the value of the error fitness MSE with proper adjustment of coefficient vector ω of the transfer function of the adaptive filter so that output responses of filter and unknown plant match closely for the same white noise input and hence the error is minimized.

Here $\omega = [a_0 a_1 \ldots a_n b_0 b_1 \ldots b_m]^T$ (5)

Table 1 Control parameters of RGA, PSO, DE and FFA.

Parameters	RGA	PSO	DE	FFA
Population size	120	120	120	120
Iteration cycles	300	300	300	300
Crossover rate	1	–	–	–
Crossover	Two Point Crossover	–	–	–
Mutation rate	0.01	–	–	–
Mutation	Gaussian Mutation	–	–	–
Selection, Selection probability	Roulette, 1/3	–	–	–
C_1, C_2	–	2.05, 2.05	–	–
v_i^{min}, v_i^{max}, w_{max}, w_{min}	–	0.01, 1.0, 1.0, 0.4	–	–
C_r, F	–	–	0.3, 0.5	–
α, γ, β_0	–	–	–	0.01, 0.2, 0.6

Table 2 Optimized coefficients for Example I (Case 1).

Run	RGA		PSO		DE		FFA	
	b_0	b_1	b_0	b_1	b_0	b_1	b_0	b_1
	b_2	b_3	b_2	b_3	b_2	b_3	b_2	b_3
	b_4	b_5	b_4	b_5	b_4	b_5	b_4	b_5
	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
	a_3	a_4	a_3	a_4	a_3	a_4	a_3	a_4
	a_5		a_5		a_5		a_5	
1	0.2256	0.1407	0.0828	0.4793	0.1062	0.4551	0.1089	0.4249
	0.4374	−0.3476	0.5830	−0.1047	0.6929	0.3767	0.5462	0.1273
	−0.0578	0.0533	−0.3976	−0.0713	−0.0312	−0.0489	−0.2393	−0.1455
	0.8299	−0.5977	−0.1250	0.3805	−0.2310	−0.4190	0.0872	−0.3154
	0.4924	−0.2698	−0.0640	0.1657	0.0451	0.0461	0.3576	0.0170
	0.1303		0.0885		−0.0408		0.0326	
2	0.0407	0.5534	0.1123	0.3938	0.1033	0.4286	0.1085	0.4845
	0.0648	0.0107	0.5357	0.2142	0.4924	0.0212	0.8181	0.6130
	−0.4341	−0.0213	−0.1108	0.0309	−0.3301	−0.1173	0.1583	−0.0150
	0.6056	−0.2885	0.1211	−0.4256	0.1599	−0.1188	−0.4547	−0.6528
	0.4413	0.0693	0.2379	−0.1558	0.2592	0.1313	−0.0187	−0.0508
	−0.0743		0.0269		−0.0385		0.0113	
3	0.4028	0.1908	0.0798	0.4557	0.1011	0.4465	0.1087	0.4432
	0.2742	−0.3148	0.4133	0.0604	0.5640	0.1361	0.6280	0.2771
	−0.2642	0.1595	−0.2631	−0.0140	−0.2889	−0.1619	−0.1128	−0.1020
	0.6993	−0.1842	0.2697	−0.2198	0.0095	−0.1985	−0.0754	−0.4265
	0.1830	0.0176	0.1739	0.1088	0.3130	0.0366	0.2420	−0.0070
	−0.0403		−0.1036		0.0505		0.0259	
4	0.1147	0.3990	0.1147	0.4280	0.1081	0.4040	0.1088	0.4423
	0.5351	0.0252	0.6061	0.2885	0.4875	0.2087	0.6223	0.2681
	−0.0329	0.4040	0.0075	0.2293	0.0134	0.1203	−0.1200	−0.1025
	0.2121	−0.1491	0.0254	−0.4524	0.3041	−0.7208	−0.0616	−0.4229
	−0.3846	0.0483	−0.0874	−0.0459	0.2568	−0.2069	0.2505	−0.0092
	−0.1381		−0.0362		0.0204		0.0270	
5	0.0893	0.3111	0.1182	0.3820	0.1086	0.4250	0.1086	0.4236
	0.4776	0.1200	0.5228	0.0796	0.6172	0.1799	0.5380	0.1144
	−0.2187	−0.1194	−0.1743	−0.0471	−0.2680	−0.1601	−0.2490	−0.1464
	0.1047	−0.1836	0.1941	−0.3554	−0.1388	−0.0956	0.1030	−0.3086
	0.3750	−0.1831	0.2587	−0.0181	0.1502	0.1889	0.3671	0.0170
	0.0711		−0.0467		−0.0423		0.0332	

Table 3 MSE values and Run times (in Second) for Example I (Case 1).

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.0356	8.907633	0.0277	4.953836	6.8820e−004	6.661808	5.6061e−006	2.907472
2	0.0507	8.166661	0.0103	4.053498	0.0014	6.529037	1.8737e−006	2.869395
3	0.0991	8.082030	0.0068	4.098620	0.0022	6.485849	5.7630e−006	2.887492
4	0.0307	8.042549	0.0177	4.115985	9.7176e−004	6.699796	4.5938e−006	2.871119
5	0.0556	8.983041	0.0035	4.987767	0.0016	6.491667	4.7569e−006	2.872375

3. Evolutionary algorithm employed

Evolutionary algorithms stand upon the platform of meta-heuristic optimization methods, which are characterized as stochastic, adaptive and learning in order to produce intelligent optimization schemes. Such schemes have the potential to adapt to their ever changing dynamic environment through the previously acquired knowledge. The novel algorithm, FFA is discussed here for the identification of some benchmark IIR systems. The other algorithms RGA, PSO and DE considered in this paper are well known and not discussed here.

3.1. Firefly algorithm (FFA)

Yang (Yang, 2009) has originally developed FFA. FFA is inspired by the flash pattern and characteristics of fireflies. The basic rules for FFA are:

Table 4 Statistical analysis of MSE (dB) values for Example I (Case 1).

MSE statistics	RGA	PSO	DE	FFA
Best	−15.1286	−24.5593	−31.6229	−57.2730
Worst	−10.0393	−15.5752	−26.5758	−52.3935
Mean	−13.0305	−19.8403	−28.9641	−53.7570
Variance	3.1409	9.8419	3.0622	3.2385
Standard Deviation	1.7723	3.1372	1.7499	1.7996

- One firefly may be attracted to other fireflies regardless of their sex;
- For any two flashing fireflies, the less bright one will move towards the brighter one. If there is no brighter one than a particular firefly, it will move randomly;
- The brightness of a firefly is affected or determined by the landscape of the objective function. For a minimization problem, the brightness can simply be inversely proportional to the value of the objective function. In this work, the objective function is the mean square error (MSE) fitness value. So, the firefly with less MSE is brighter than the one with more MSE. So, the second firefly will be attracted towards the first firefly. Again, if the second firefly has less MSE being brighter than the first one with more MSE, then, the first one will be attracted towards the second firefly.

Thus, in the present work, the brightness I of a firefly at a particular location x is chosen as $B(x) = 1/MSE$. However, the attractiveness β is relative; it should be seen in the eyes

Table 5 Optimized coefficients for Example I (Case 2).

Run	RGA		PSO		DE		FFA	
	b'_0	b'_1	b'_0	b'_1	b'_0	b'_1	b'_0	b'_1
	b'_2	b'_3	b'_2	b'_3	b'_2	b'_3	b'_2	b'_3
	b'_4		b'_4		b'_4		b'_4	
	a'_1	a'_2	a'_1	a'_2	a'_1	a'_2	a'_1	a'_2
	a'_3	a'_4	a'_3	a'_4	a'_3	a'_4	a'_3	a'_4
1	0.1815	0.4475	−0.0157	0.3993	0.0760	0.4542	0.1088	0.4975
	0.5830	−0.2736	0.6304	0.4561	0.4860	0.1841	0.8728	0.7005
	−0.5156		0.1018		−0.1390		0.2210	
	−0.0003		0.3300	−0.3673	−0.5760	−0.4788	−0.5685	−0.6908
	0.2626	0.0227	−0.1215	−0.0711	0.2730	−0.0715	−0.0951	−0.0475
2	−0.1502	0.5225	0.0098	0.4728	0.0956	0.5056	0.1083	0.4978
	0.0666	0.3265	0.3972	−0.2509	0.9586	0.8021	0.8764	0.7062
	0.0436		−0.1260		0.2477		0.2234	
	0.3863	−1.0578	0.2984	0.1922	−0.8238	−0.5551	−0.5800	−0.6899
	0.2969	−0.2930	−0.2890	0.2830	−0.2552	0.1098	−0.1020	−0.0455
3	0.5103	0.7060	0.1597	0.2824	0.1044	0.4281	0.1084	0.4981
	0.9617	0.4676	0.5018	−0.0761	0.4942	0.1270	0.8791	0.7100
	0.0149		0.0266		−0.1337		0.2256	
	−0.5030	−0.3230	0.5376	−0.6945	0.1874	−0.3916	−0.5864	−0.6898
	−0.0511	−0.1135	0.3970	−0.1655	0.1795	0.0074	−0.1054	−0.0443
4	0.2050	0.2061	0.1101	0.4649	0.1384	0.5693	0.1083	0.4971
	0.8765	−0.1663	0.6804	0.2190	1.0556	0.9097	0.8735	0.7017
	0.0203		−0.1443		0.2887		0.2209	
	0.4377	−0.5219	−0.2896	0.0224	−0.9989	−0.5853	−0.5733–0.6878	
	0.4131	−0.1788	−0.1999	0.1775	−0.3876	0.0560	−0.0986–0.0448	
5	−0.0379	0.1407	−0.0993	0.5564	0.0996	0.3711	0.1084	0.4973
	0.5280	0.2857	0.0367	−0.0242	0.3010	−0.1258	0.8739	0.7024
	−0.0624		−0.2271		−0.3059		0.2210	
	−0.5968	−0.4457	0.9407	−0.7832	0.6507	−0.4276	−0.5744	−0.6883
	−0.2872	−0.4449	0.7529	−0.2802	0.5468	−0.1234	−0.0991	−0.0453

Table 6 MSE values and Run times (in Second) for Example I (Case 2).

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.1087	7.056505	0.0261	4.896845	0.0027	6.914889	6.6309e−006	3.634302
2	0.1130	7.958515	0.0678	4.903141	0.0028	6.964220	6.9083e−006	3.390380
3	0.1955	7.901888	0.0127	4.877134	0.0039	6.920229	5.5835e−006	3.377936
4	0.1753	7.905592	0.0152	4.768323	0.0037	6.921007	5.7288e−006	3.368403
5	0.2395	7.926118	0.0497	4.771008	0.0028	6.926389	7.0606e−006	3.313850

Table 7 Statistical analysis of MSE (dB) values for Example I (Case 2).

MSE Statistics	RGA	PSO	DE	FFA
Best	−9.6377	−18.9620	−25.6864	−52.5309
Worst	−6.2069	−11.6877	−24.0894	−51.5116
Mean	−7.9929	−15.5403	−25.0301	−51.9705
Variance	1.8156	7.9764	0.4639	0.1787
Standard Deviation	1.3474	2.8242	0.6811	0.4227

of the beholder or judged by the other fireflies. Thus, it will vary with the distance r_{ij} between firefly i and firefly j. For a given medium with a fixed light absorption coefficient γ, the light intensity B varies with the distance r. That is given by

$$B = B_0 e^{-\gamma r^2}, \tag{6}$$

where B_0 is the original light intensity at $r = 0$; r is the Euclidian distance between the fireflies; γ is the absorption coefficient. As a firefly's attractiveness is proportional to the light intensity seen by adjacent fireflies, the attractiveness β of a firefly can be defined by

$$\beta = \beta_0 e^{-\gamma r^2}, \tag{7}$$

where β_0 is the attractiveness at $r = 0$.

The distance between any two fireflies i and j at x_i and x_j, respectively, is the Euclidian distance.

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^{D} (x_{i,k} - x_{j,k})^2}, \tag{8}$$

where $x_{i,k}$ is the kth component solution of the ith firefly (x_i); D is the number of component solutions of each x_i and x_j.

The movement of ith firefly is attracted to another more attractive (brighter) jth firefly and is determined by

$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}^2}(x_j - x_i) + \alpha\left(rand - \frac{1}{2}\right); \ \beta_0 \text{ is} + \text{ve}. \tag{9}$$

where the second term is due to the attraction. If the ith firefly is brighter than the jth firefly, then the jth firefly will move

Table 8 Optimized coefficients for Example II (Case 1).

Run	RGA		PSO		DE		FFA	
	b_0	b_1	b_0	b_1	b_0	b_1	b_0	b_1
	b_2	b_3	b_2	b_3	b_2	b_3	b_2	b_3
	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
	a_3	a_4	a_3	a_4	a_3	a_4	a_3	a_4
1	0.4455	−1.0393	1.1484	−0.5971	1.0033	−0.9680	1.0000	−0.9000
	0.5202	−0.0479	1.1646	−0.5174	0.8020	−0.8528	0.8100	−0.7290
	0.2255	0.3577	−0.2520	−0.5885	0.0222	−0.2159	−0.0400	−0.2775
	0.4146	−0.1457	−0.1095	−0.2076	0.3080	−0.0863	0.2101	−0.1400
2	0.9645	−0.8443	0.8774	−1.0592	1.0093	−0.7694	1.0000	−0.9000
	0.2034	−0.1885	0.8081	−0.4333	0.7358	−0.7281	0.8100	−0.7290
	0.0153	0.3180	0.1108	−0.1911	−0.1661	−0.3141	−0.0400	−0.2775
	0.2691	−0.1529	0.0425	−0.3248	0.2410	−0.0672	0.2101	−0.1400
3	0.7255	−0.4600	0.9982	−1.0219	0.9951	−0.7995	1.0000	−0.9000
	0.0899	0.1883	0.6286	−0.6268	0.8823	−0.6127	0.8100	−0.7290
	−0.5440	0.1317	0.0808	−0.0229	−0.1320	−0.4245	−0.0400	−0.2775
	0.4522	0.1273	0.3270	−0.1685	0.0452	−0.2107	0.2101	−0.1400
4	0.4371	−0.4814	1.0031	−1.0782	0.9878	−0.9980	1.0000	−0.9000
	−0.6779	−0.4024	0.6215	−0.5740	0.8417	−0.7638	0.8100	−0.7290
	−1.6489	−1.4568	0.1131	0.0584	0.0464	−0.2312	−0.0400	−0.2775
	−0.5748	−0.0937	0.3672	−0.1163	0.2310	−0.1556	0.2101	−0.1400
5	1.0300	−1.1999	0.9899	−0.7888	1.0223	−0.7849	1.0000	−0.9000
	−0.2034	0.6163	0.6624	−0.5784	0.8133	−0.6794	0.8100	−0.7290
	0.3786	0.7242	−0.1372	−0.2647	−0.1482	−0.3681	−0.0400	−0.2775
	−0.1522	−0.4506	0.1489	−0.2065	0.1457	−0.1419	0.2101	−0.1400

Table 9 MSE values and Run times (in Second) for Example II (Case 1).

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.5253	10.416564	0.0491	3.152210	0.0072	5.623845	7.5058e−014	2.228058
2	0.2580	10.464885	0.0688	2.963749	0.0055	5.322271	7.5058e−014	2.026183
3	0.3429	10.270444	0.0315	2.950188	0.0058	5.356064	2.8091e−014	2.008325
4	0.4248	10.225402	0.0311	2.962914	0.0026	5.379739	3.4546e−013	2.010614
5	0.3745	10.210907	0.0221	2.914437	0.0037	5.352835	2.2340e−014	1.986317

Table 10 Statistical analysis of MSE (dB) values for Example II (Case 1).

MSE Statistics	RGA	PSO	DE	FFA
Best	−5.8838	−16.5561	−25.8503	−136.5090
Worst	−2.7959	−11.6241	−21.4267	−131.2460
Mean	−4.2623	−14.2717	−23.3114	−131.8260
Variance	1.0449	2.9646	2.4834	17.6385
Standard Deviation	1.0222	1.7218	1.5759	4.1998

towards the ith firefly. In that case, subscripts i and j will be interchanged. The third term is randomized with a control parameter α, which makes the exploration of search space more efficient. Usually, $\beta_0 = 1, \alpha \in [0, 1]$ for most applications.

There are two important limiting cases when absorption coefficient $\gamma \to 0$ and $\gamma \to \infty$. For $\gamma \to 0$, the attractiveness is constant $\beta = \beta_0$ and the length scale $\Gamma = 1/\sqrt{\gamma} \to \infty$. This is equivalent to saying that the light intensity does not decrease in an idealized sky. Thus, a flashing firefly can be seen anywhere in the domain. Thus, a global optimum value can easily be reached.

Again $\gamma \to \infty$ leads to $\Gamma = 0$ and $\beta(r) = \delta(r)$ (the Dirac delta function), which means that the attractiveness is almost zero in the sight of other fireflies or the fireflies are short-sighted. This is equivalent to the case where the fireflies roam in a very foggy region randomly. No other fireflies can be seen, and each firefly roams in a completely random way. Therefore, this corresponds to the completely random search method. By adjusting the parameters γ, α and β_0, the performance of the algorithm can be improved. In this work, γ has been chosen as 0.2, yielding the near-global best solutions as investigated thorough several trial runs.

Steps of FFA are as follows:

Step 1: Initialize the real coded fireflies/particles/vectors (ω) of n_p population, each consisting of a number of numerator and denominator filter coefficients b_k and a_k, respectively; dimension of the search space, D is equal to the number of adaptive filter coefficients of each firefly vector need to be optimized; minimum and maximum values of adaptive filter coefficients, hmin = −2, hmax = 2; number of samples = 256; firefly parameters: $\beta_0 = 0.6$, $\gamma = 0.2$, $\alpha = 0.01$; these parameters are determined to be the best

Table 11 Optimized coefficients for Example II (Case 2).

Run	RGA			PSO			DE			FFA		
	b'_0 a'_1	b'_1 a'_2	b'_2 a'_3	b'_0 a'_1	b'_1 a'_2	b'_2 a'_3	b'_0 a'_1	b'_1 a'_2	b'_2 a'_3	b'_0 a'_1	b'_1 a'_2	b'_2 a'_3
1	0.8556,	−1.1226,	0.4962	1.1440	−0.1232	0.4343	1.0173	−0.0015	0.6369	0.9968	0.0997	0.5836
	0.1514,	0.1956,	0.0682	−0.5901	−0.2116	0.0739	−0.8653	−0.6680	−0.0488	−1.0172	−0.8171	−0.1411
2	0.5134,	−0.8072,	0.3698	0.9141	−0.7485	0.5413	0.9839	0.1320	0.5392	1.0063	0.1058	0.6202
	−0.0185,	0.3342,	0.3084	−0.2517	0.0022	0.2887	−1.0698	−0.8553	−0.1616	−0.9962	−0.8067	−0.1335
3	0.4001,	−0.9281,	0.4600	0.7342	−1.0924	0.3800	0.9998	0.0989	0.5911	0.9971	0.1026	0.5880
	0.0244,	0.3262,	0.6283	−0.0306	0.2553	0.4292	−1.0038	−0.8034	−0.1319	−1.0168	−0.8189	−0.1386
4	0.9843,	−1.2931,	0.4704	0.7652	−0.1753	0.4358	0.9696	0.0735	0.5285	1.0252	0.1195	0.6573
	0.5478,	0.1762,	−0.2570	−0.9601	−0.7569	−0.1072	−1.0263	−0.8098	−0.1348	−0.9762	−0.7885	−0.1183
5	1.2153,	−0.1985,	1.1833	0.9256	−0.3838	0.6838	1.0037	0.0791	0.6295	1.0196	0.1145	0.6431
	−0.4670,	−0.7004,	−0.3556	−0.5465	−0.3809	0.0990	−0.9696	−0.7787	−0.1161	−0.9995	−0.8193	−0.1414

Table 12 MSE values and Run times (in Second) for Example II (Case 2).

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.5023	9.362615	0.2439	2.001095	0.0736	4.183314	0.0033	1.613161
2	0.6957	9.271273	0.1821	1.959428	0.0742	4.272986	0.0039	1.489001
3	0.4426	9.723126	0.2107	1.988298	0.0634	4.183729	0.0033	1.616858
4	0.5476	9.500304	0.1938	1.994808	0.0625	4.152417	0.0035	1.607337
5	0.4707	9.273422	0.1899	1.964135	0.0536	4.136579	0.0035	1.552438

Table 13 Statistical analysis of MSE (dB) values for Example II (Case 2).

MSE Statistics	RGA	PSO	DE	FFA
Best	−3.5399	−7.3969	−12.7084	−24.8149
Worst	−1.5758	−6.1279	−11.2960	−24.0894
Mean	−2.7988	−6.9259	−11.8712	−24.5675
Variance	0.4680	0.2018	0.2728	0.0702
Standard Deviation	0.6841	0.4492	0.5223	0.2650

Table 14 Optimized coefficients for Example III (Case 1).

Run	RGA		PSO		DE		FFA	
	b_1	b_2	b_1	b_2	b_1	b_2	b_1	b_2
	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
1	0.2888	−0.6827	0.0645	−0.4585	0.0435	−0.3871	0.0500	−0.4000
	−1.0373	0.1755	−0.9596	0.0912	−1.1509	0.2679	−1.1310	0.2500
2	−0.0764	−0.4239	0.0131	−0.3583	0.0590	−0.4108	0.0500	−0.4000
	−0.5655	−0.2752	−1.1530	0.2691	−1.1237	0.2436	−1.1310	0.2500
3	−0.0892	−0.4069	0.0982	−0.4637	0.0659	−0.4270	0.0500	−0.4000
	−0.6710	−0.1705	−1.0705	0.1955	−1.1050	0.2277	−1.1310	0.2500
4	−0.0247	−0.2479	0.0134	−0.4012	0.0223	−0.3734	0.0500	−0.4000
	−1.3363	0.4315	−1.0177	0.1454	−1.1340	0.2519	−1.1310	0.2500
5	0.2536	−0.6584	0.0614	−0.4384	0.0659	−0.4270	0.0500	−0.4000
	−0.9675	0.1112	−1.0541	0.1798	−1.1050	0.2277	−1.1310	0.2500

Table 15 MSE values and Run times (in Second) for Example III (Case 1).

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.0529	7.259305	0.0051	1.074006	9.4261e−005	2.194798	5.0315e−011	0.838069
2	0.0733	7.262357	0.0013	1.107948	8.2321e−005	2.177833	3.2338e−011	0.757718
3	0.0472	7.217446	0.0028	1.058614	3.7453e−004	2.181863	1.6311e−011	0.763078
4	0.0139	7.205715	0.0054	1.056336	9.0273e−004	2.220801	1.7411e−011	0.658609
5	0.0464	7.209177	0.0016	1.092462	3.7453e−004	2.191015	2.0466e−011	0.663983

Table 16 Statistical analysis of MSE (dB) values for Example III (Case 1).

MSE Statistics	RGA	PSO	DE	FFA
Best	−18.5699	−28.8606	−40.8449	−107.8750
Worst	−11.349	−22.6761	−30.4444	−102.9830
Mean	−13.8559	−25.5896	−36.0153	−106.0490
Variance	6.0642	6.3817	15.6950	3.4271
Standard Deviation	2.4626	2.5262	3.9617	1.8513

Table 17 Optimized coefficients for Example III (Case 2).

Run	RGA		PSO		DE		FFA	
	b	a	b	a	b	a	b	a
1	−0.4336	−0.7181	−0.3278	−0.8998	−0.3330	−0.8867	−0.3461	−0.8915
2	−0.1634	−0.9449	−0.2948	−0.9096	−0.3242	−0.9031	−0.3057	−0.9090
3	−0.3877	−0.7963	−0.3174	−0.9123	−0.3064	−0.8509	−0.3283	−0.8974
4	−0.4091	−0.8787	−0.3239	−0.9142	−0.3289	−0.8994	−0.3365	−0.9025
5	−0.7182	−0.8092	−0.3153	−0.9038	−0.3104	−0.9098	−0.3013	−0.9082

Table 18 MSE values and Run times (in Second) for Example III (Case 2).

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.4495	6.435756	0.2397	1.050031	0.0681	1.974343	0.0034	0.720564
2	0.3431	6.931287	0.2297	0.969721	0.0794	1.981879	0.0037	0.684716
3	0.3723	6.459677	0.2373	0.959971	0.0955	1.962799	0.0037	0.671865
4	0.2736	6.456051	0.2021	1.008698	0.0623	1.969290	0.0036	0.674621
5	0.6260	6.429260	0.2418	0.951343	0.0439	1.994566	0.0038	0.721414

Table 19 Statistical analysis of MSE (dB) values for Example III (Case 2).

MSE Statistics	RGA	PSO	DE	FFA
Best	−5.6288	−6.9443	−13.5754	−24.6852
Worst	−2.0343	−6.1654	−10.2000	−24.2022
Mean	−4.0145	−6.3897	−11.7002	−24.3921
Variance	1.4592	0.0826	1.2763	0.0270
Standard Deviation	1.2080	0.2874	1.1297	0.1643

after many trials for the determination of near-global best coefficient values; maximum generation cycles = 300.
Step 2: Generate initial firefly vectors of the entire population with appropriate number of adaptive filter coefficients randomly within limits for each vector; Computation of initial error fitness functions (MSE) of the total population, n_p.
Step 3: Computation of the initial population based best solution (h_{gbest}) vector corresponding to the historical population best firefly and least MSE value.

Table 22 Statistical analysis of MSE (dB) values for Example IV (Case 1).

MSE Statistics	RGA	PSO	DE	FFA
Best	−7.3025	−13.8934	−21.9382	−82.9491
Worst	−3.6947	−12.6043	−20.7572	−62.7344
Mean	−5.3754	−13.4570	−21.2907	−70.7260
Variance	1.5230	0.2155	0.1594	54.8293
Standard Deviation	1.2341	0.4642	0.3993	7.4047

Step 4: Computation for the fireflies: consider two fireflies x_i and x_j, then,

(a) Compute square root (rsqrt) of Euclidian distance between the first firefly and the second firefly as per (8).
(b) Compute β with the help of β_0 as per (7).
(c) If MSE of second firefly x_j is < MSE of first firefly x_i, then, the second firefly x_j is brighter, update the first firefly x_i as per (9) with $+\beta_0$ (case of attraction

Table 20 Optimized coefficients for Example IV (Case 1).

Run	RGA		PSO		DE		FFA	
	b_0	b_1	b_0	b_1	b_0	b_1	b_0	b_1
	b_2		b_2		b_2		b_2	
	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
	a_3		a_3		a_3		a_3	
1	−0.0471	−0.1900	−0.1274	−0.6637	−0.1946	−0.5377	−0.1999	−0.4001
	0.1822		−0.2175		0.1560		0.5002	
	−0.2163	−0.1193	−0.5264	−0.2188	−0.0344	−0.3199	0.6001	−0.2497
	−0.4565		−0.0656		−0.0725		0.2000	
2	−0.2862	−0.0699	−0.3173	−0.2746	−0.1810	−0.4074	−0.2000	−0.4000
	0.2352		0.2298		0.4712		0.5001	
	−0.0325	0.1102	0.3573	−0.3622	0.4841	−0.1393	0.6001	−0.2499
	−0.0694		−0.0650		0.0662		0.2000	
3	−0.4886	−0.5664	−0.1274	−0.6637	−0.2000	−0.5083	−0.1996	−0.4011
	−0.3977		−0.2175		0.2875		0.4995	
	0.0645	−0.1555	−0.5264	−0.2188	0.1816	−0.2333	0.5981	−0.2498
	−0.0460		−0.0656		0.0278		0.1989	
4	−0.0832	−0.2618	−0.2383	−0.3220	−0.1971	−0.3862	−0.1998	−0.4005
	0.2736		0.3107		0.4217		0.4999	
	−0.0752	−0.4531	0.2965	−0.1804	0.5844	−0.3930	0.5988	−0.2493
	−0.2080		−0.1633		0.2439		0.1992	
5	0.3577	−0.6459	−0.3814	−0.6817	−0.2014	−0.4853	−0.1996	−0.4005
	0.2523		−0.1458		0.3401		0.4997	
	0.3976	0.1358	−0.4187	−0.3618	0.2515	−0.2123	0.5993	−0.2503
	0.1909		−0.1811		0.0110		0.1999	

Table 21 MSE values and Run times (in Second) for Example IV (Case 1).

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.2472	4.897522	0.0424	1.426705	0.0078	3.327881	2.9073e−008	1.103049
2	0.3033	4.850501	0.0549	1.428853	0.0084	3.360007	5.0709e−009	1.114289
3	0.4271	4.859162	0.0424	1.439448	0.0076	3.295443	5.3279e−007	1.093677
4	0.1861	4.872295	0.0464	1.462885	0.0071	3.374069	2.3890e−007	1.057885
5	0.3442	4.850222	0.0408	1.418772	0.0064	3.377174	2.3102e−007	1.055683

Table 23 Optimized coefficients for Example IV (Case 2).

Run	RGA		PSO		DE		FFA	
	b'_0 a'_1	b'_1 a'_2	b'_0 a'_1	b'_1 a'_2	b'_0 a'_1	b'_1 a'_2	b'_0 a'_1	b'_1 a'_2
1	−0.3826	0.2049	−0.3818	−0.6029	−0.2082	−0.5021	−0.2090	−0.5657
	0.7783	−0.1056	−0.0899	−0.1757	−0.1511	−0.3326	−0.1597	−0.3927
2	0.0804	−0.1459	−0.0970	−0.5176	−0.2131	−0.6077	−0.2161	−0.5710
	−1.1377	−0.6174	−0.1073	−0.2041	−0.1845	−0.3889	−0.1582	−0.3911
3	−0.4050	−1.0205	−0.1387	−0.5367	−0.2379	−0.5751	−0.2067	−0.5680
	−0.2940	−0.0713	−0.4544	−0.3710	−0.0967	−0.2676	−0.1717	−0.3928
4	0.0673	−0.1082	−0.1615	−0.5649	−0.2117	−0.5627	−0.2201	−0.5809
	−0.6519	−0.1326	−0.4358	−0.3394	−0.1595	−0.3961	−0.1587	−0.3819
5	−0.3296	0.1456	−0.1270	−0.5104	−0.2071	−0.5682	−0.2187	−0.5771
	1.0482	−0.6442	−0.2241	−0.3377	−0.1661	−0.3931	−0.1617	−0.3873

Table 24 MSE values and Run times (in Second) for Example IV (Case 2).

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.3336	6.404650	0.0747	1.778472	0.0075	4.324747	9.9991e−004	1.495513
2	0.3289	6.438330	0.0527	1.811410	0.0050	4.330562	9.3783e−004	1.421581
3	0.2416	6.472368	0.0490	1.812248	0.0071	4.287799	9.2150e−004	1.419128
4	0.3274	6.425385	0.0430	1.808315	0.0053	4.295133	8.0205e−004	1.502671
5	0.2273	6.415799	0.0322	1.808293	0.0051	4.397553	9.3588e−004	1.512523

Table 25 Statistical analysis of MSE (dB) values for Example IV (Case 2).

MSE Statistics	RGA	PSO	DE	FFA
Best	−6.4340	−14.9214	−23.0103	−30.9580
Worst	−4.7677	−11.2668	−21.2494	−30.0004
Mean	−5.4099	−13.1467	−22.2857	−30.3760
Variance	0.5378	1.4176	0.5733	0.0995
Standard deviation	0.7333	1.1906	0.7572	0.3154

for x_i firefly towards x_j firefly). On the other hand, if MSE of second firefly x_j > MSE of first firefly x_i, then update the second firefly x_j as per the following equation $x_j = x_j + \beta_0 e^{-\gamma r_{ij}^2}(x_i - x_j) + \alpha(rand - \frac{1}{2})$ (case of attraction for x_j firefly towards x_i firefly).

(d) Repeat steps (a), (b) and (c) for the whole population and then repeat from Step 3 till the maximum iteration cycles or the near-global convergence of minimum MSE values; finally, h_{gbest} is the vector having same or reduced number of coefficients of the adaptive IIR filter.

4. Simulation results and discussions

Extensive MATLAB simulation studies have been performed for the performance comparison of four algorithms RGA, PSO, DE, and the proposed FFA for unknown system identification problem. Nine different benchmark plants which are already reported in different literatures have been considered in this paper.

For each plant under consideration, different cases are studied, one with the same filter order and the other with a reduced filter order. For all cases b s and a s are considered as numerator and denominator coefficients, respectively, for the same and reduced order models. In each case, independent fifty runs each of 300 iteration cycles are performed using all four algorithms for analysing the consistency and usefulness of the results obtained. The results for the best five runs are reported in this work. The values of the control parameters used for the algorithms are given in Table 1. All optimization programs are run in MATLAB 7.5 version on core (TM) 2 duo processor, 3.00 GHz with 2 GB RAM.

4.1. Example I

In this example, a fifth order IIR plant is considered and is taken from (Panda et al., 2011; Krusienski and Jenkins, 2004). The transfer function is shown in (10).

$$H_s(z) = \frac{0.1084 + 0.5419z^{-1} + 1.0837z^{-2} + 1.0837z^{-3} + 0.5419z^{-4} + 0.1084z^{-5}}{1 + 0.9853z^{-1} + 0.9738z^{-2} + 0.3864z^{-3} + 0.1112z^{-4} + 0.0113z^{-5}} \quad (10)$$

Table 26 Optimized coefficients for Example V (Case 1).

Run	RGA		PSO		DE		FFA	
	b_0	b_2	b_0	b_2	b_0	b_2	b_0	b_2
	b_4	b_6	b_4	b_6	b_4	b_6	b_4	b_6
	a_2	a_4	a_2	a_4	a_2	a_4	a_2	a_4
	a_6		a_6		a_6		a_6	
1	0.9310	−0.1148	0.9308	−0.4207	0.9155	−0.2397	1.0000	−0.4002
	0.1204	−0.1874	−0.1991	0.1162	−0.1617	−0.0977	−0.6500	0.2601
	0.6074	0.0078	0.7917	0.4188	0.5922	0.4339	0.7702	0.8497
	−0.2221		0.3106		0.0767		−0.6488	
2	0.6139	−0.1314	0.8717	−0.3294	0.9358	−0.1877	1.0000	−0.4001
	0.5322	−0.3720	−0.0215	−0.0841	−0.6138	0.1874	−0.6500	0.2600
	0.4983	−0.1153	0.6586	0.2894	0.5630	0.8824	0.7700	0.8498
	−0.4428		0.0249		0.4936		−0.6486	
3	0.9935	0.1947	0.9877	0.1551	1.0000	−0.3383	1.0000	−0.4002
	0.1964	−0.1032	0.0655	−0.2683	−0.2830	0.0431	−0.6500	0.2600
	−0.0516	0.4113	0.1764	0.3803	0.6812	0.5070	0.7701	0.8497
	−0.4227		−0.2953		0.2648		−0.6487	
4	0.5028	0.2695	0.9162	0.1397	0.9419	0.4739	0.9999	−0.3978
	0.0443	−0.2906	0.1329	−0.2267	−0.0934	−0.1772	−0.6493	0.2592
	0.1570	0.3649	0.2003	0.4068	−0.1054	0.6380	0.7678	0.8499
	−0.3789		−0.2201		−0.2876		−0.6468	
5	0.6399	0.1936	0.9842	−0.7823	0.9667	0.6263	1.0000	−0.3988
	−0.1298	−0.0238	−0.9822	0.3705	−0.0208	−0.2205	−0.6496	0.2597
	0.1647	0.6216	0.9985	0.9399	−0.2671	0.6424	0.7688	0.8499
	−0.0082		0.9025		−0.4070		−0.6477	

Table 27 MSE values and Run times (in Second) for Example V (Case 1).

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.1393	8.390696	0.0286	2.401580	0.0237	5.581869	2.8343e−009	1.997802
2	0.2406	8.280219	0.0537	2.296299	0.0048	5.523165	8.2324e−010	1.953687
3	0.1024	8.131639	0.0420	2.243625	0.0190	5.522960	1.2867e−009	2.005883
4	0.2872	8.150794	0.0642	2.212568	0.0088	5.497758	1.2840e−008	1.978769
5	0.1503	8.123629	0.0619	2.245647	0.0111	5.478768	1.1256e−008	1.958286

Table 28 Statistical analysis of MSE (dB) values for Example V (Case 1).

MSE Statistics	RGA	PSO	DE	FFA
Best	−9.8970	−15.4363	−23.1876	−90.8447
Worst	−5.4182	−11.9246	−16.2525	−78.9143
Mean	−7.6586	−13.1824	−19.3509	−84.7252
Variance	2.6672	1.6891	6.0763	23.3396
Standard Deviation	1.6331	1.2997	2.4650	4.8311

Case 1. This fifth order plant $H_s(z)$ can be modelled using fifth order IIR filter $H_{af}(z)$.Hence the transfer function of the adaptive IIR filter model is assumed as (11).

$$H_{af}(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5}}{1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3} - a_4 z^{-4} - a_5 z^{-5}} \quad (11)$$

Case 2. In this case a higher order plant is modelled by a reduced order filter. With this nexus a fifth order plant as in (10) is modelled by a fourth order IIR filter given in (12).

$$H_{af}(z) = \frac{b'_0 + b'_1 z^{-1} + b'_2 z^{-2} + b'_3 z^{-3} + b'_4 z^{-4}}{1 - a'_1 z^{-1} - a'_2 z^{-2} - a'_3 z^{-3} - a'_4 z^{-4}} \quad (12)$$

4.2. Example II

In this example, a fourth order IIR plant is considered from (Panda et al., 2011; Majhi et al., 2008) and the transfer function is shown in (13).

$$H_s(z) = \frac{1 - 0.9z^{-1} + 0.81z^{-2} - 0.729z^{-3}}{1 + 0.04z^{-1} + 0.2775z^{-2} - 0.2101z^{-3} + 0.14z^{-4}} \quad (13)$$

Table 29 Optimized coefficients for Example V (Case 2).

Run	RGA		PSO		DE		FFA	
	b'_0	b'_1	b'_0	b'_1	b'_0	b'_1	b'_0	b'_1
	b'_2	b'_3	b'_2	b'_3	b'_2	b'_3	b'_2	b'_3
	b'_4	b'_5	b'_4	b'_5	b'_4	b'_5	b'_4	b'_5
	a'_1	a'_2	a'_1	a'_2	a'_1	a'_2	a'_1	a'_2
	a'_3	a'_4	a'_3	a'_4	a'_3	a'_4	a'_3	a'_4
	a'_5		a'_5		a'_5		a'_5	
1	0.9601	−0.5228	0.8256	−0.1669	0.9489	−0.3308	0.9967	−0.1898
	0.4939	−0.4814	−0.3022	−0.1525	0.1527	−0.2845	0.3341	−0.0788
	0.0485	0.2293	0.3542	0.2980	−0.1127	0.2054	−0.3836	0.0645
	−0.5557	0.4872	−0.0694	−0.5724	−0.2701	−0.2116	−0.2005	−0.0115
	−0.6541	−0.1409	−0.1381	−0.2700	−0.0902	−0.6341	−0.0021	−0.8539
	−0.0050		0.2340		0.2900		0.1724	
2	1.1007	−0.0739	0.8479	−0.0830	0.8884	−0.2491	1.0031	−0.0416
	0.6934	0.1519	−0.2502	−0.0361	0.2485	0.0545	0.3465	−0.0181
	0.1998	0.0409	0.0351	0.2926	−0.0386	−0.1347	−0.4075	0.0124
	0.0694	0.2199	0.0340	−0.6859	−0.2275	−0.1387	−0.0450	0.0002
	0.3775	−0.2902	−0.2400	−0.2091	0.1628	−0.6724	0.0010	−0.8661
	0.0910		0.2590		−0.0134		0.0381	
3	0.5935	−0.1381	1.2239	−0.6695	1.0407	0.4450	0.9967	−0.1898
	0.1643	−0.3046	−0.8226	0.5371	−0.6995	−0.3966	0.3341	−0.0788
	0.3040	0.1382	0.2998	−0.0096	0.1895	0.1798	−0.3836	0.0645
	−0.1755	−0.5417	−0.4348	−0.9576	0.3570	−0.9780	−0.2005	−0.0115
	−0.1124	−0.2760	0.6025	0.0959	−0.3663	0.0540	−0.0021	−0.8539
	0.2381		−0.1470		0.0430		0.1724	
4	0.9659	−0.7096	0.9252	−0.5404	0.9258	0.2240	1.0031	−0.0416
	0.6918	−0.5039	0.3061	−0.6424	−0.0739	−0.0223	0.3465	−0.0181
	0.5122	−0.4622	0.1333	0.2117	−0.0487	0.0677	−0.4075	0.0124
	−0.7352	0.2400	−0.4254	−0.1841	0.2895	−0.4072	−0.0450	0.0002
	−0.1393	−0.1788	−0.3832	−0.3898	−0.1292	−0.4759	0.0010	−0.8661
	−0.1191		0.4401		−0.1185		0.0381	
5	0.9690	0.0424	0.8390	−0.2955	0.9835	0.4533	0.9982	0.2043
	0.2493	0.4077	−0.2772	0.0731	0.3121	−0.0977	0.2972	0.0619
	0.2574	0.2316	0.3500	−0.2949	−0.3410	0.0277	−0.3589	−0.0712
	0.1030	−0.0419	−0.6174	−0.4508	0.4791	−0.0880	0.2226	−0.0417
	0.4907	−0.0465	0.5137	−0.3982	−0.2768	−0.7762	−0.0063	−0.8242
	0.1998		0.0105		−0.1463		−0.1893	

Table 30 MSE values and Run times (in Second) for Example V (Case2).

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.5104	17.036003	0.2238	7.199082	0.0526	11.300139	0.0002	5.943288
2	0.5304	16.951918	0.1915	7.245923	0.0758	11.613527	0.0003	5.987810
3	0.4682	17.017063	0.2636	7.209689	0.0656	11.402017	0.0002	6.085984
4	0.5185	16.597373	0.2325	7.107593	0.0358	11.468651	0.0003	5.999833
5	0.3520	16.949727	0.2168	7.119361	0.0336	11.158695	0.0004	6.041347

Table 31 Statistical analysis of MSE (dB) for Example V (Case2).

MSE Statistics	RGA	PSO	DE	FFA
Best	−4.5346	−7.1783	−14.7366	−36.9897
Worst	−2.7540	−5.7906	−11.2033	−33.9794
Mean	−3.2715	−6.4891	−13.0044	−35.6833
Variance	0.4325	0.2018	1.9579	1.3460
Standard Deviation	0.6576	0.4493	1.3993	1.1602

Table 32 Optimized coefficients for Example VI, same order.

Run	RGA			PSO			DE			FFA		
	b_0	a_1	a_2	b_0	a_1	a_2	b_0	a_1	a_2	b_0	a_1	a_2
1	0.8352	−1.2693	0.6608	1.0419	−1.1854	0.5867	1.0047	−1.1965	0.5968	1.0000	−1.2000	0.6000
2	1.1063	−1.0481	0.4692	1.0207	−1.1825	0.5846	1.0036	−1.1968	0.5966	1.0000	−1.2000	0.6000
3	1.1450	−1.1468	0.5554	1.0288	−1.1748	0.5767	0.9968	−1.2024	0.6022	1.0000	−1.2000	0.6000
4	1.0794	−1.1313	0.5321	1.0248	−1.1781	0.5774	1.0051	−1.1972	0.5973	1.0000	−1.2000	0.6000
5	1.1530	−1.1283	0.5253	1.0194	−1.1827	0.5814	1.0021	−1.1982	0.5980	1.0000	−1.2000	0.6000

Table 33 MSE values and Run times (in Second) for Example VI, same order.

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.0657	9.410385	0.0038	1.238679	8.1044e−005	2.733340	5.9934e−022	1.002477
2	0.0935	9.731646	0.0023	1.289035	8.5757e−005	2.687126	6.2086e−023	1.068678
3	0.0317	9.595427	0.0038	1.240124	5.0934e−005	2.702923	6.9964e−023	1.049990
4	0.0325	9.498112	0.0031	1.239264	7.2212e−005	2.711819	1.6860e−022	1.083509
5	0.0600	9.332241	0.0023	1.255030	3.3328e−005	2.728694	9.1208e−023	1.072146

Table 34 Statistical analysis of MSE (dB) for Example VI, same order.

MSE Statistics	RGA	PSO	DE	FFA
Best	−14.9894	−26.3827	−44.7719	−222.0700
Worst	−10.2919	−24.2022	−40.6673	−212.2230
Mean	−12.8411	−25.2512	−42.1392	−218.7950
Variance	3.3394	0.9578	2.3506	13.0434
Standard Deviation	1.8274	0.9787	1.5332	3.6116

Table 35 Optimized coefficients for Example VII, reduced order.

Run	RGA		PSO		DE		FFA	
	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
1	−1.2884	0.4193	−1.3851	0.5179	−1.3834	0.5143	−1.3838	0.5135
2	−1.2407	0.3739	−1.3817	0.5149	−1.3828	0.5146	−1.3829	0.5125
3	−1.1783	0.3155	−1.3858	0.5188	−1.3835	0.5156	−1.3830	0.5152
4	−1.0723	0.2068	−1.3837	0.5146	−1.3832	0.5138	−1.3838	0.5135
5	−1.3569	0.4899	−1.3820	0.5166	−1.3682	0.5000	−1.3814	0.5149

Table 36 MSE values and Run times (in Second) for Example VII, reduced order.

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.1855	8.472056	0.0455	0.884828	0.0042	2.916377	4.0793e−005	0.663111
2	0.4616	8.526644	0.0333	0.930829	0.0039	2.995895	3.6976e−005	0.674374
3	0.2039	8.512866	0.0334	0.887783	0.0042	2.938087	3.7169e−005	0.655201
4	0.4028	8.347573	0.0433	0.894658	0.0032	2.947194	4.0793e−005	0.675324
5	0.2086	8.526392	0.0422	0.900955	0.0038	2.938690	4.9272e−005	0.659644

Table 37 Statistical analysis of MSE (dB) values for Example VII, reduced order.

MSE Statistics	RGA	PSO	DE	FFA
Best	−7.3166	−14.7756	−24.9485	−44.3208
Worst	−3.3573	−13.4199	−23.7675	−43.0740
Mean	−5.6671	−14.0680	−24.1550	−43.8963
Variance	2.7681	0.3387	0.1873	0.2036
Standard Deviation	1.6638	0.5820	0.4328	0.4512

Table 38 Optimized coefficients for Example VIII, same order.

Run	RGA		PSO		DE		FFA	
	b_1 a_1	b_2 a_2	b_1 a_1	b_2 a_2	b_1 a_1	b_2 a_2	b_1 a_1	b_2 a_2
1	1.1386	0.1957	1.2622	−0.4700	1.2562	−0.2994	1.2500	−0.2500
	−0.0067	0.3867	−0.4292	0.3965	−0.3346	0.4004	−0.3000	0.4000
2	1.2505	0.3092	1.2504	−0.1030	1.2221	−0.2321	1.2500	−0.2500
	0.0627	0.3406	−0.1969	0.3919	−0.2944	0.4063	−0.3000	0.4000
3	1.2901	0.4619	1.2538	−0.1694	1.2416	−0.2432	1.2500	−0.2500
	0.1450	0.2996	−0.2495	0.3979	−0.3000	0.4122	−0.3000	0.4000
4	1.2800	0.4512	1.2593	−0.0322	1.2781	−0.2464	1.2500	−0.2500
	0.1344	0.2890	−0.1535	0.3856	−0.2940	0.3931	−0.3000	0.4000
5	1.2907	0.5482	1.2622	−0.0782	1.2425	−0.2637	1.2500	−0.2500
	0.2278	0.2041	−0.1771	0.3841	−0.3111	0.4088	−0.3000	0.4000

Table 39 MSE values and Run times (in Second) for Example VIII, same order.

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.0450	6.756453	0.0080	0.979533	4.2797e−004	2.559533	3.6491e−019	0.693990
2	0.0516	6.726393	0.0045	0.982021	8.4319e−004	2.562146	5.4619e−018	0.710570
3	0.0763	6.734847	0.0012	1.004172	3.9576e−004	2.773202	4.6631e−019	0.695857
4	0.0744	6.782670	0.0079	0.995696	7.0471e−004	2.549921	3.3652e−018	0.698573
5	0.0857	6.727623	0.0052	1.001587	2.6918e−004	2.555368	4.4222e−018	0.689753

Table 40 Statistical analysis of MSE (dB) values for Example VIII, same order.

MSE Statistics	RGA	PSO	DE	FFA
Best	−13.4679	−29.2082	−35.6996	−184.3780
Worst	−10.6702	−20.9691	−30.7407	−172.6270
Mean	−11.8941	−23.5018	−33.1344	−177.7180
Variance	1.1647	9.1115	3.2030	25.5883
Standard Deviation	1.0792	3.0185	1.7897	5.0585

Case 1. This fourth order plant $H_s(z)$ can be modelled by using a fourth order IIR filter $H_{af}(z)$.Hence the transfer function of the adaptive IIR filter model is assumed as (14).

$$H_{af}(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3} - a_4 z^{-4}} \quad (14)$$

Case 2. In this case the fourth order plant as in (13) is modelled by a third order IIR filter presented in (15).

$$H_{af}(z) = \frac{b'_0 + b'_1 z^{-1} + b'_2 z^{-2}}{1 - a'_1 z^{-1} - a'_2 z^{-2} - a'_3 z^{-3}} \quad (15)$$

4.3. Example III

In this example, a second order IIR plant is considered and is taken from (Dai et al., 2010; Panda et al., 2011; Karaboga, 2005, 2009; Rashedi et al., 2011; Chen and Luk, 2010; Majhi et al., 2008; Durmus and Gun, 2011; Fang et al., 2006, 2009). The transfer function is shown in (16).

Table 41 Optimized coefficients for Example IX (Case 1).

Run	RGA		PSO		DE		FFA	
	b_0	b_1	b_0	b_1	b_0	b_1	b_0	b_1
	b_2		b_2		b_2		b_2	
	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
	a_3		a_3		a_3		a_3	
1	−0.4997	0.2674	−0.2710	0.2062	−0.2830	0.3661	−0.2998	0.3994
	−0.2969		−0.2721		−0.4683		−0.4994	
	0.5833	0.2049	0.9890	0.0908	1.1291	−0.3135	1.2002	−0.4999
	−0.0036		−0.2556		−0.0174		0.0999	
2	−0.4646	−0.2778	−0.3493	0.3568	−0.2680	0.3139	−0.3003	0.4004
	0.0476		−0.5155		−0.4277		−0.5002	
	−0.3786	0.6997	0.8477−	0.1698	1.1267	−0.3171	1.1998	−0.4994
	0.4009		0.0888		−0.0063		0.0997	
3	−0.1010−	0.1050	−0.2683	0.2123	−0.2612	0.3437	−0.3012	0.4017
	−0.0780		−0.4383		−0.4699		−0.5008	
	1.0792	0.0150	0.7581	0.0093	1.3106	−0.7430	1.2000	−0.5001
	−0.2497		0.0032		0.2451		0.1001	
4	−0.3753−	0.1301	−0.2861	0.2902	−0.3325	0.4388	−0.3000	0.4000
	−0.2632		−0.4866		−0.5287		−0.5002	
	−0.0093	0.2352	0.9197	−0.3881	1.0930	−0.3416	1.2002	−0.5012
	0.4266		0.2430		0.0380		0.1009	
5	0.0202−	0.0720	−0.2938	0.3465	−0.2904	0.3898	−0.3004	0.4007
	−0.4208		−0.4609		−0.5024		−0.5011	
	0.2354	0.4981	0.9737	−0.0461	1.2697	−0.6848	1.1981	−0.4981
	0.0177		−0.1377		0.2195		0.0996	

Table 42 MSE values and Run times (in Second) for Example IX (Case 1).

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.1162	12.569719	0.0330	5.142259	0.0037	8.224979	3.1366e−007	4.681101
2	0.2257	12.235222	0.0234	5.073310	0.0047	8.231835	1.1226e−007	4.678935
3	0.1168	12.264891	0.0422	5.122462	0.0071	8.033279	1.7477e−006	4.709319
4	0.1385	12.154483	0.0248	5.195176	0.0038	8.287764	2.6156e−007	4.845975
5	0.2469	12.670946	0.0109	5.220253	0.0034	8.187429	1.1067e−006	4.649065

Table 43 Statistical analysis of MSE (dB) values for Example IX (Case 1).

MSE Statistics	RGA	PSO	DE	FFA
Best	−9.3479	−19.6257	−24.6852	−69.4977
Worst	−6.0748	−13.7469	−21.4874	−57.5753
Mean	−7.9597	−16.1102	−23.5944	−63.4985
Variance	1.9945	3.9329	1.3243	18.8721
Standard Deviation	1.4123	1.9831	1.1508	4.3442

$$H_s(z) = \frac{0.05 - 0.4z^{-1}}{1 - 1.131z^{-1} + 0.25z^{-2}} \quad (16)$$

Case 1. This second order plant $H_s(z)$ can be modelled using the second order IIR filter $H_{af}(z)$.Hence the transfer function of the adaptive IIR filter model is assumed as in (17).

$$H_{af}(z) = \frac{b_1 + b_2 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (17)$$

Case 2. In this case a higher order plant is modelled by a reduced order filter. For the situation under consideration a second order plant as in (16) is modelled by a first order IIR filter given in (18).

$$H_{af}(z) = \frac{b}{1 + az^{-1}} \quad (18)$$

4.4. Example IV

In this example, a third order IIR plant is considered from (Panda et al., 2011; Luitel and Venayagamoorthy, 2010; Fang et al., 2009) and the transfer function is given in (19).

Table 44 Optimized coefficients for Example IX (Case 2).

Run	RGA		PSO		DE		FFA	
	b'_0 a'_1	b'_1 a'_2	b'_0 a'_1	b'_1 a'_2	b'_0 a'_1	b'_1 a'_2	b'_0 a'_1	b'_1 a'_2
1	−0.2516	−0.0352	−0.3785	0.0287	−0.3958	−0.0609	−0.3906	−0.0813
	1.0023	−0.1344	0.5228	0.3178	0.2322	0.5599	0.2236	0.5718
2	−0.2439	−0.0136	−0.3942	−0.0457	−0.3958	−0.0420	−0.4067	−0.0749
	−0.7190	−0.3431	0.2289	0.5675	0.3321	0.4728	0.2066	0.5762
3	−0.3996	−0.0575	−0.4346	−0.0945	−0.3959	−0.1141	−0.3683	−0.0764
	0.2748	0.5313	0.2312	0.5528	0.1750	0.6194	0.2084	0.5946
4	−0.3254	−0.1106	−0.4025	−0.0858	−0.4012	−0.1041	−0.3906	−0.0813
	0.2312	0.5666	0.2236	0.5573	0.1983	0.5796	0.2236	0.5718
5	−0.3797	−0.0305	−0.4204	0.0456	−0.3955	−0.1663	−0.1609	0.0024
	0.2975	0.5239	0.4335	0.3923	0.0634	0.6824	1.5530	−0.6373

Table 45 MSE values and Run times (in Second) for Example IX (Case 2).

Run	RGA		PSO		DE		FFA	
	MSE	Run time	MSE	Run time	MSE	Run time	MSE	Run time
1	0.1625	11.307345	0.0164	4.793721	0.0168	7.805651	6.9238e−004	4.280845
2	0.4407	11.105872	0.0165	4.745398	0.0080	7.354091	7.2863e−004	4.301533
3	0.1624	11.038437	0.0312	4.699015	0.0207	7.752912	7.6944e−004	4.205301
4	0.1715	11.144016	0.0265	4.917291	0.0197	7.553740	6.9238e−004	4.283310
5	0.2022	11.101674	0.0234	4.702526	0.0171	7.712270	7.2005e−004	4.270882

Table 46 Statistical analysis of MSE (dB) values for Example IX (Case 2).

MSE Statistics	RGA	PSO	DE	FFA
Best	−7.8941	−17.8516	−20.9691	−31.5966
Worst	−3.5586	−15.0585	−16.8403	−31.1383
Mean	−6.7887	−16.5621	−18.0563	−31.4265
Variance	2.7299	1.2430	2.2420	0.0287
Standard Deviation	1.6523	1.1149	1.4973	0.1695

$$H_s(z) = \frac{-0.2 - 0.4z^{-1} + 0.5z^{-2}}{1 - 0.6z^{-1} + 0.25z^{-2} - 0.2z^{-3}} \quad (19)$$

Case 1. This third order plant $H_s(z)$ can be modelled using the third order IIR filter $H_{af}(z)$.Hence the transfer function of the model is assumed as (20).

$$H_{af}(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3}} \quad (20)$$

Case 2. In this case a higher order plant is modelled by a reduced order filter. For the situation under consideration a third order plant as in (19) is modelled by a second order IIR filter presented in (21).

$$H_{af}(z) = \frac{b'_0 + b'_1 z^{-1}}{1 - a'_1 z^{-1} - a'_2 z^{-2}} \quad (21)$$

4.5. Example V

In this example, a sixth order IIR plant is considered from (Karaboga, 2009; Luitel and Venayagamoorthy, 2010) and the transfer function is shown in (22).

$$H_s(z) = \frac{1 - 0.4z^{-2} - 0.65z^{-4} + 0.26z^{-6}}{1 - 0.77z^{-2} - 0.8498z^{-4} + 0.6486z^{-6}} \quad (22)$$

Case 1. This sixth order plant $H_s(z)$ can be modelled using sixth order IIR filter $H_{af}(z)$.Hence the transfer function of the adaptive IIR filter model is assumed as (23).

$$H_{af}(z) = \frac{b_0 + b_2 z^{-2} + b_4 z^{-4} + b_6 z^{-6}}{1 - a_2 z^{-2} - a_4 z^{-4} + a_6 z^{-6}} \quad (23)$$

Case 2. In this case the sixth order plant as in (22) is modelled by a fifth order IIR filter presented in (24).

$$H_{af}(z) = \frac{b'_0 + b'_1 z^{-1} + b'_2 z^{-2} + b'_3 z^{-3} + b'_4 z^{-4} + b'_5 z^{-5}}{1 + a'_1 z^{-1} - a'_2 z^{-2} + a'_3 z^{-3} - a'_4 z^{-4} + a'_5 z^{-5}} \quad (24)$$

4.6. Example VI

In this example, a second order IIR plant is considered from (Karaboga, 2009; Rashedi et al., 2011; Durmus and Gun, 2011) and the transfer function is shown in (25).

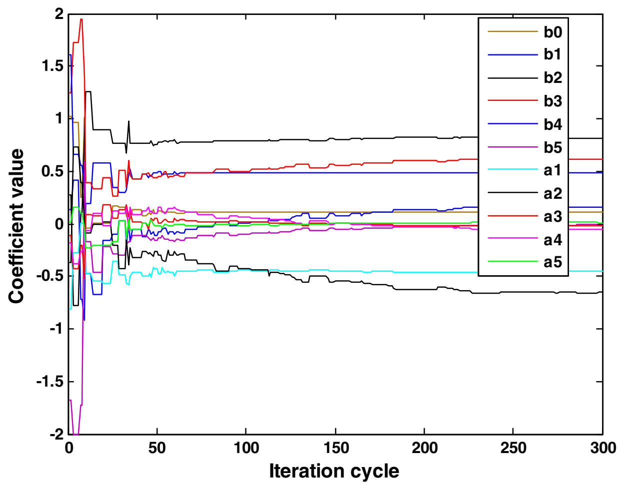

Figure 2 Coefficient convergence profiles of FFA for the best run for Example I (Case 1).

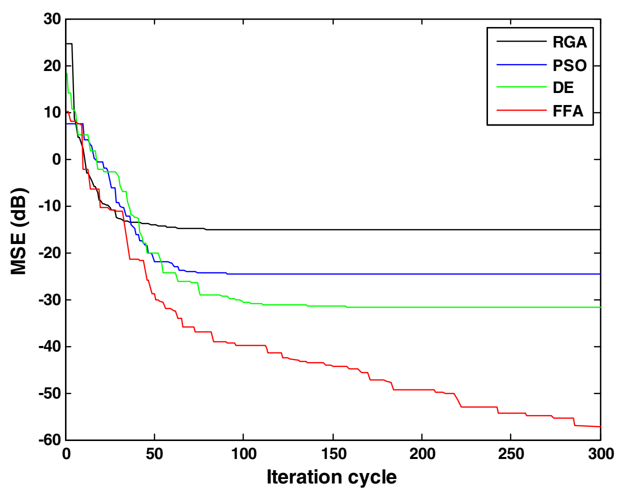

Figure 3 Algorithms' best convergence profiles for Example I (Case 1).

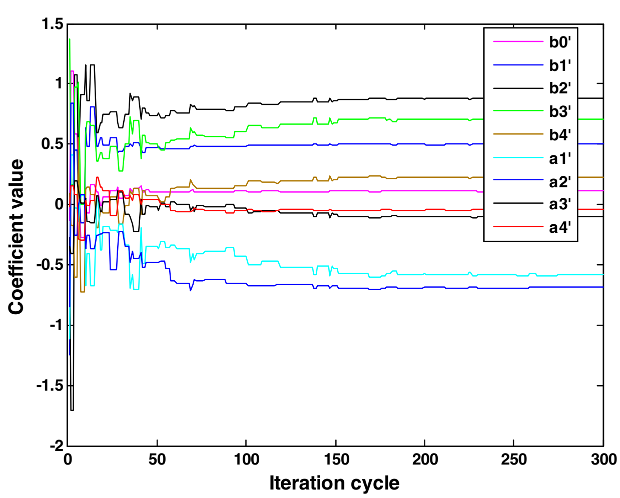

Figure 4 Coefficient convergence profiles of FFA for the best run for Example I (Case 2).

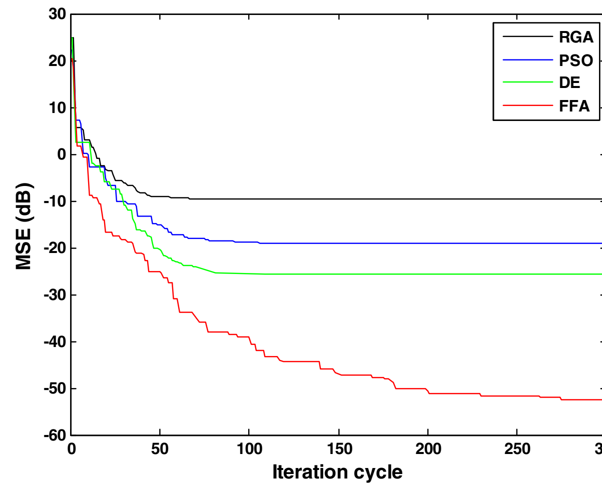

Figure 5 Algorithms' best convergence profiles for Example I (Case 2).

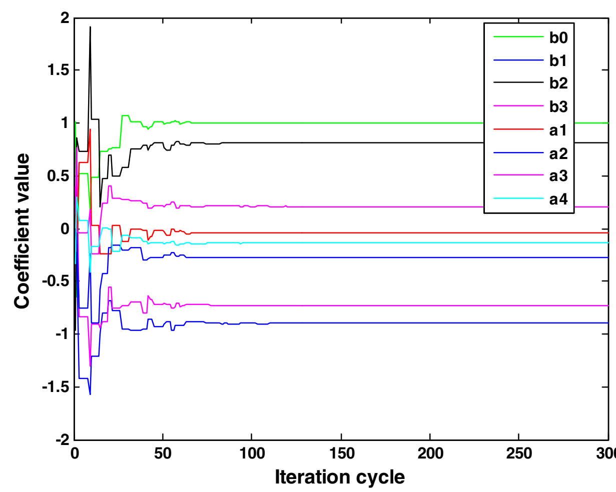

Figure 6 Coefficient convergence profiles of FFA for the best run for Example II (Case 1).

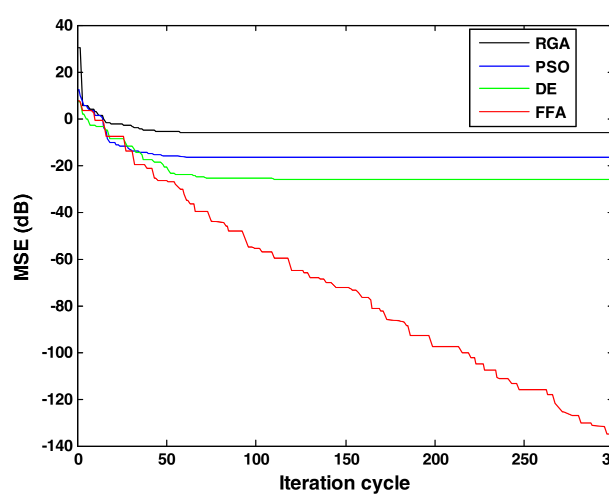

Figure 7 Algorithms' best convergence profiles for Example II (Case 1).

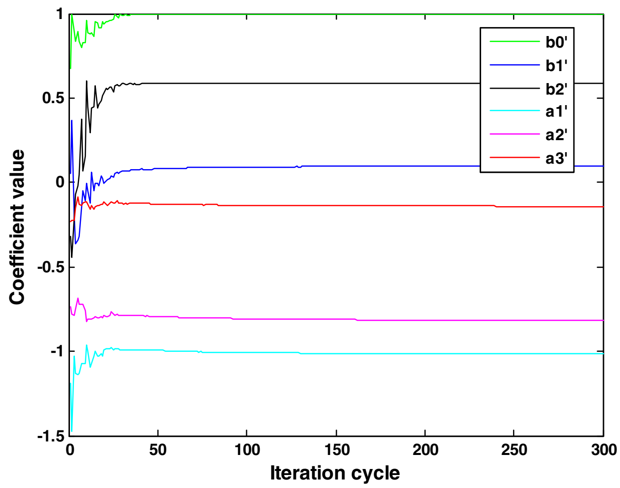

Figure 8 Coefficient convergence profiles of FFA for the best run for Example II (Case 2).

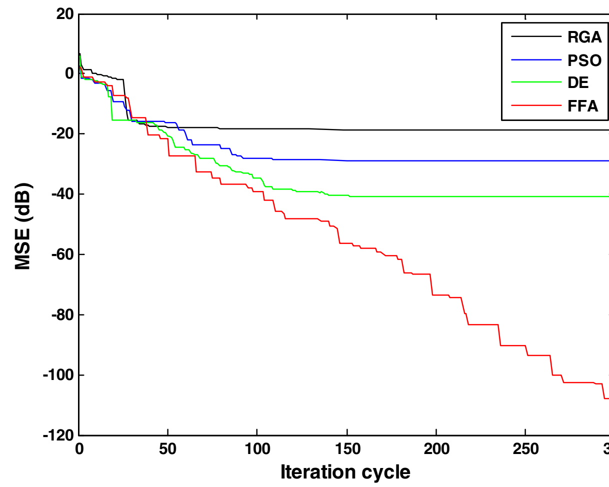

Figure 11 Algorithms' best convergence profiles for Example III (Case 1).

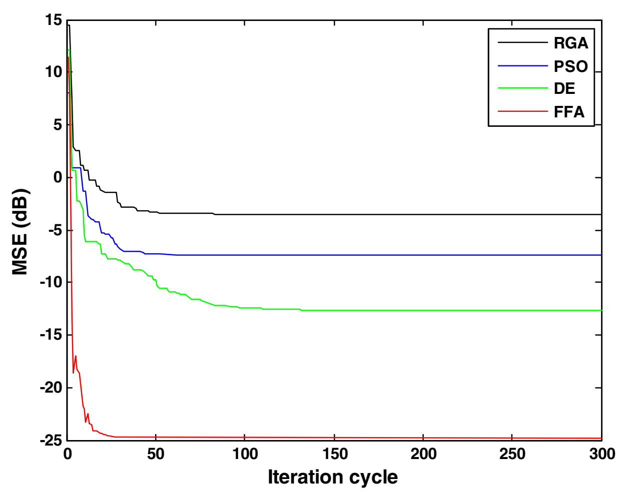

Figure 9 Algorithms' best convergence profiles for Example II (Case 2).

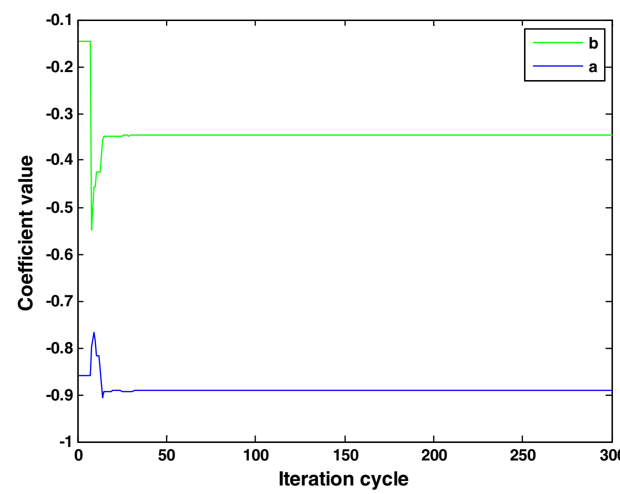

Figure 12 Coefficient convergence profiles of FFA for the best run for Example III (Case 2).

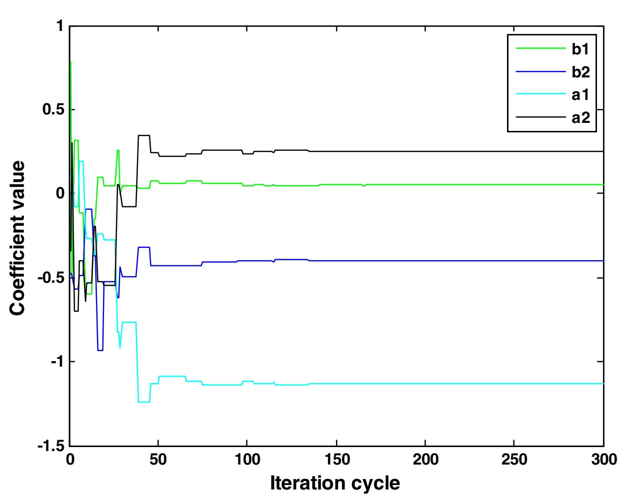

Figure 10 Coefficient convergence profiles of FFA for the best run for Example III (Case 1).

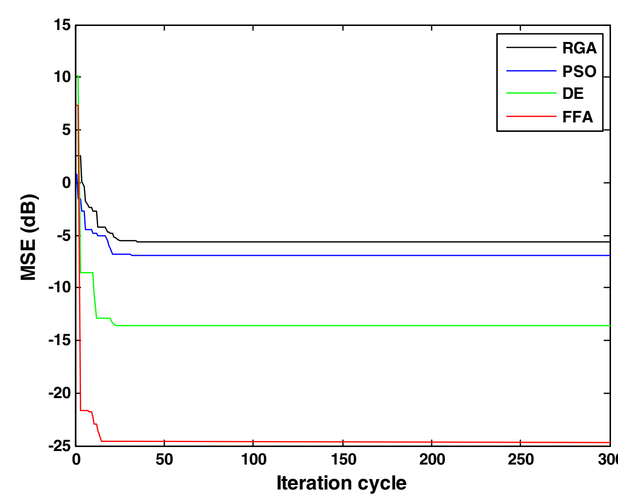

Figure 13 Algorithms' best convergence profiles for Example III (Case 2).

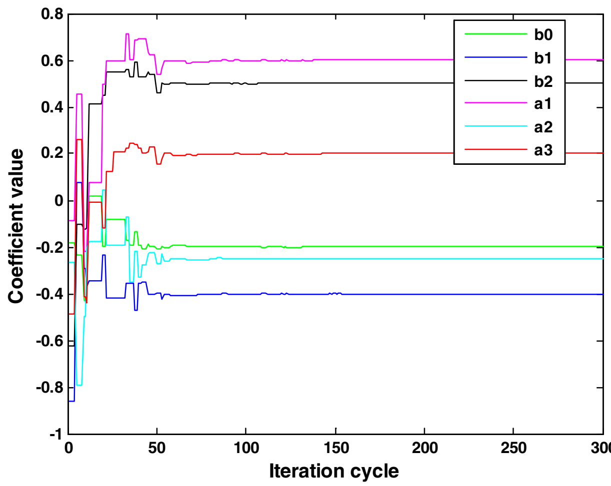

Figure 14 Coefficient convergence profiles of FFA for the best run for Example IV (Case 1).

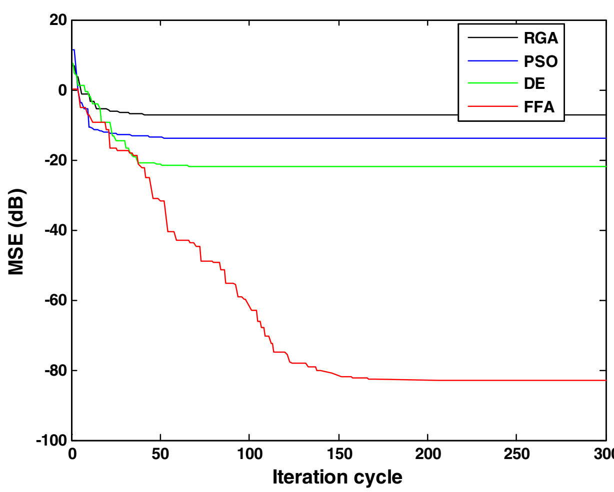

Figure 15 Algorithms' best convergence profiles for Example IV (Case 1).

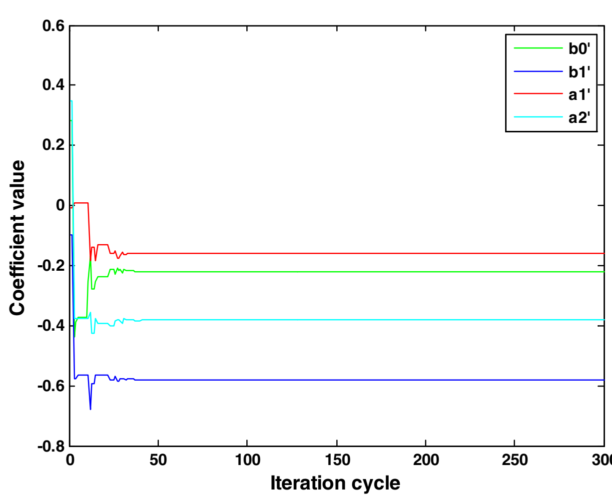

Figure 16 Coefficient convergence profiles of FFA for the best run for Example IV (Case 2).

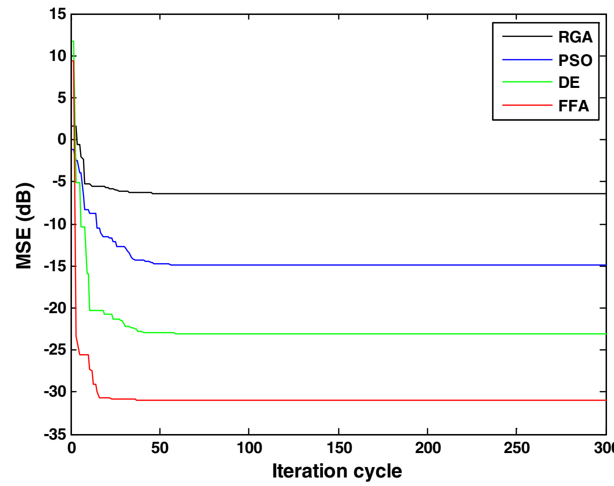

Figure 17 Algorithms' best convergence profiles for Example IV (Case 2).

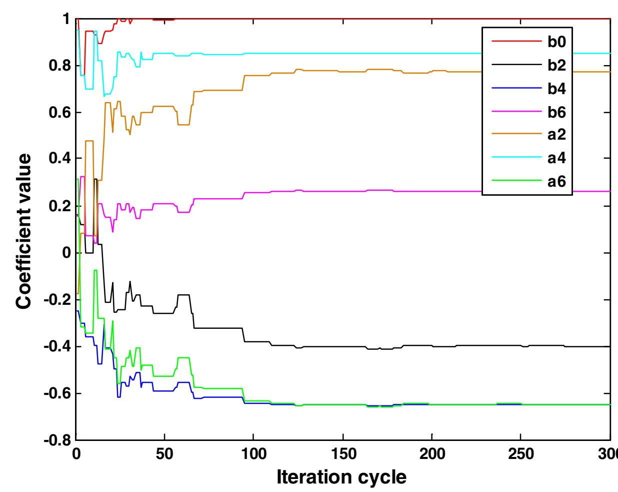

Figure 18 Coefficient convergence profiles of FFA for the best run for Example V (Case 1).

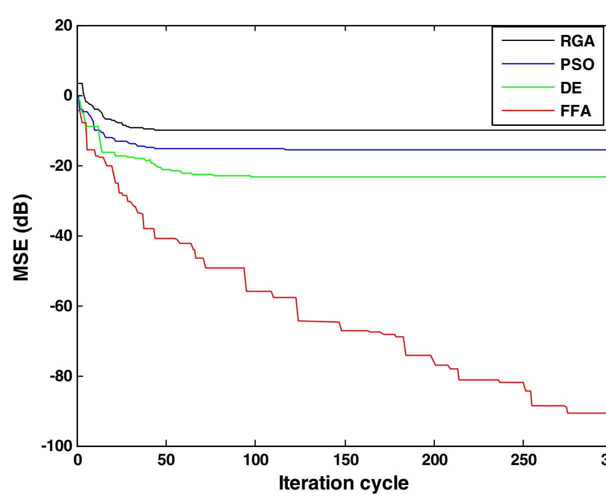

Figure 19 Algorithms' best convergence profiles for Example V (Case 1).

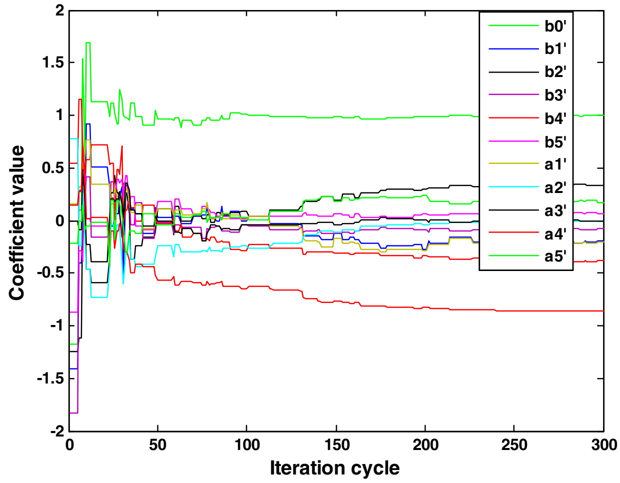

Figure 20 Coefficient convergence profiles of FFA for the best run for Example V (Case 2).

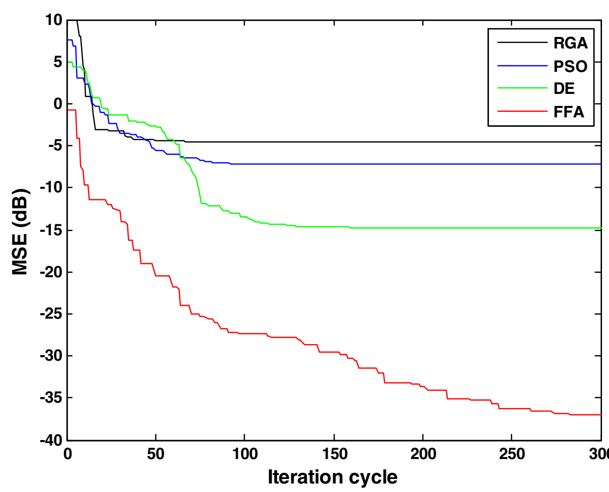

Figure 21 Algorithms' best convergence profiles for Example V (Case 2).

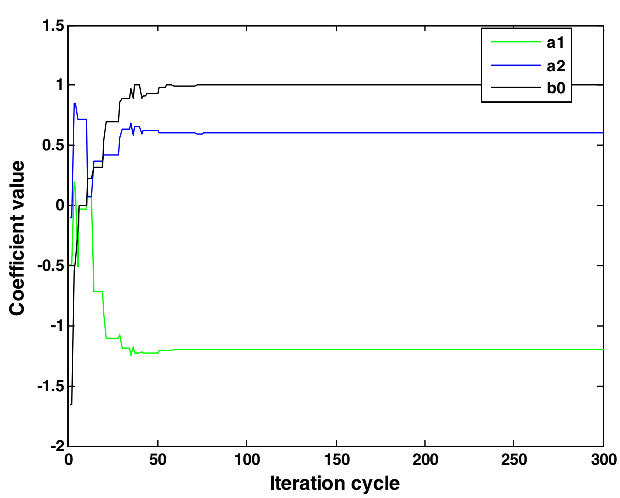

Figure 22 Coefficient convergence profiles for the best run of FFA for Example VI, same order.

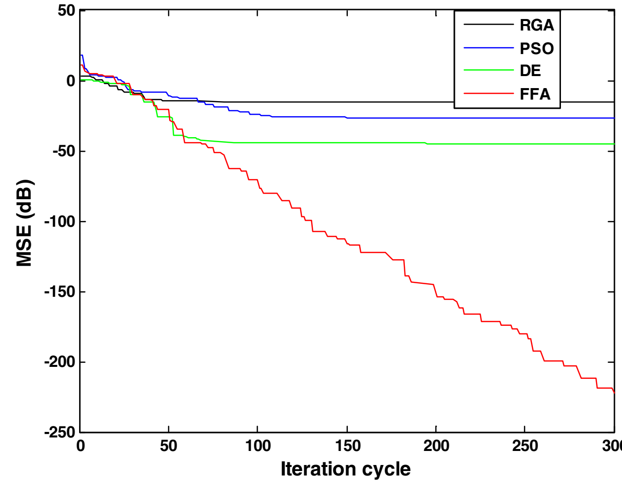

Figure 23 Algorithms' best convergence profiles for Example VI, same order.

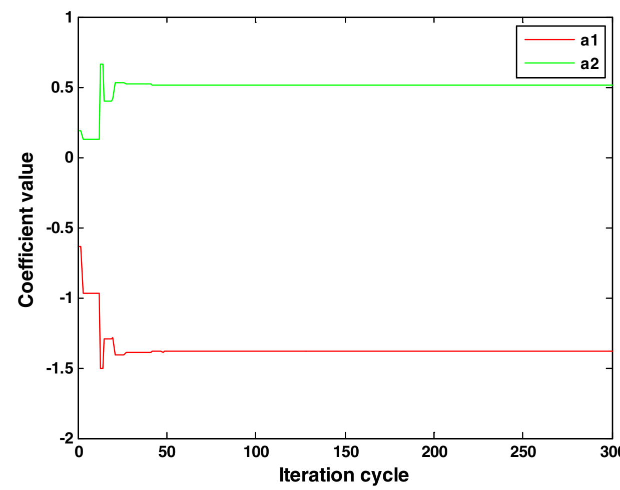

Figure 24 Coefficient convergence profiles for the best run of FFA for Example VII, reduced order.

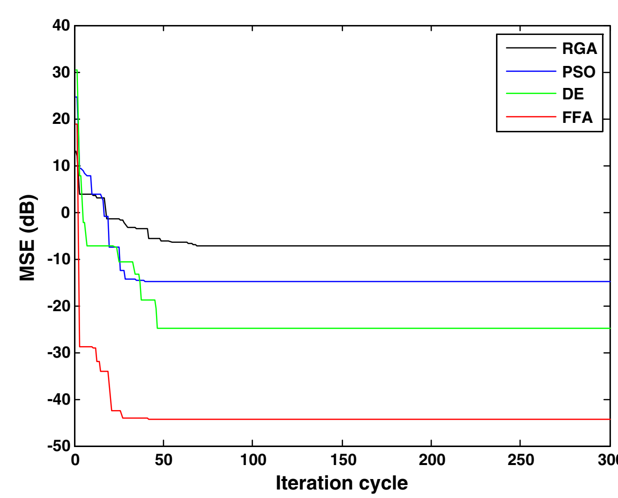

Figure 25 Algorithms' best convergence profiles for Example VII, reduced order.

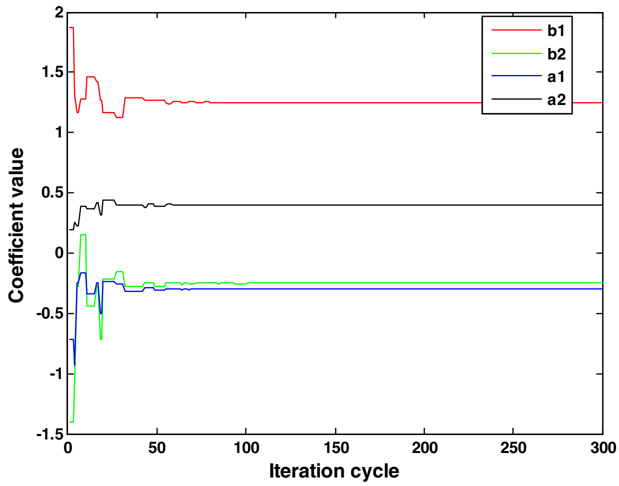

Figure 26 Coefficient convergence profiles for the best run of FFA for Example VIII, same order.

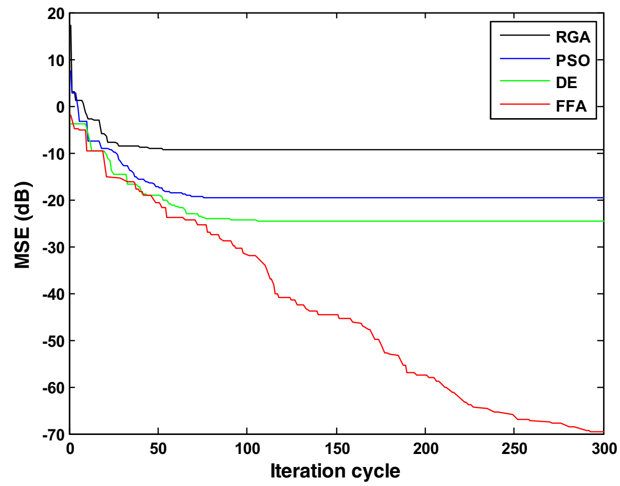

Figure 29 Algorithms' best convergence profiles for Example IX (Case 1).

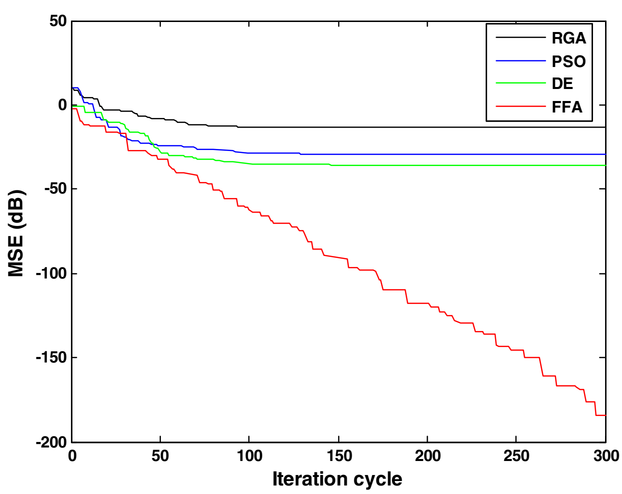

Figure 27 Algorithms' best convergence profiles for Example VIII, same order.

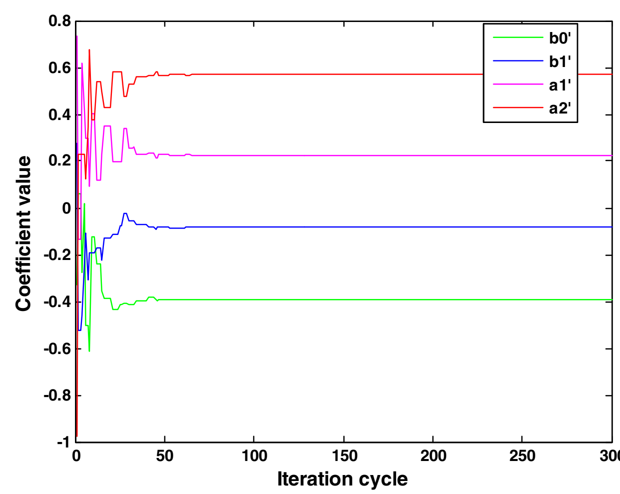

Figure 30 Coefficient convergence profiles of FFA for the best run for Example IX (Case 2).

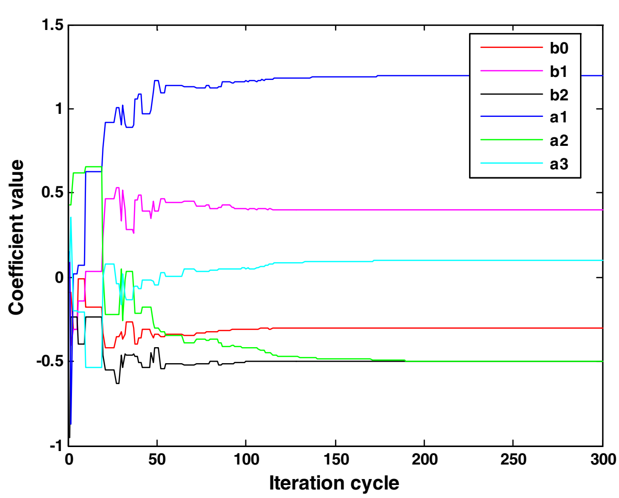

Figure 28 Coefficient convergence profiles of FFA for the best run for Example IX (Case 1).

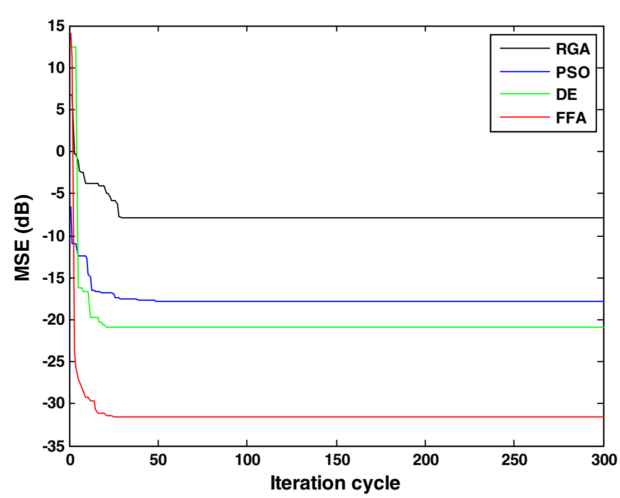

Figure 31 Algorithms' best convergence profiles for Example IX (Case 2).

$$H_s(z) = \frac{1}{1 - 1.2z^{-1} + 0.6z^{-2}} \quad (25)$$

This second order plant $H_s(z)$ can be modelled using the second order IIR filter $H_{af}(z)$.Hence the transfer function of the adaptive IIR filter model is assumed as (26).

$$H_{af}(z) = \frac{b_0}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (26)$$

4.7. Example VII

In this example, a third order IIR plant is considered from (Yu et al., 2009) and the transfer function is shown in (27).

$$H_s(z) = \frac{1}{(1 - 0.5z^{-1})^3} \quad (27)$$

This third order IIR plant is modelled by a reduced order adaptive IIR filter of second order as in (28).

$$H_{af}(z) = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (28)$$

4.8. Example VIII

In this example, a second order IIR plant is considered from (Dai et al., 2010; Krusienski and Jenkins, 2003; Krusienski and Jenkins, 2004; Luitel and Venayagamoorthy, 2010; Yu et al., 2009) and the transfer function is shown in (29).

$$H_s(z) = \frac{1.25z^{-1} - 0.25z^{-2}}{1 - 0.3z^{-1} + 0.4z^{-2}} \quad (29)$$

This second order IIR plant is modelled by a same order adaptive IIR filter as in (30).

$$H_{af}(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (30)$$

4.9. Example IX

In this example, a third order IIR plant is considered from (Dai et al., 2010; Chen and Luk, 2010; Fang et al., 2006, 2009) and the transfer function is shown in (31).

$$H_s(z) = \frac{-0.3 + 0.4z^{-1} - 0.5z^{-2}}{1 - 1.2z^{-1} + 0.5z^{-2} - 0.1z^{-3}} \quad (31)$$

Case 1. This third order plant $H_s(z)$ can be modelled using the third order IIR filter $H_{af}(z)$.Hence the transfer function of the model is assumed as in (32).

$$H_{af}(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3}} \quad (32)$$

Case 2. In this case a higher order plant is modelled by a reduced order filter. For the situation under consideration a third order plant as in (31) is modelled by a second order IIR filter given in (33).

$$H_{af}(z) = \frac{b'_0 + b'_1 z^{-1}}{1 - a'_1 z^{-1} - a'_2 z^{-2}} \quad (33)$$

Optimized coefficient values obtained over five best independent runs for four optimization techniques are presented in Table 2 (Example I, Case 1), Table 5 (Example I, Case 2), Table 8 (Example II, Case 1), Table 11 (Example II, Case 2), Table 14 (Example III, Case 1), Table 17 (Example III, Case 2), Table 20 (Example IV, Case 1), Table 23 (Example IV, Case 2), Table 26 (Example V, Case 1), Table 29 (Example V, Case 2), Table 32 (Example VI, same order), Table 35 (Example VII, reduced order), Table 38 (Example VIII, same order), Table 41 (Example IX, Case 1) and Table 44 (Example IX, Case 2). Optimized coefficient values computed by FFA are near-global optimal, as compared to suboptimal coefficient values yielded by RGA, PSO and DE.

Run times (in Second) and MSE values are reported in Table 3 (Example I, Case 1), Table 6 (Example I, Case 2), Table 9 (Example II, Case 1), Table 12 (Example II, Case 2), Table 15 (Example III, Case 1), Table 18 (Example III, Case 2), Table 21 (Example IV, Case 1), Table 24 (Example IV, Case 2), Table 27 (Example V, Case 1), Table 30 (Example V, Case 2), Table 33 (Example VI, same order), Table 36 (Example VII, reduced order), Table 39 (Example VIII, same order), Table 42 (Example IX, Case 1) and Table 45 (Example IX, Case 2). Study of all tabular results reveal that both the MSE values and run times of FFA are very much less than those of RGA, PSO and DE.

Statistically analysed results, render a ground of judgement of performance for four optimization techniques under consideration and are presented in Table 4 (Example I, Case 1), Table 7 (Example I, Case 2), Table 10 (Example II, Case 1), Table 13 (Example II, Case 2), Table 16 (Example III, Case 1), Table 19 (Example III, Case 2), Table 22 (Example IV, Case 1), Table 25 (Example IV, Case 2), Table 28 (Example V, Case 1), Table 31 (Example V, Case 2), Table 34 (Example VI, same order), Table 37 (Example VII, reduced order), Table 40 (Example VIII, same order), Table 43 (Example IX, Case 1) and Table 46 (Example IX, Case 2). From the statistically analysed results it is revealed that not only the best and mean MSE (dB) values, even the worst MSE (dB) values yielded by FFA are the lowest, as compared to those of other algorithms. Variances and standard deviations are moderately low for FFA indicating fewer fluctuations during the whole process of convergence.

Coefficient convergence profiles for the best run of FFA technique which produces the lowest MSE value among five best runs for a particular case of an unknown system under consideration are presented in Figs. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28 and 30. Algorithm convergence characteristics for all examples are also shown in Figs. 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29 and 31. All these figures depict that for FFA, coefficients are converging very well to their corresponding near-global counteragent values.

Superior performance of the FFA is established by comparing its results with the reported results of the examples cited in this paper for IIR system identification problem. For Example I, Krusinski et al. suggested the PSO algorithm for the Case 1 model and the best MSE level (dB) of −35 dB is reported in (Krusienski and Jenkins, 2004). Panda et al. in (Panda et al., 2011) proposed the CSO for Case 1 and Case 2 models with the best MSE levels of 6.35514e−5 and 6.9475e−5, respectively. Whereas, FFA based approach suggested by the authors yields the best and the least MSE (dB) values of −57.2730 dB and −52.5309 dB for Case 1 and Case 2 models, respectively.

For Example-II, Majhi et al. suggested the PSO technique (Majhi et al., 2008) for Case 1 model with MSE (dB) value of −38 dB. Panda et al. (Panda et al., 2011) achieved MSE values of 5.94209e−5 and 0.006705056 for Case 1 and Case 2 models, respectively, with the CSO technique. FFA technique as suggested for Case 1 and Case 2 models yields the least and the best MSE (dB) levels of −136.5090 dB and −24.8149 dB for Case 1 and Case 2 models, respectively.

Chen et al. used the Case 2 model for Example-III with PSO and a MSE value of 0.275 is reported in (Chen and Luk, 2010). Majhi et al. in (Majhi et al., 2008) applied PSO and a MSE (dB) level of −38 dB is achieved for the Case 1 model. PSO algorithm is applied for the Case 2 model by Durmus et al. in (Durmus and Gun, 2011) and a MSE level of 0.015 is reported. In (Fang et al., 2006), Fang et al. proposed the QPSO for the Case 2 model and the best MSE value of 0.173 is reported. Again Fang et al. suggested MuQPSO for the Case 2 model and MSE of 0.206 is reported in (Fang et al., 2009). In (Karaboga, 2005), Karaboga has applied the DE algorithm and MSE level of 0.0685 for the Case 2 model. In (Karaboga, 2009) Karaboga also suggested the ABC optimization technique for the Case 2 model and the best MSE level of 0.0706 is reported. Rashedi et al. suggested the GSA technique for the Case 2 model with MSE level of 0.172 in (Rashedi et al., 2011). CSO technique is applied by Panda et al. in (Panda et al., 2011) for Case 1 and Case 2 models with reported MSE levels of 6.36395e−5 and 0.0175154, respectively. In (Dai et al., 2010) Dai et al. suggested the SOA technique for the Case 2 model and best MSE level of 8.2773e−2 is reported. The proposed FFA technique for the Case 1 and

Table 47 Performance Comparison of Different reported MSE values.

Example	Reference	Proposed Algorithm	MSE Value Same Order	Reduced Order
Example I	Krusinski et al. (2004)	PSO	−35 dB	NR*
	Panda et al. (2011)	CSO	6.35514e−5	6.9475e−5
	Present work	FFA	1.8737e−6 (=−57.2730 dB)	5.5835e−6 (=−52.5309 dB)
Example II	Majhi et al. (2008)	PSO	−38 dB	NR*
	Panda et al. (2011)	CSO	5.94209e−5	0.006705056
	Present work	FFA	2.2340e−14 (=−136.5090 dB)	0.0033 (=−24.8149 dB)
Example III	Chen et al. (2010)	PSO	NR*	0.275
	Majhi et al. (2008)	PSO	−38 dB	NR*
	Durmus et al. (2011)	PSO	NR*	0.015
	Fang et al. (2006)	QPSO	NR*	0.173
	Fang et al. (2009)	MuQPSO	NR*	0.206
	Karaboga (2005)	DE	NR*	0.0685
	Karaboga (2009)	ABC	NR*	0.0706
	Rashedi et al. (2011)	GSA	NR*	0.172
	Panda et al. (2011)	CSO	6.36395e−5	0.0175154
	Dai et al. (2010)	SOA	NR*	8.2773e−2
	Present work	FFA	1.6311e−11 (=−107.8750 dB)	0.0034 (=−24.6852 dB)
Example IV	Panda et al. (2011)	CSO	6.35201e−5	0.001393846
	Luitel et al. (2010)	PSO-QI	7.791e−4	0.004
	Fang et al. (2009)	MuQPSO	2.041e−3	NR*
	Present work	FFA	5.0709e−9 (=−82.9491 dB)	8.0205e−4 (=−30.9580 dB)
Example V	Karaboga (2009)	ABC	NR*	0.0144
	Luitel et al. (2010)	PSO-QI	7.984e−4	0.001
	Present work	FFA	8.2324e−10 (=−90.8447 dB)	0.0002 (=−36.9897 dB)
Example VI	Durmus et al. (2011)	PSO	1.33e−14	NR*
	Karaboga (2009)	ABC	5.1410e−16	NR*
	Present work	FFA	6.2086e−23 (=−222.0700 dB)	NR*
Example VII	Yu et al. (2009)	AIWPSO	NR*	−32 dB
	Present work	FFA	NR*	3.6976e−5 (=−44.3208 dB)
Example VIII	Krusinski et al. (2004)	PSO	−39 dB	NR*
	Yu et al. (2009)	AIWPSO	−59 dB	NR*
	Krusinski et al. (2003)	MPSO	−130 dB	NR*
	Luitel et al. (2010)	PSO-QI	7.102e−4	0.006
	Present work	FFA	3.6491e−19 (=−184.3780 dB)	NR*
Example IX	Dai et al. (2010)	SOA	NR*	5.1821e−3
	Chen et al. (2010)	PSO	NR*	−17.4036 dB
	Fang et al. (2006)	QPSO	NR*	0.013
	Fang et al. (2009)	MuQPSO	NR*	0.01374
	Present work	FFA	1.1226e−7 (=−69.4977 dB)	6.9238e−4 (=−31.5966 dB)

* NR: not reported in the refereed literature/present work.

Case 2 models results in the best and the least MSE (dB) levels of −107.875 dB and −24.6852 dB, respectively.

For Example IV, Panda et al. suggested the CSO technique (Panda et al., 2011) for Case 1 and Case 2 models with MSE values of 6.35201e−5 and 0.001393846, respectively. Luitel et al. also suggested MSE values of 7.791e−4 and 0.004 for Case 1 and Case 2 models, respectively; with the PSO-QI technique as reported in (Luitel and Venayagamoorthy, 2010). Fang et al. in (Fang et al., 2009), suggested the MuQPSO for the Case 1 model with the best MSE level of 2.041e−3. FFA technique results in the best and the least MSE (dB) levels of −82.9491 dB and −30.9580 dB for Case 1 and Case 2 models, respectively.

For Example V, Karaboga suggested the ABC algorithm for the Case 2 model and the best MSE level of 0.0144 is reported in (Karaboga, 2009). Luitel et al. in (Luitel and Venayagamoorthy, 2010) proposed PSO-QI for Case 1 and Case 2 models with the best MSE levels of 7.984e−4 and 0.001, respectively. In this work, FFA results in the best and the least MSE (dB) values of −90.8447 dB and −36.9897 dB for Case 1 and Case 2 models, respectively.

For Example VI, Durmus et al. suggested the PSO technique for the Case 1 model and the best MSE of 1.33e−14 is reported in (Durmus and Gun, 2011). Karaboga in (Karaboga, 2009) suggested the ABC for the Case 1 model with the best and the least MSE level of 5.1410e−16. FFA for the Case 1 model yields the least and the best MSE (dB) value of −222.07 dB as reported in this paper.

For Example VII, Yu et al. suggested the AIWPSO algorithm for the Case 2 model with best MSE (dB) value of −32 dB reported in (Yu et al., 2009). FFA as suggested by the authors for the Case 2 model results in the least and the best MSE (dB) value of −44.3208 dB.

For Example VIII, Krusinski et al. suggested the PSO technique (Krusienski and Jenkins, 2004) for Case 1 model with an MSE value of −39 dB. Yu et al. also suggested an MSE (dB) value of −59 dB for the Case 1 model with the AIWPSO technique as reported in Yu et al. (2009). Again in (Krusienski and Jenkins, 2003), Krusinski et al. suggested the MPSO technique for the Case 1 model with the best MSE level of −130 dB. Luitel et al. in (Luitel et al., 2010) suggested PSO-QI for Case 1 and Case 2 models with the best MSE levels of 7.102e−4 and 0.006, respectively. FFA technique for the Case 1 model results in the best and the least MSE (dB) level of −184.3780 dB.

For Example IX, Dai et al. suggested the SOA technique for the Case 2 model and best MSE level of 5.1821e−3 is reported in (Dai et al., 2010). In (Chen et al., 2010), Chen et al. proposed the PSO algorithm with an MSE value of −17.4036 dB for the Case 2 model. Fang et al. proposed QPSO and MuQPSO for the Case 2 model and best MSE levels of 0.013 and 0.01374 are reported, respectively, in Fang et al. (2006, 2009). FFA algorithm for Case 1 and Case 2 models yields the best and the least MSE (dB) levels of −69.4977 dB and −31.5966 dB, respectively. All the above comparative results, given above for the comparative study among the algorithms are presented in Table 47.

5. Conclusions

In this paper, the proposed Firefly Algorithm (FFA) for finding optimal sets of adaptive IIR filter coefficients for both same order and reduced order models is used for unknown system identification problem. Firefly's behaviour for finding brighter mate results in a noticeable improvement in mimicking the unknown plant in terms of minimum error fitness value and algorithm's optimal convergence profile. From the simulation study it is established that the proposed optimization technique for adaptive filtering is efficient in finding an optimal solution in multidimensional search space where the rest algorithms are entrapped to suboptimal solutions. Hence it can be concluded that the proposed technique is good enough to handle unknown system identification problem.

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