Thermo-damage-viscoelastic constitutive model of HTPB composite propellant


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A R T I C L E   I N F O

Article history:
Received 19 December 2013
Received in revised form 20 April 2014
Available online 2 June 2014

Keywords:
Viscoelastic
Relaxation test
 Constitutive model
Damage
Composite propellant

A B S T R A C T

The main goal of this work is to establish a thermo-damage-viscoelastic model for Hydroxyl-Terminated Polybutadiene (HTPB) composite propellant based on the thermodynamic theory and elastic–viscoelastic correspondence principles. The model will also consider the influence of temperatures. The parameter \( a \) which represent the damage evolution rate and the material constants \( a, b \) are defined as exponential functions of temperature \( T \), i.e. \( a(T), b(T) \). Relaxation tests and uniaxial constant rate tensile tests are used to acquire the model parameters, and \( C(T) \) curves of different rates under the same temperature states are considered to be overlapped in this paper, while noncoincidence under the different temperature states. Then, uniaxial constant rate tensile tests and multi-step tensile-relaxation tests are used to verify the accuracy of the model. The results show that, the model is highly accurate in describing the mechanical property of HTPB under various loading conditions, but some drawback in describing the relaxation property inside of the HTPB's nonlinear viscoelastic segments.

1. Instruction

HTPB composite propellant is a polymer material with amount of solid particles, such as aluminum (Al), hexogen (RDX), ammonium perchlorate (AP), etc., which embedded in a lightly-crosslinked HTPB binder (Park and Schapery, 1997; Kakavas, 2014). Results from a mass of experiments show that, the HTPB composite propellant have a classic viscoelastic characteristic as the HTPB binder, and express an approximate linear viscoelastic property when the structures or materials in an especial range of deformation, while expressing a nonlinear viscoelastic property. Combining the microcosmic-test, microdefects (debonding, microvoid and microcracks, et al.) will be formed gradually when such materials have been loaded, and becoming more deformed, more microdestructive. Thereby, the nonlinear viscoelastic property of such materials will be increased while deformation increased. The mechanical property of HTPB binder and bonded interface are temperature-dependent (Zalewski and Wolszakiewicz, 2011). This is because the relaxation time of molecular movement is longer under the lower temperature states, while shorter under the higher temperature states. Therefore, it is necessary to establish a thermo-damage-viscoelastic constitutive model to precisely describe the nonlinear viscoelastic and temperature-dependent mechanical property of HTPB composite propellant.

There are usually two methods to establish a damage model. One way is to measure the formation and evolution of microdefect using CT scanning, acoustic emission (AE) or X-ray, et al. Another way, take the density, elastic modulus or strain energy of the material as research objects, and base on the macro variation of the research objects during the loading process to establish the damage evolution models. Since the first method should meet a certain experiment conditions, the second method are widely used. Based on the cycle loading–unloading tests, the linear cumulative damage model (LCDM) was applied commonly (Duncan and Margetson, 1998; Stigh, 2006). By defining the damage as the function of frequency and damage, the LCDM was improved (Desmorat, 2006), and the specific damage evolution model was acquired (Chaboche, 2009). To obtain the functional relationship of cycle index \( N_c \) and loading \( C \) conveniently, the relationship between damage evolutionary rate and cycle loading can be imported into the model in an implicit way (Jardin et al., 2010). The damage variable can also be defined by the density variation during the loading process, and the damage model was simplified based on amount of experiment studies (Bernasconi and Piatti, 1978), the simplified process was verified by creep tests at different stress levels (Cozzarelli and Bernasconi, 1981). The crosslinking degree molecular structure in propellant material can also be defined as aging damage variable (Deng et al., 2014).
Make use of the elastic–viscoelastic correspondence principles (Schapery, 1975, 1984), a viscoelastic continuum damage model (VECD) was established by importing the pseudo strain (Schapery, 1999) and it was applied to study the property of mechanical, fatigue, relaxation or creep for various of materials, include asphalt, HTPB, PMMA, et al. (Park and Schapery, 1997; Lundstrom and Ekblad, 2006; Hinterhoelzl and Schapery, 2004). Usually, two damage variable $S_1$ and $S_2$ will be imported into the damage evolution law (Kim et al., 2008; Kim and West, 2010), while to isotropic incompressible materials only one damage variable can be sufficient (Mun et al., 2005). Based on the VECD model, a viscoelastoplastic continuum damage (VEPCD) model which considering the plastic behavior were proposed (Mun, 2009; Underwood et al., 2006).

The setup process of the VEC model can be summed up as coming under three categories: the first one is the elastic–viscoelastic correspondence principles based on the concept of pseudo strain, which is applied to describe the linear viscoelastic mechanical property; the second one is the cumulative damage mode based on the pseudo strain energy, which is applied to calculate the damage evolution; the last one is the time–temperature superposition principle (TTSP) which is used to consider the stress temperature-dependence.

In this paper, only one damage variable will be considered in VEC model (Mun et al., 2005), while the parameter $a$ which represent the damage evolution rate and the material constants $a, b$ were defined as exponential functions of temperature $T$, i.e. $a(T), a(T)$ and $b(T)$, which were used to consider the temperature-dependence of the damage evolution. The way to acquire the model parameters were studied base on a series of uniaxial constant rate tensile tests, and the thermo-damage-viscoelastic constitutive model was used to predict the mechanical property of the uniaxial constant rate tensile tests and the multi-step tensile-relaxation tests, and the predicting results are highly accurate in describing the mechanical property under various loading conditions, but some drawback in describing the relaxation property inside the nonlinear viscoelastic segments. To improve the accuracy of the model, some improvement, such as more loading conditions, more times of repetition and more suitable C(S) functional should be carried out.

2. Constitutive model

2.1. Thermodynamics damage theory

Following Schapery (1999) and Park, the thermodynamic state of the materials and structures is expressed by the generalized displacements $q_j (j = 1, 2, \ldots, J)$, internal states variables (ISVs) $S_m (m = 1, 2, \ldots, M)$, temperature and entropy, and internal states variables $S_m$ are supposed to be the key parameters which described the nonlinear mechanical properties of materials. For an elastic medium, the microstructure and mechanical properties are determined by generalized forces $Q_j (j = 1, 2, \ldots, J)$ and generalized displacements $q_j$, following the equation below

$$Q_j = \frac{\partial W}{\partial q_j}$$

where, $W$ is Helmholtz free energy (when temperature is used as an independent state variable) or internal energy (when entropy is used as an independent state variable), and $W$ equal to strain energy density when taking no account of thermal dissipation. Therefore, the generalized forces $Q_j$ and the generalized displacements $q_j$ can be defined as either the forces and displacements on structures or defined as the stresses and strains on elements.

When materials or structures have flaw inside (such as Microcrack, Microvoid, Dewet, et al.), the microstructural changes need to be described via internal states variables. For an arbitrary infinitesimal process, the changing of strain energy density can be described by the changes in $q_j$ and $S_m$ as

$$dW = \frac{\partial W}{\partial q_j} dq_j + \frac{\partial W}{\partial S_m} dS_m = Q_j dq_j - f_m dS_m$$

where $f_m = -\frac{\partial W}{\partial S_m}$, it is called thermodynamic force, defining the evolution law of ISVs as

$$f_m = \frac{\partial W_S}{\partial S_m} (S_m = 0)$$

where, $W_S$ is a state function of single or multi-ISVs. The left side of Eq. (3) is the available force for producing changes in $S_m$, while the right side of Eq. (3) is the required force (as the crack growth equation). For a certain specific process, Eq. (3) is not apply to the whole ISVs, but to $r$ ISVs ($r < m$), and the remaining $m - r$ ISVs are constants.

For the changing process of a certain $S_m$, the total work done by the generalized forces $Q_j$ is

$$W_T = \int Q_j dq_j$$

where $j = 1, 2, \ldots, J$, the summation convention is used here. From Eqs. (2)–(4), $W_T$ is a function of the generalized displacements $q_j$ and damage variable $S_m$, as

$$W_T = W + W_S$$

In Eq. (5), it is indicated that there is no damage inside when $W = W_S = 0$, and consider the state as the reference state. Therefore, define $W_S$ as the portion of the total work called dissipated energy which contributes to changes in the structure. It is a irreversible procedure, according to the second law of thermodynamics only those changes in $S_m$ that correspond to a non-negative entropy production rate $S'$ are possible, get

$$W_S = TS > 0$$

where $T$ is absolute temperature, $S$ is the growth rate of entropy.

According to the definition of damage variable, $S_m$ is also the function of the generalized displacements $q_j$ thus, $W_T = W(q_j, S_m(q_j)) + W_S(S_m(q_j))$ can be simplified as $W_T = W_T(q_j)$. The total work $W_T$ is not only a state function of $S_m$ and $q_j$ but also a potential in $q_j$ during inelastic processes. Then from Eq. (4) we get

$$Q_j = \frac{\partial W_T}{\partial q_j}$$

2.2. Damage model

2.2.1. Correspondence principles

There is no energy dissipation in elastic deformation which is a reversible procedure, thus, the elastic strain energy can be fully released after unloading. For isotropous elastic materials, the strain energy density can be described as

$$W = \frac{1}{2} \sigma_{ij} e_{ij}$$

where $\sigma_{ij}$ is the volumetric strain, $e_{ij} = e_{11} + e_{22} + e_{33}$, $G$ is the shear relaxation modulus, $G = E/(2(1 + v))$, and $v$ is the Poisson’s ratio.

Schapery (1984) established a stress–strain relationship for viscoelastic materials as the same form as elastic materials by defining a pseudo strain. Following the corresponding principle, both the time dependence of mechanical property of viscoelastic materials and the quasi-static solution of nonlinear viscoelastic
boundary value can be resolved. For linear viscoelastic materials, the stress–strain relationship can be described as

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl}^e$$

where

$$C_{ijkl} = \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) - \frac{1}{2} \delta_{ij} \delta_{kl}$$

$$\epsilon_{ij}^e = \frac{1}{E_k} \int_0^t E_k(t - \tau) \frac{\partial \sigma_{ij}}{\partial \tau} d\tau$$

(9)

where, $C_{ijkl}$ are material constants, $\epsilon_{ij}^e$ are strain pseudo tensor, $E_k$ is the reference elastic modulus which can be arbitrarily selected, usually we select $E_k = 1$. Meanwhile, the pseudo strain $\epsilon_{ij}^e$ can be nondimensionalized by introducing the reference elastic modulus. $\lambda$ and $\mu$ are Lamb constant, that can be described as

$$\lambda = \frac{vE_k}{(1+v)(1-2v)} \quad \mu = \frac{E_k}{2(1+v)}$$

Referring Eq. (9), the pseudo strain has a convolution integral form, which considered the time dependence of the mechanic properties of viscoelastic materials. In the case of one dimension, $C_k = E_k$, taking the pseudo strain $\epsilon_{ii}^e$ into linear viscoelastic theory, the constitutive become

$$\sigma = E_k \epsilon^e$$

(10)

Referring Eq. (10), the linear viscoelastic constitutive model have the same form as the linear elastic model (Hooke’s law). The pseudo strain $\epsilon_{ij}^e$ equal to linear viscoelastic stress response when $E_k = 1$. Furthermore, there is a linear relationship between the pseudo strain $\epsilon_{ij}^e$ and the linear viscoelastic stress response $\sigma_{ij}^e$, the slope is just the reference modulus $E_k$.

2.2.2. Damage evolution law

For the most viscoelastic materials, not only the available force for growth in $S$ is rate-dependent, but also the resistance against the growth of $S$ is rate-dependent. Thus, the damage evolution for elastic materials cannot directly be translated into an evolution law for viscoelastic materials. According to correspondence principles, the evolution law proposed has the same form as the crack growth equation for viscoelastic materials, specifically

$$\dot{S}_m = \left( - \frac{\partial W^e}{\partial S_m} \right) \dot{S}_m \quad \text{(no sum on m)}$$

(11)

where $W^e = W(e_{ii}^e, S_m)$ is pseudo strain energy density, $\dot{S}_m$ are material constants. Using the evolution law to simulate the viscoelastic materials that have solid particle inside, the accuracy of the mechanical property have been verified (Park et al., 1996). In this paper, only one damage variable $S$ is considered, then the damage evolution model is

$$\dot{S} = \left( - \frac{\partial W^e}{\partial S} \right) \dot{S}$$

(12)

2.3. Constitutive model for three-dimension loading

According to correspondence principles, take the pseudo strain $\epsilon_{ij}^e$ to replace the physical strain $e_{ij}$, and proposed the concept of pseudo strain energy density $W^e$, as

$$W^e = \frac{1}{2} \left[ A_{11} (\epsilon_{11}^e)^2 + A_{22} (\epsilon_{22}^e)^2 + 2A_{12} \epsilon_{12}^e \epsilon_{21}^e + A_{44} (\epsilon_{13}^e)^2 - (\epsilon_{23}^e)^2 \right]$$

$$+ A_{66} (\epsilon_{12}^e)^2 - (\epsilon_{21}^e)^2 \right]$$

(13)

where

$$\epsilon_{11}^e = \epsilon_{11}^v + \epsilon_{12}^v + \epsilon_{13}^v \quad \epsilon_{22}^e = \epsilon_{22}^v - \epsilon_{11}^v \quad \gamma_{12}^e = 2\epsilon_{12}^v - 2\epsilon_{12}^v$$

$$\gamma_{13}^e = 2\epsilon_{13}^v \quad \gamma_{23}^e = 2\epsilon_{23}^v$$

(14)

When taking $x_3$ as the maximum principal stress direction, and parallel to the maximum principal pseudo strain direction, $\gamma_{12}^e = 0$, $\gamma_{13}^e = 0$, $\gamma_{23}^e = 0$, the pseudo strain energy $W^e$ becomes

$$W^e = \frac{1}{2} \left[ A_{11} (\epsilon_{11}^e)^2 + A_{22} (\epsilon_{22}^e)^2 + 2A_{12} \epsilon_{12}^e \epsilon_{21}^e + A_{66} (\epsilon_{12}^e)^2 \right]$$

(15)

According to the relationship about stress, strain and strain energy density, as

$$\sigma = \frac{\partial W^e}{\partial \epsilon_{ij}^e}$$

(16)

The principal stresses can be obtained, explicitly

$$\sigma_1 = (A_{11} - \frac{1}{2} A_{12}) \epsilon_{11}^e + (A_{12} - \frac{1}{2} A_{22}) \epsilon_{22}^e - A_{66} \epsilon_{12}^e$$

$$\sigma_2 = (A_{11} - \frac{1}{2} A_{12}) \epsilon_{22}^e + (A_{12} - \frac{1}{2} A_{22}) \epsilon_{11}^e + A_{66} \epsilon_{12}^e$$

$$\sigma_3 = (A_{11} + \frac{1}{2} A_{12}) \epsilon_{11}^e + (A_{12} + \frac{1}{2} A_{22}) \epsilon_{22}^e$$

(17)

The constitutive model (Park and Schapery, 1997) for uniaxial specimens is adopted here. When two damage variables was considered, the pseudo strain energy $W^e$ can be written as

$$W_{SD}(e^v, p, S_1, S_2) = C_{11}(S_1)C_{12}(S_2) (\epsilon_{11}^v)^2 + C_{12}(S_1) \epsilon_{22}^v p$$

$$+ \frac{1}{2} C_{22}(S_1) p^2$$

(18)

According to Eq. (16), differentiating Eq. (18) with respect to the pseudo strain and pressure, the stress strain relations give

$$\sigma_1 = \frac{\partial W_{SD}}{\partial \epsilon_{11}^v} = C_{11}(S_1)C_{12}(S_2) \epsilon_{11}^v + C_{12}(S_1) \epsilon_{22}^v + C_{22}(S_1) p$$

(19)

$$\sigma_2 = \frac{\partial W_{SD}}{\partial \epsilon_{22}^v} = C_{11}(S_1)C_{12}(S_2) \epsilon_{22}^v + C_{12}(S_1) \epsilon_{11}^v$$

$$\sigma_3 = \frac{\partial W_{SD}}{\partial p} = C_{22}(S_1) p$$

(20)

By taking Eq. (15) to compare with Eq. (18), which two have different forms about the pseudo stress energy. The relationship between $A_{ij}$ and $C_{ij}$ are

$$A_{11}(S_1, S_2) = \frac{1}{2} \left[ C_{11}(S_1)C_{12}(S_2) - \frac{(C_{12}(S_1)C_{22}(S_2))}{2} \right]$$

$$A_{12}(S_1, S_2) = \frac{1}{2} \left[ C_{11}(S_1)C_{12}(S_2) + C_{12}(S_1)C_{22}(S_2) \right] (1 - C_{22}(S_2))$$

$$A_{22}(S_1, S_2) = C_{11}(S_1)C_{12}(S_2) - \frac{(C_{12}(S_1)C_{22}(S_2))}{2}$$

$$A_{44} = A_{66} = C_{G}$$

(20)

There is no damage at the initial time for viscoelastic materials. The soften parameter $C_{ij}$ is the function of modulus $C$ and Poisson’s ratio $v$ for elastic materials (Hinterhoelzl and Schapery, 2004). According to the corresponding principle, for viscoelastic materials, $C_{ij}$ is the function of reference modulus $E_k$ instead, as

$$C_{11} = E_k, \quad C_{12} = 1 - 2v, \quad C_{22} = \frac{2}{E_k} (1 + v) (1 - 2v)$$

(21)

It is worth noting that, transverse strain ($\epsilon_{22}^e$, $\epsilon_{33}^e$) can be expressed by axial strain $\epsilon_{11}^v$ for isotropy elastic materials as $\epsilon_{22}^e = \epsilon_{33}^e = \epsilon_{11}^v$, when the specimen be loaded along axial, thus, the model should import the Poisson’s ratio $v$ which consider the transverse effect. However, the research objects change from $\sigma - \epsilon$ to $\sigma - d^e$ for viscoelastic materials, and there is no direct relationship between the transverse pseudo strain ($\epsilon_{22}^e$, $\epsilon_{33}^e$) and axial pseudo strain $\epsilon_{11}^e$, thus, it is considered to $\nu = 0$ in viscoelastic materials study. Actually, the transverse effect has been considered in the pseudo strain solving. Then Eq. (21) become

$$C_{11} = E_k, \quad C_{12} = 1, \quad C_{22} = -\frac{2}{E_k}$$

(22)
There is only one ISVs $S_2$ considered in this paper, and $S_1$ is constant as 0. Thus, the soften parameter $C_4$ which considered $S_1$ are constant as 1. According to Eqs. (20) and (22)

$$
\begin{align*}
A_{11}(S_1, S_2) &= \frac{e^4}{C_25/C_0} [C(S_2) + 2] \\
A_{12}(S_1, S_2) &= \frac{e^4}{C_25/C_0} [C(S_2) - 1] \\
A_{22}(S_1, S_2) &= E_R [C(S_2) + \frac{e^4}{C_25/C_0}] \\
A_{44} &= A_{04} = \frac{e^4}{C_25/C_0}
\end{align*}
$$

From Eq. (17), the principal stresses are

$$
\sigma_1 = E_R e^4 \\
\sigma_2 = E_R e^4 \\
\sigma_3 = E_R C(S) e^4
$$

2.4. Study of parameters acquirement

The model parameters will be acquired by stress relaxation tests and constant-rate elongation tests in this section. The data used here is all from one dimensional experiment due to the test equipment limitations. According to the three dimensional constitutive model, the one dimensional form can be acquired after some reasonable simplifications. From Eq. (24), the stress response of one dimensional elongation tests can be written as

$$
\sigma(t) = E_R C(S) e^4(t)
$$

where, pseudo strain $e^4(t)$ is defined as

$$
e^4(t) = \frac{1}{E_R} \int_0^t E(R(t) - \tau, \partial e^4/\partial \tau) d\tau
$$

The reference modulus $E_R$ is selected as 1, then soften parameter, $C(S) = \varepsilon(t)/e^4(t)$. According to Eqs. (15) and (18), only one damage variable is considered, the one dimensional pseudo strain energy density is

$$
W^d = \frac{1}{2} C(S)(e^4)^2
$$

Damage evolution law is

$$
\dot{S} = \left( -\frac{\partial W^d}{\partial S} \right)^2
$$

From Eqs. (26)–(28), the parameters need to be acquired include the relaxation modulus $E(t)$, material constant $\alpha$ which express the damage evolution rate have not physical meanings. The relaxation modulus $E(t)$ can be acquired by stress relaxation tests, and the other parameters will be acquired by the constant-rate elongation tests. Take Eq. (27) into Eq. (29) give

$$
\dot{S} = \left( -\frac{1}{2} \frac{\partial C(S)}{\partial S} (e^4)^2 \right)^2
$$

Small time increment step can be got by high data collecting frequency, then $\dot{S}$ and $\partial C(S)/\partial S$ can be expressed by the incremental form, specifically

$$
\frac{\Delta S}{\Delta t} \approx \left( -\frac{1}{2} \frac{AC(S)}{\Delta S} (e^4)^2 \right)^2
$$

Then the delta value of damage variable $\Delta S$ is

$$
\Delta S = \left( -\frac{1}{2} \frac{AC(S)}{\Delta S} (e^4)^2 \right)^2 \Delta t e^4
$$

$$
S(t + \Delta t) - S(t) = \left( -\frac{1}{2} (C(t + \Delta t) - C(t))(e^4)^2 \right)^2 \Delta t e^4
$$

In Eq. (32), the pseudo strain $e^4$ and soften parameter $C$ can be calculated by Eqs. (26) and (25) individually, then the damage variable $S$ can be calculated after $\alpha$ be confirmed, and the relationship between $C$ and $S$ can be established, that is, $C(S)$ equation.

The damage evolution law Eq. (28) which is rate-dependent. That is to say, the value of $\alpha$ should be invariant in different rates of uniaxial tensile test. According to Park's work (Park et al., 1996), we assume that the damage occurs in HTPB material is closely related to the growth of microcracks, then the value of $\alpha$ can be calculated by the relationship (Schapery, 1975) as

$$
\alpha = 1 + 1/n
$$

where, $n = \log D(t)/\log t$ or $n \approx -\log E(t)/\log t, D(t)$ is the creep compliance, $E(t)$ is the relaxation modulus.

In this paper, the initial value of $\alpha$ is determined by the ratio of the logarithm of relaxation modulus to the logarithm of time. Then $\alpha$ will be adjusted appropriately on the basis of test results under different rate of extension, and the final value of $\alpha$ will be ensured until the C–S curves in different rate of extension overlapped. The specific fitting process of the constitutive parameters are as follows:

1. The master relaxation curves which expressed by Prony Series can be obtained by relaxation test under different temperature field, for the specific process see reference (Xu et al., 2013);
2. Select reference modulus $E_R = 1$, the pseudo strain can be calculated based on Eq. (26);
3. Based on the pseudo strain value and experiment results, the value of soften function $C$ can be obtained by Eq. (25);
4. The initial value of $\alpha$ can be given based on the ratio of the logarithm of relaxation modulus to the logarithm of time, then the damage value $S$ can be calculated by Eq. (32), and C–S relationship can be ensured;
5. Put all of the C–S curves under different rates of extension in the same coordinate system, the value of $\alpha$ will be adjusted appropriately until the C–S curves have a good contact ratio;
6. According to the final C–S curve, fitting all of the $C(S)$ functions under different temperature states.

Specific fitting flow chart can be expressed by Fig. 1:

3. Experiment and validation

3.1. Experiment and parameters fitting

3.1.1. Experiment research

The material studied in this paper is Hydroxyl-Terminated Polybutadiene (HTPB) propellant, the test specimens were designed as a dumbbell-slat shape according to the aerospace industry standard of PRC, QJ 924-85, as illustrated in Fig. 2. In this paper, the same test specimen was used for relaxation as well as tensile strength test. Experimental facilities included Temperature Cabinet, QJ211B type Electronic Universal Testing Machines (EUTM), Data Acquisition System (DAS) and the special fixtures, as shown in Fig. 2.

To obtain the HTPB mechanic property under different loading conditions, the specimens which packaged hermetically will be put in Temperature Cabinet until the temperature state stabilized inside specimens, and then constant velocity tensile experiments can be conducted with the help of QJ211B EUTM. The temperature levels were selected as 223, 238, 253, 273, 293, 308 and 323 K, tensile velocities were 1, 5, 20, 100 and 500 mm/min, and the corresponding strain rates were 0.0002469, 0.001235, 0.004938, 0.02469 and 0.1235 s⁻¹. Each test was repeated at least 5 times,
curves for 100 mm/min (293 K) was shown in Fig. 3, the overstriking line is the average result.

According to the constant velocity tensile experiment results, we select 0.1 as the strain level for relaxation tests, at this point the mechanical property of HTPB propellant still obeys the linear viscoelastic theory. The master relaxation curves which expressed in Prony series were as shown in Fig. 4, and the time–temperature shift factors were shown in Fig. 5, the specific fitting process (Xu et al., 2013) is ignored in this paper.

3.1.2. Model parameter acquirement

According to the master relaxation modulus, the $\sigma$–$\varepsilon$ curves and C–t curves can be obtained. The initial value of $a$ was selected as 4, and the damage value $S$ can be calculated by Eqs. (31) and (32). Then, the C–S curves can be obtained, under any one temperature level, the value of $a$ will be adjusted appropriately until all of the C–S curves overlapped. Meanwhile, function $C(S)$ will be fitted to the form as follows:

$$C(S) = \frac{\exp(a \cdot S) + \exp(b \cdot S)}{2}$$

(34)

where, the model parameters $a$ and $b$ are functions of temperature, and have not physical meanings, just as $a = a(T), b = b(T)$. The
superposition results of $C-S$ results under different tensile rates are as shown in Fig. 6.

According to Eq. (34), the fitting curves expressed by solid lines are as shown in Fig. 6, the specific parameter values are given in Table 1. As shown here, the fitting results are good, while as a result of temperature influence, the material constant $a$ and model parameters $a$ and $b$ are the functions of $T$, as shown in Fig. 7, the $C(S)$ curves under different temperature levels are not overlapped. Through observation we find that, parameters $a$, $a$ and $b$ decreased exponentially with the temperature increase, and the fitted exponential functions are as shown in Fig. 7.

From Eq. (34), the partial derivative of soften function $C$ to damage parameter $S$ is

$$\frac{\partial C}{\partial S} = \frac{a \exp(a \cdot S) + b \exp(b \cdot S)}{2}$$

(35)

Then, according to Eq. (30), the updates of damage value $S$ is

$$\Delta S = \left[ - \frac{a \exp(a \cdot S) + b \exp(b \cdot S) (e^g)^2}{4} \right] \Delta t$$

(36)

So far we have obtained all of the model parameters and the updates of damage variable $S$, with the help of MATLAB, the stress response of the HTPB specimens under different temperatures and rates can be predicted.

3.2. Model validation

In this section, uniaxial tensile tests and tensile-relaxation tests will be used to validate the accuracy of the constitutive model. The specific validation process include pseudo strain calculation,
3.2.1. Uniaxial constant rate tensile tests

According to Eq. (26), the pseudo strain $e^R$ response of uniaxial constant rate tensile tests can be described as:

$$e^R = \frac{E_0}{E_1} t + \sum_{i=1}^{n} E_i \tau_i \left(1 - \exp \left( -\frac{t}{\tau_i} \right) \right)$$

(37)

Fig. 9 shows the comparison between the uniaxial constant rate tensile test results and the predicted curves under different temperature states. As shown in Fig. 9, the overlap ratio between test results and predicted curves are generally well but some forecast
error under specific loading conditions, this is because that there are large scattered errors in HTPB's mechanical property, which may caused the model parameters do not have insufficient accuracy. Besides, the C(S) functional form may also have to be improved. To solve the problems above, more test levels and more number of replications should be considered to reduce the negative influence of the large variance; then, according to the amount of test results, select the more suitable C(S) function form.

3.2.2. Multi-step tensile-relaxation tests

The multi-step tensile-relaxation verified experiments were tested in 323, 293 and 253 K, select the same specimen structure as that for uniaxial constant rate tensile tests. The speed for each tensile process is different, while the time for each relaxation process is 60 s). The specific experimental procedures are as follows:

1) Stretch specimen as the speed of 5 mm/min, hold on for \( t_1 \) seconds and stop it once the tensile displacement achieve 5 mm. At this moment, the strain is about 8%, and the HTPB specimen can still be considered as linear viscoelastic materials because of little damage inside. The pseudo strain \( \varepsilon^p \) response during this period can be described as:

\[
\varepsilon^p = \dot{\varepsilon}_1 \left[ E_T t + \sum_{i=1}^{n} E_i \tau_i \left(1 - \exp \left( -\frac{t}{\tau_i} \right) \right) \right] \quad 0 < t < t_1
\]  
(38)

2) Relax for 60 s (i.e. \( t_2 - t_1 = 60 \) s), through which the relaxation characteristics of linear viscoelastic can be verified. The pseudo strain \( \varepsilon^p \) response during this period can be described as:

\[
\varepsilon^p = \dot{\varepsilon}_1 \left[ E_T t + \sum_{i=1}^{n} E_i \tau_i \left(\exp \left( -\frac{t-t_1}{\tau_i} \right) - \exp \left( -\frac{t}{\tau_i} \right) \right) \right]
\]
\[ t_1 < t < t_2 \]  
(39)

3) Continue stretching specimen as the speed of 20 mm/min, hold on for \( t_3 - t_2 \) seconds and stop it once the tensile displacement achieve 20 mm. At this moment, the strain is about 30%, and the specimen becomes nonlinear viscoelastic materials due to lots of damage inside. The pseudo strain \( \varepsilon^p \) response during this period can be described as:

\[
\varepsilon^p = \dot{\varepsilon}_1 \left[ E_T t_1 + \sum_{i=1}^{n} E_i \tau_i \left(\exp \left( -\frac{t-t_1}{\tau_i} \right) - \exp \left( -\frac{t}{\tau_i} \right) \right) \right] + \dot{\varepsilon}_2 \left[ E_T (t_2 - t_1) + \sum_{i=1}^{n} E_i \tau_i \left(1 - \exp \left( -\frac{t-t_1}{\tau_i} \right) \right) \right] \quad \text{for} \ t_2 < t < t_3
\]

(40)

4) Relax for 60 s (i.e. \( t_4 - t_3 = 60 \) s), through which the relaxation characteristics of nonlinear viscoelastic can be verified. The pseudo strain \( \varepsilon^p \) response during this period can be described as:

\[
\varepsilon^p = \dot{\varepsilon}_1 \left[ E_T t_1 + \sum_{i=1}^{n} E_i \tau_i \left(\exp \left( -\frac{t-t_1}{\tau_i} \right) - \exp \left( -\frac{t}{\tau_i} \right) \right) \right] + \dot{\varepsilon}_2 \left[ E_T (t_2 - t_1) + \sum_{i=1}^{n} E_i \tau_i \left(\exp \left( -\frac{t-t_1}{\tau_i} \right) - \exp \left( -\frac{t-t_2}{\tau_i} \right) \right) \right] + \dot{\varepsilon}_3 \left[ E_T (t_4 - t_2) + \sum_{i=1}^{n} E_i \tau_i \left(1 - \exp \left( -\frac{t-t_2}{\tau_i} \right) \right) \right] \quad \text{for} \ t_3 < t < t_4
\]

(41)

5) Stretch the specimen to destruction as the speed of 100 mm/min. The pseudo strain \( \varepsilon^p \) response during this period can be described as:

\[
\varepsilon^p = \dot{\varepsilon}_1 \left[ E_T t_1 + \sum_{i=1}^{n} E_i \tau_i \left(\exp \left( -\frac{t-t_1}{\tau_i} \right) - \exp \left( -\frac{t}{\tau_i} \right) \right) \right] + \dot{\varepsilon}_2 \left[ E_T (t_2 - t_1) + \sum_{i=1}^{n} E_i \tau_i \left(\exp \left( -\frac{t-t_1}{\tau_i} \right) - \exp \left( -\frac{t-t_2}{\tau_i} \right) \right) \right] + \dot{\varepsilon}_3 \left[ E_T (t_4 - t_2) + \sum_{i=1}^{n} E_i \tau_i \left(1 - \exp \left( -\frac{t-t_2}{\tau_i} \right) \right) \right] + \dot{\varepsilon}_4 \left[ E_T (t_4 - t_2) + \sum_{i=1}^{n} E_i \tau_i \left(1 - \exp \left( -\frac{t-t_2}{\tau_i} \right) \right) \right] \quad \text{for} \ t > t_4
\]

(42)

Fig. 10. The validation of multi-step tensile-relaxation tests.
According to Eqs. (38)–(42), we can obtain the pseudo strain under each loading condition, then the stress response can be calculated by the Eqs. (36), (34), and (25). The prediction and test results are as shown in Fig. 10.

As shown in Fig. 10, the proposed constitutive model can describe the mechanical property of HTPB under multi-step tensile-relaxation conditions basically, since some errors during the second relaxation process. The damage increased slowly which even can be ignored during the first relaxation process, the phenomenon illustrates that HTPB specimen still obeyed the linear viscoelastic theory. However, the damage increased quickly during the second relaxation process, and HTPB specimen obeyed the non-linear viscoelastic mechanical property.

From Fig. 10, the damage value increased quickly during the second tensile process \( t_2 < t < t_3 \) while it is very slowly during the second relaxation process \( t_3 < t < t_4 \). But, in fact, the damage increased cannot be so slowly like this, it should be more quickly, and brought out larger damage value. Thus, the main weakness is that the damage evolution law used in this paper may not be suited greatly, caused the soften value \( C(S) \) is bigger than the actual value, and caused the stress prediction are bigger than the test results. In the future, the damage evolution model will be improved to take the relaxation process into consideration for the mechanical property of HTPB.

4. Conclusions

In this work, based on thermodynamic theory and elastic–viscoelastic correspondence principles, a thermo-damage-viscoelastic model of HTPB was proposed, and the model parameters fitting methods were studied on the basis of relaxation tests and one dimension constant rate tensile tests. The \( C(S) \) curves of different rates under the same temperature states were considered to be overlapped, while noncoincidence under the different temperature states. Thus, parameter \( x \) which represent the damage evolution rate and the material constants \( a, b \) were defined as exponential functions of temperature \( T \). Uniaxial constant rate tensile tests and multi-step tensile-relaxation tests were carried out to verify the accuracy of the thermo-damage-viscoelastic constitutive model. The results show that, the model proposed in this paper is highly accurate in describing the mechanical property of HTPB under various loading conditions, but have some drawbacks in describing the relaxation property inside the nonlinear viscoelastic segments. To improve the accuracy of the model, the damage evolution model should be improved by considering the damage influence from relaxation. Besides, more loading conditions of experiments and more times of repetition should be conducted to reduce the negative influence that caused by scattered errors of HTPB's mechanical property, as well as acquiring a more accuracy constitutive model. Furthermore, a more suitable \( C(S) \) functional form should be considered according to a mount of test results, then the prediction accuracy under different loading conditions can be resolved.

References

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