Multiple Fading Factors Kalman Filter for SINS Static Alignment Application

GAO Weixi a, MIAO Lingjuan a,*, NI Maolin b

a School of Automation, Beijing Institute of Technology, Beijing 100081, China
b National Key Laboratory of Science and Technology on Space Intelligent Control, Beijing Institute of Control Engineering, Beijing 100190, China

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Abstract

To solve the problem that the standard Kalman filter cannot give the optimal solution when the system model and stochastic information are unknown accurately, single fading factor Kalman filter is suitable for simple systems. But for complex systems with multi-variable, it may not be sufficient to use single fading factor as a multiplier for the covariance matrices. In this paper, a new multiple fading factors Kalman filtering algorithm is presented. By calculating the unbiased estimate of the innovation sequence covariance using fenestration, the fading factor matrix is obtained. Adjusting the covariance matrix of prediction error \( P_{k|k-1} \) using fading factor matrix, the algorithm provides different rates of fading for different filter channels. The proposed algorithm is applied to strapdown inertial navigation system (SINS) initial alignment, and simulation and experimental results demonstrate that, the alignment accuracy can be upgraded dramatically when the actual system noise characteristics are different from the pre-set values. The new algorithm is less sensitive to uncertainty noise and has better estimation effect of the parameters. Therefore, it is of significant value in practical applications.

Keywords: inertial navigation systems; strapdown; initial alignment; fading filter; multiple fading factors; fenestration

1. Introduction

Inertial navigation system is an autonomous one. Due to its small size, high precision, good concealment, the strapdown inertial navigation system (SINS) has developed rapidly in military and civilian fields in recent years. Today, SINS is being used more widely for the positioning and navigation of planes, ships, vehicles, and rockets, etc. Initial alignment is a key technology for SINS, and it directly affects the navigation performance of the system [1-6]. The purpose of the initial alignment is to get a coordinate transformation matrix from body frame to navigation frame, and drive the misalignment to zero. The process of initial alignment for SINS consists of two stages, the coarse alignment and the precise alignment. In the coarse alignment stage, a coordinate transformation matrix from body frame to navigation frame is approximately estimated through gravity vector \( g \) and the measurement value of the Earth’s rotation rate \( \Omega \). In the precise alignment stage, the small misalignment angles between reference frame and true frame are computed accurately through processing the information of various sensors, and the accurate initial transformation matrix is formulated. Kalman filtering algorithm is often used in this stage. At present, the method of continuous rotation alignment has become a hot research topic. This method can eliminate the constant drifts of the inertial sensors and improve the precision of alignment, but the rotation...
mechanism design increases the complexity and reduces the reliability of the system [1-2].

In the precise alignment process, the setting of Kalman filter parameters is very important. Before the setting of the filter parameters, the test of the inertial sensors is necessary. But in actual environmental conditions, the noise characteristics are often difficult to get accurately (test results are not consistent with the current environmental conditions). If we still use the test parameters, alignment accuracy may be affected, and the filter even gets to divergence, leading to the failure of the alignment.

Kalman filter is a kind of linear minimum variance estimator. When the system model is accurate and noise characteristics meet the Gaussian conditions, it is an optimal filter. For decades, this well-known Kalman filtering technique has been widely employed in inertial navigation, target tracking and industrial processes. However, if the prior statistical information is inadequate to represent the real statistic noise levels, the Kalman filter estimation is not optimal and may cause unreliable results, sometimes even lead to filtering divergence. Therefore, many scholars have proposed some improved Kalman filtering algorithms, such as adaptive Kalman filtering algorithm, $H_s$ filtering algorithm, etc [6-24].

Fading Kalman filtering algorithm is a kind of adaptive Kalman filtering algorithm. By using exponential fading of past data via factor $\lambda$, a method to limit the memory of the Kalman filter is initiated in Refs. [7]-[8]. In Ref. [9], a variable fading factor algorithm is proposed, in which fading factors are determined based on “memory length”. Rapid fading occurs when data give poor fit with the model, and slow fading for a good fit. Several years later, an optimal fading factor Kalman filtering algorithm is presented, which uses the variable exponential weighting approach to compensate the model errors and unknown drifts [10-13]. Ref. [14] shows the use of adaptive filtering techniques to improve the speed of the dynamic alignment of a micro-electro-mechanical systems inertial measurement unit (MEMS IMU) with real-time kinematic global positioning system (RTK GPS) for a marine application. In Ref. [15], real-time adaptive algorithms are applied to GPS data processing. Fading memory algorithm and variance component estimation adaptive algorithm are discussed. To avoid causing the problem that the matrix covariance of prediction error $P_{k|k-1}$ is asymmetric, an improved algorithm is proposed, which makes use of the filter residuals. Along with the statistical evaluation of the filter residuals using a Chi-square test, the fading factors are computed separately to increase the predicted variance components of the state vector [19-20]. Ref. [21] analyzes the stability of the adaptive fading extended Kalman filter (EKF). In Ref. [22], an adaptive unscented Kalman filter (UKF) with multiple fading factors-based gain correction is introduced and tested on the attitude estimation system of a pico-satellite by the use of simulations.

In this paper, multiple fading factors Kalman filter is proposed. Making use of the characteristics of the innovation sequence vector, the new algorithm calculates the fading factor of the corresponding channel adaptively. Any channel of the observation is independent of each other. It has been successfully applied to the SINS initial alignment.

The structure of the paper is as follows. Section 2 briefly introduces the SINS error model for the alignment. Detailed information is available in Refs. [3]-[4]. Section 3 presents the multiple fading factors Kalman filter. Computer simulation of testing algorithm is presented in Section 4. Section 5 gives the experimental results and discussion. Finally, Section 6 summarizes and concludes the work.

2. SINS Error Model for Alignment

2.1. Coordinate frames

Coordinate frames related in the case study of initial alignment of the SINS are described as follows [9].

Inertial frame (i frame): Its origin is at the Earth center; $z_i$ axis is normal to the equatorial plane; $x_i$ axis lies in equatorial plane, its direction can be specified arbitrarily; $y_i$ axis complements the right handed system.

Navigation frame (n frame): It is a local geographic coordinate frame: $x_n$ axis points towards east, and $y_n$ axis points towards north; $z_n$ axis is parallel to the upward vertical at the local Earth surface referenced position location.

Body frame (b frame): Its origin is at the centroid of body; $x_b$ axis is along transverse axis; $y_b$ axis is along longitudinal axis; $z_b$ axis is perpendicular to longitudinal plan of symmetry.

Earth fixed frame (e frame): It is a frame fixed to Earth and the origin is at its center. Its $z_e$ axis is coincident with the Earth’s polar axis while the other two axes are fixed to the Earth within the equatorial plane.

2.2. SINS error model

In the SINS error model [1-4], the state vector consists of 12 system states, i.e., three velocity errors, three attitude errors, three accelerometer biases, and three gyro drifts. The state equations in the navigation frame can be represented as

$$\ddot{X} = AX + Gw$$

where

$$X = [\delta V_E \ \delta V_N \ \delta V_U \ \phi_E \ \phi_N \ \phi_U \ V_x \ V_y \ V_z]$$

$$w = [w_x \ w_y \ w_z \ \omega_x \ \omega_y \ \omega_z]^T$$

$$A = \begin{bmatrix}
A_1 & A_2 & 0_{3x3} \\
0_{3x3} & A_3 & -C_b \n \end{bmatrix}$$

$$G = \begin{bmatrix}
0_{3x3} & A_4 & \ C_b \\
0_{3x3} & 0_{3x3} & 0_{3x3} \\
0_{3x3} & 0_{3x3} & 0_{3x3}
\end{bmatrix}$$
\[ A_\pi = \begin{bmatrix} 0 & 2\omega_{ag} \sin L & -2\omega_{ag} \cos L \\ -2\omega_{ag} \sin L & 0 & 0 \\ 2\omega_{ag} \cos L & 0 & 0 \end{bmatrix} \]

\[ A_\sigma = \begin{bmatrix} 0 & -g & 0 \\ g & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} C_b & 0_{b*3} \\ 0_{b*3} & C_b \end{bmatrix} \]

\[ A_\tau = \begin{bmatrix} 0 & \omega_{ag} \sin L & -\omega_{ag} \cos L \\ -\omega_{ag} \sin L & 0 & 0 \\ \omega_{ag} \cos L & 0 & 0 \end{bmatrix} \]

where \( \omega_{ag} \) is the Earth’s rotation rate, \( g \) the local acceleration of gravity, \( L \) the local latitude; \( \omega_{ag} \) and \( V_e \) are the biases of three accelerometers respectively, \( \varepsilon, \varepsilon_\pi \) and \( \varepsilon_\tau \) the drifts of three gyros respectively, \( \delta V_{\pi_e}, \delta V_{\pi_n}, \delta V_{\pi_iu} \) the local north and vertical velocity errors respectively, \( \phi_{\pi_e}, \phi_{\pi_n}, \phi_{\pi_iu} \) the local north and azimuth misalignment angles respectively; \( C_b \) is the transformation matrix between the body frame and the navigation frame, and \( w \) the system noise vector.

Taking the velocity errors as the observation, the corresponding observation equation is

\[ Z = HX + v \] (2)

where \( H = [I_{3*3}, 0_{b*3}] \), \( v = [V_e, V_n, V_l]^T \) is the observation noise vector.

### 3. Multiple Fading Factors Kalman Filter

#### 3.1. Development of single fading factor Kalman Filter

Discrete linear system state equation and observation equation can be expressed as follows [5]:

\[ X_k = \Phi_{k-1}X_{k-1} + \Gamma_{k-1}W_{k-1} \tag{3} \]

\[ Z_k = H_kX_k + V_k \tag{4} \]

where \( X_k \) is the state vector at epoch \( k \), \( \Phi_{k-1} \) the state transition matrix, \( \Gamma_{k-1} \) the system disturbance matrix, \( Z_k \) the observation at epoch \( k \), \( H_k \) the observation matrix, \( W_{k-1} \) the dynamic model noise vector, and \( V_k \) the observation noise vector.

When the system noise and measurement noise are uncorrelated Gaussian white noise, the standard Kalman filter equations are given as

\[ \hat{X}_{k|k-1} = \Phi_{k|k-1} \hat{X}_{k-1} \]

\[ \hat{X}_k = \hat{X}_{k|k-1} + K_k(Z_k - H_k \hat{X}_{k|k-1}) \]

\[ K_k = P_{k|k-1}H_k^T(H_kP_{k|k-1}H_k^T + R_k)^{-1} \]

\[ P_{k|k-1} = \Phi_{k|k-1}P_{k-1}\Phi_{k|k-1}^T + \Gamma_{k-1}Q_{k-1}\Gamma_{k-1}^T \]

\[ P_k = (I - K_kH_k)P_{k|k-1}(I - K_kH_k)^T + K_kR_kK_k^T \]

where \( \hat{X}_{k|k-1} \) is the predicted state vector, \( P_{k|k-1} \) the covariance matrix for \( \hat{X}_{k|k-1} \), \( K_k \) the gain matrix, \( \hat{X}_k \) the estimated state vector, \( P_k \) the covariance matrix for the estimated states, \( Q_k \) the system noise covariance matrix, \( R_k \) the measurement noise covariance matrix, and \( I \) the identity matrix.

When the system state model and observation model are accurate enough and the noise characteristics are known accurately, the Kalman filter will produce optimal estimates for system states. However, it is usually difficult to meet the above conditions in the actual system. The filter estimation depends highly upon the past data, and the system model degrades the observation information from the distant past, and the heavy reliance on the past data may cause state estimation to diverge. In order to overcome this problem, by using exponential fading of past data via factor \( \lambda_k \), a method to limit the memory of the Kalman filter is initiated [7-8]. Compared with standard Kalman filter equations, this single fading factor Kalman filter is different only in the time propagation error covariance equation, which can be expressed as

\[ P_{k|k-1} = \lambda_k\Phi_{k|k-1}P_{k-1}\Phi_{k|k-1}^T + \Gamma_{k-1}Q_{k-1}\Gamma_{k-1}^T \] (10)

where fading factor \( \lambda_k > 1 \).

When the unpredictable disturbance exists in the system, the fixed constant fading factor is often difficult to achieve the desired performance. A variable fading factor algorithm is proposed, in which fading factors are determined based on “memory length” [9]. Rapid fading occurs when data give poor fit with the model, and slow fading for a good fit. An optimal fading factor Kalman filtering algorithm is presented in Ref. [10], which can be expressed as Algorithm 1.

Algorithm 1 Give that

\[ A_k = \lambda_k H_k \Phi_{k|k-1}P_{k|k-1}\Phi_{k|k-1}^T \]

\[ M_k = H_k \Phi_{k|k-1}P_{k|k-1}H_k^T \]

\[ B_k = H_k \Gamma_{k-1}Q_{k-1}\Gamma_{k-1}^T + R_k \]

\[ N_k = \hat{\delta}_k - B_k \]

\[ J_k = \Phi_{k|k-1}P_{k|k-1}\Phi_{k|k-1}^T \]

where \( \hat{\delta}_k \) is the estimation of the innovation sequence covariance. Then, the optimal fading factor can be expressed as \( \lambda_k = \max\{1, \text{tr}(N_kM_k^{-1})/m\} \), \( m \) is the dimension of the observation vector.

In all these algorithms, covariance matrix is multiplied by only one fading factor. However, the estimation effect of each state is different, and these methods cannot keep the filter optimal absolutely, in some cases, even cause the divergence of the filter.

#### 3.2. Multiple fading factors Kalman filter

It is not enough to only use one fading factor to multiply the covariance matrix. In order to provide different rates of fading for different filter channels, we consider using the fading factors matrix instead of single
fading factor. At the same time, to avoid causing the problem that the matrix \( P_{k\mid k-1} \) is asymmetric, the improved time propagation error covariance equation can be expressed as follows \(^{[19,20]}\):

\[
P_{k\mid k-1} = A_k \Phi_\text{k-1} P_{k-1} \Phi^\text{T}_\text{k-1} A_k^T + \Gamma_k \Omega_k \Gamma^T_k + R_k \tag{11}
\]

where \( A_k \) is the fading factors matrix, which is the diagonal matrix.

When a filter is stable, the innovation sequence is the Gaussian white noise sequence \(^{[24]}\) and meets \( N(0, H_k P_{k\mid k-1} H^T_k + R_k) \) distribution. Considering Eq. (11), we can get

\[
\delta_{k} = H_k (A_k \Phi_\text{k-1} P_{k-1} \Phi^T_\text{k-1} A_k^T + \Gamma_k \Omega_k \Gamma^T_k + R_k) H^T_k + R_k = A_k + B_k \tag{12}
\]

where

\[
A_k = H_k A_k \Phi_\text{k-1} P_{k-1} \Phi^T_\text{k-1} A_k^T H^T_k
\]

\[
B_k = H_k \Gamma_k \Omega_k \Gamma^T_k H^T_k + R_k
\]

The unbiased estimate of the innovation sequence covariance can be expressed as

\[
\hat{\delta}_{k} = \frac{1}{k-1} \sum_{i=1}^{k} v_i v_i^T \tag{13}
\]

where \( v_i \) is the innovation sequence. Thus,

\[
\hat{\delta}_{k} = A_k + B_k \tag{14}
\]

Generally, observation matrix can often be expressed as the following form in navigation and positioning applications.

\[
H_k = [S_{\text{mean}} \ 0_{m \times (n-m)}]_{m \times n} \tag{15}
\]

where \( S_{\text{mean}} = \text{diag}(s_1, s_2, \ldots, s_m), m < n \).

If the observation matrix \( H_k \) satisfies Eq. (15), we can get

\[
A_k = H_k A_k \Phi_\text{k-1} P_{k-1} \Phi^T_\text{k-1} A_k^T H^T_k = [S_{\text{mean}} \ 0_{m \times (n-m)}] \begin{bmatrix} A_{\text{mean}} & 0_{m \times (n-m)} \\ 0_{(n-m) \times m} & A^2_{\text{mean}} \end{bmatrix} \tag{16}
\]

The fading factors may satisfy the following conditions:

\[
\lambda_i = \begin{cases} \max \{1, \sqrt{\frac{\delta_{k}}{\delta_{ii}}}, \delta_{ii} \} & \delta_{ii} > \delta_{ii} \\ 1 & \delta_{ii} \leq \delta_{ii} \end{cases} \tag{19}
\]

Then, we can get the following algorithm, which is described as Algorithm 2.

**Algorithm 2** If the observation matrix satisfies the Eq. (15), given that

\[
A_k = H_k A_k \Phi_\text{k-1} P_{k-1} \Phi^T_\text{k-1} A_k^T H^T_k
\]

\[
B_k = H_k \Gamma_k \Omega_k \Gamma^T_k H^T_k + R_k
\]

\[
N_k = \hat{\delta}_{k} - B_k
\]

\[
J_k = \Phi_\text{k-1} P_{k-1} \Phi^T_\text{k-1}
\]

Then, the optimal fading factors matrix can be expressed as \( A_{\text{mean}} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_m, 1, 1, \ldots, 1) \), and \( \lambda_i \) can be calculated according to Eq. (20).

In this algorithm, only \( \lambda_1, \lambda_2, \ldots, \lambda_m \) can be estimated adaptively, and other elements of the matrix cannot be estimated. This is determined by the dimension of the observation.

When the system noise or observation noise changes, according to the characteristic that any dimension of the innovation sequence is the white noise sequence, multiple fading factors Kalman filtering algorithm calculates the corresponding fading factor adaptively. Several fading factors make up of fading factors matrix. By adjusting the covariance matrix \( P_{k\mid k-1} \) using fading factors matrix, the objective of adjusting the gain matrix \( K_k \) is achieved.
3.3. Comparison of single fading factor Kalman filter and multiple fading factors Kalman filter

Algorithm 2 calculates the fading factors of each channel independently, so it is superior to other single factor fading algorithms.

**Theorem 1** Based on the conditions of Algorithm 1 and Algorithm 2, under the optimal criterion function

\[ J = \min \sum_{i=1}^{n}(U_{ii} - N_{ii})^2, \quad U_2 = H_iA_iJ_iA_i^TH_i^T \]

than \( U_1 = \lambda_i H_iJ_iH_i^T (\lambda_i > 1) \).

**Proof**

\[ J_1 = \sum_{i=1}^{m}(U_{ii}^1 - N_{ii})^2 \geq 0 \]

\[ J_2 = \sum_{i=1}^{m}(U_{ii}^2 - N_{ii})^2 = (a_{11} - N_{11})^2 + (a_{22} - N_{22})^2 + \cdots + (a_{mn} - N_{mn})^2 = 0 \]

where \( U_{ii}^1 \) and \( U_{ii}^2 \) are the \( i \)th diagonal elements of matrix \( U_1 \) and \( U_2 \) respectively. Thus, \( J_1 \geq J_2 \).

If the observation matrix satisfies Eq. (15), when the dimension of the observation is 1, Algorithm 1 and Algorithm 2 are equivalent; when the dimension of the observation is greater than 1, Algorithm 1 is to obtain the average fading factor of different observation, and the Algorithm 2 is to provide different rates of fading for different filter channels. Besides, according to Theorem 1, under the optimal criterion function \( J \), we get \( J_2 = 0 \). Thus, it is optimal.

In practical applications, \( \hat{\delta}_i \) can be estimated by fenestration:

\[ \hat{\delta}_i = \frac{1}{N}\sum_{j=0}^{N-k}v_{k-j}v_{k-j}^T \]

where \( N \) is the window width.

The selection of the window width is often difficult to determine. If \( N \) is too small, lacking historical information, the unbiased estimate of the innovation sequence covariance cannot be calculated; if \( N \) is too big, the amount of information is too large, and the short-term characteristics of the innovation sequence covariance are hard to reflect.

When using velocity errors as the observation, the observation matrix of the SINS alignment model can be expressed as \( H_i = [I_{3\times3}, 0_{1\times3}] \), so the optimal fading factors matrix can be expressed as

\[ A_i = \text{diag}\{\lambda_1, \lambda_2, \lambda_3, 1, 1, \ldots, 1\} \]

where

\[ \lambda_i = \max \left\{ 1, \frac{\delta_{e_i}^u - b_i}{J_i^u} \right\} \quad \delta_{e_i}^u > b_i \quad (i = 1, 2, 3) \]

\[ 1 \quad \delta_{e_i}^u \leq b_i \]

4. Computer Simulation

Simulation conditions are set as follows: for a moderate accuracy SINS, the constant and random drifts of each gyro are both chosen as 0.02 °/h; and the constant and random biases of each accelerometer are both chosen as 1×10⁻⁶ g; the true attitude angles of the system are 0°, 0°, 0°; the local latitude of SINS place is 39.96°. After the coarse alignment, the horizontal accuracy is 0.1°, and the azimuth accuracy is 0.3°, which meet the applicability demand of linear model.

In order to verify the effectiveness of the multiple fading factors Kalman filtering algorithm, due to the presence of external interference, we assume that between the interval 100-150 s, system noise \( Q = 10Q \); between the interval 200-250 s, system noise \( Q = 12Q \). Standard Kalman filtering algorithm, Algorithm 1, and Algorithm 2 are applied to SINS initial alignment respectively.

The initial state vector \( X(0) \) is assumed to be \( 0 \). The initial state covariance matrix is defined as

\[ P_0 = \text{diag}\{(0.1 \text{ m/s})^2, (0.1 \text{ m/s})^2, \}

\[ (1^\circ)^2, (1^\circ)^2, (1^\circ)^2, (0.02 (\circ)/h)^2, (0.02 (\circ)/h)^2, \]

\[ (0.02 (\circ)/h)^2, (1\times10^{-4} \text{ g})^2, (1\times10^{-4} \text{ g})^2 \}

System noise \( Q \) is set as \( R = \text{diag}\{(0.1 \text{ m/s})^2, (0.1 \text{ m/s})^2, \}

\[ (0.1 \text{ m/s})^2 \} \). When using Algorithm 1 and Algorithm 2, the window width \( N \) is taken as 100.

According to the observability analysis of the SINS error model on stationary base [3-5], two horizontal accelerometer biases \( V_{xi}, V_{yi} \) and the east gyro constant drift \( \delta e_k \) are inevitably unobservable. The east misalignment angle \( \phi_E \), north misalignment angle \( \phi_N \), and vertical accelerometer bias \( V_{zi} \) converge fast. Azimuth misalignment angle \( \phi_I \), and north gyro constant drift \( \delta a_n \) get convergent very slow. The estimation speed of the vertical gyro constant drift \( \delta a_z \) is the slowest, and the estimation effect is poor.

The observability of the system is determined by the system model, and it has nothing to do with the filtering algorithm. Therefore, different filtering algorithms cannot change the number of states which are observable. According to the results of the observability analysis above, part curves of the states \( V_z, \varepsilon_z, \phi_E, \phi_N \) and \( \phi_U \) which are observable are given. The estimation of \( V_z, \varepsilon_z \) and the estimation errors of \( \phi_E, \phi_N \), and \( \phi_U \) (\( \delta \phi_E, \delta \phi_N \) and \( \delta \phi_U \)) are shown in Figs. 1-5 respectively. The true attitude angles of the SINS are 0°, 0°, 0°, so \( V_z \) and \( \varepsilon_z \) are approximate to \( V_z \) and \( \varepsilon_z \) respectively.

From Figs. 1-5, we can conclude that

(1) When the external interference exists, using the standard Kalman filtering algorithm and Algorithm 1, the estimation effect of some state variables is bad, but it is perfect when using Algorithm 2. At the same time, from the curves of some state variables we can find that Algorithm 1 is no better than the Kalman filtering algorithm.
(2) With the external interference existing, it is clear that when estimating $V_U$ and $\phi_N$, there are a lot of disturbances with the standard Kalman filtering algorithm and Algorithm 1, but Algorithm 2 is more stable.

(3) Seeing from the final estimation accuracy of misalignment angles, we can find that the estimation error of $\phi_E$ and $\phi_N$ are nearly the same when using different algorithms (arc second level). However, when using the standard Kalman filtering algorithm and Algorithm 1, the final estimation accuracy of $\phi_U$ has deviated from the extreme accuracy which is determined by the constant drifts of the inertial sensors. Algorithm 2 can still have a perfect estimation effect. The azimuth alignment accuracy can be upgraded by 50% dramatically.

When the system noise changes, the standard Kalman filtering algorithm cannot calculate the gain matrix $K_t$ at present adaptively. When Algorithm 1 is applied to the SINS alignment, it aims to obtain the average fading factor of different observation (three velocity errors). The essence is that it takes the three-dimension innovation sequence vector as one-dimension white noise sequence. However, the influence of these three channels by the noise may be quite different. Therefore, this method of averaging will certainly affect the alignment results. In Algorithm 2, the fading factors are calculated separately, any channel of the observation is independent of each other, and adaptivity is enhanced significantly.
5. Experimentation Results and Discussion

To validate and corroborate the proposed algorithm in this paper, actual fiber optic gyroscope strapdown inertial navigation system (FOG-SINS) alignment test is conducted. FOG-SINS which is developed by Navigation Research Center, Beijing Institute of Technology is shown in Fig. 6. FOG-SINS contains three fiber optic gyros and three quartz accelerometers. After calibration and compensation, the constant drifts of the gyros are about 0.02 (°)/h, and the biases of the accelerometers are about 3×10^{-4} g.

![Fig. 6 FOG-SINS.](image)

FOG-SINS is placed on the test vehicle. This FOG-SINS has an in-house rotation mechanism, so it has a high alignment precision. Before and after collecting data, two alignment tests are carried out. Alignment results are shown in Table 1. The average of these four alignment results is considered as the true attitude angles.

<table>
<thead>
<tr>
<th>Number</th>
<th>Azimuth(°)</th>
<th>Pitch(°)</th>
<th>Roll(°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>95.693</td>
<td>-1.882</td>
<td>0.313</td>
</tr>
<tr>
<td>2</td>
<td>95.700</td>
<td>-1.874</td>
<td>0.312</td>
</tr>
<tr>
<td>3</td>
<td>95.693</td>
<td>-1.878</td>
<td>0.312</td>
</tr>
<tr>
<td>4</td>
<td>95.705</td>
<td>-1.876</td>
<td>0.312</td>
</tr>
<tr>
<td>Mean</td>
<td>95.698</td>
<td>-1.877</td>
<td>0.312</td>
</tr>
</tbody>
</table>

Fig. 7 Angular velocity measurement of gyros.

Fig. 8 Specific force measurement of accelerometers.

After the two alignment tests, data collection is needed for about 20 min. Between the interval 240-480 s, we keep the engine vibration. During other time, the vehicle engine is shut down. A total of five sets of data are collected. One of these five sets is shown in Figs. 7-8. Both figures illuminate that when keeping engine shut down or vibration, the outputs of the inertial sensors are quite different, especially for the outputs of gyros. The vehicle engine vibration brings a large number of system noises. These five sets of collected data are saved on the hard drive of computer.

After the data collection, alignment tests using these collected data are carried out. Standard Kalman filtering algorithm, Algorithm 1, and Algorithm 2 are applied to precise alignment respectively. The alignment results are summarized in Table 2.

<table>
<thead>
<tr>
<th>Number</th>
<th>Kalman filtering algorithm</th>
<th>Algorithm 1</th>
<th>Algorithm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Azimuth(°)</td>
<td>Pitch(°)</td>
<td>Roll(°)</td>
</tr>
<tr>
<td>1</td>
<td>95.714</td>
<td>-1.904</td>
<td>0.332</td>
</tr>
<tr>
<td>2</td>
<td>95.801</td>
<td>-1.872</td>
<td>0.325</td>
</tr>
<tr>
<td>3</td>
<td>95.967</td>
<td>-1.885</td>
<td>0.334</td>
</tr>
<tr>
<td>4</td>
<td>95.895</td>
<td>-1.893</td>
<td>0.333</td>
</tr>
<tr>
<td>5</td>
<td>95.940</td>
<td>-1.902</td>
<td>0.323</td>
</tr>
<tr>
<td>Error standard deviation</td>
<td>0.104</td>
<td>0.013</td>
<td>0.005</td>
</tr>
</tbody>
</table>
Table 2 clearly indicates that the new algorithm dramatically improves the alignment results by providing fast and accurate estimation of the attitude angles. The repeatability of azimuth accuracy is no more than 0.04°, and the repeatability of level accuracy is no more than 0.015°, which meets the requirements of terrestrial navigation vehicle. Nevertheless, when using other algorithms, the alignment accuracy is clearly poor.

6. Conclusions

Multiple fading factors Kalman filtering algorithm is presented in this paper. It uses the innovation sequence to compute multiple fading factors to scale the predicted covariance matrix. The new algorithm thoroughly considers both the optimism and convergence of the filter at the same time. The implementation process of the new algorithm is derived in detail, and theoretical analysis shows that this algorithm is superior to single fading Kalman filtering algorithm. It has been tested through computer simulation and practical experiment. Compared with the standard Kalman filter and the single factor fading Kalman filter, this algorithm has higher predictive accuracy. It overcomes the crucial shortcomings of the previous filtering algorithms and has strong robustness and adaptability.

References


Biographies:

GAO WEIXI Born in 1981, he received the B.S. degree (2006) from School of Information & Control Engineering, Weifang University, China. Now, he is a Ph.D. candidate in School of Automation, Beijing Institute of Technology. His main research interests include strapdown inertial navigation technology, INS/GPS integrated navigation, etc. E-mail: gaowei0811@163.com

MIAO LINGJUAN Born in 1965, she received the Ph.D. degree (2001) from China Academy of Launching-vehicle Technology. She is currently a professor in School of Automation, Beijing Institute of Technology. Her main research interests include GPS, inertial navigation system, INS/GPS integrated navigation and multi-sensor fusion technique. E-mail: miaolijingjuan@bit.edu.cn