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Compactifications of the $\mathcal{N} = 2^*$ flow

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Abstract

In [hep-th/0004063] Pilch and Warner (PW) constructed $\mathcal{N} = 2$ supersymmetric RG flow corresponding to the mass deformation of the $\mathcal{N} = 4$ $SU(N)$ Yang–Mills theory. In this Letter we present exact deformations of PW flow when the gauge theory 3-space is compactified on S^3. We consider also the case with the gauge theory world-volume being dS_4 instead of $R^{3,1}$. The solution is constructed in five-dimensional gauged supergravity and is further uplifted to 10d.

1. Introduction

Probably the most intriguing aspect of the gauge theory/string theory duality [1] (see [2] for a review) is the fact that it provides a dynamical principle for the nonperturbative definition of string theory in the asymptotically anti-de Sitter space–time, where there is no notion of an S-matrix. The best understood example of this duality is for the $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang–Mills theory. Given the original correspondence [1], new examples can be constructed by deforming the gauge theory by relevant operators. By now there is an extensive literature on such, renormalization group (RG) flow deformations [2]. In [3] it was suggested that the duality can be extended to cases when one deforms the gauge theory space–time. Furthermore, in [4,5] it was suggested that gauge theories on nondynamical de Sitter backgrounds might be relevant for understanding string theory in backgrounds with cosmological horizons. Unfortunately it is difficult to use space–time deformations of [3–5] for developing a detailed gauge/string theory duality map. The main problem stems from the fact that the examples considered there typically involve gauge theory with not well understood ultraviolet properties. It seems desirable to construct nontrivial examples of such deformations for "simpler" gauge theories in the UV.

Probably the simplest candidate is to consider space–time deformations of the massive $\mathcal{N} = 4$ RG flow. In this Letter we discuss how to construct such deformations for the $\mathcal{N} = 2^*$ RG flow of Pilch and Warner [6]. We should emphasize that though we concentrate on the flow [6], the construction presented here can be applied to other RG flows.

The Letter is organized as follows. In the next section we review the Pilch–Warner RG flow in five dimensions, and discuss it's S^3 and dS_4 deformations. In Section 3 we discussed the details of the 10d uplift of the deformations. We conclude in Section 4.

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2. $\mathcal{N}=2^*$ RG flow and its deformations in five dimensions

2.1. The gauge theory story

In the language of four-dimensional $\mathcal{N}=1$ supersymmetry, the mass deformed $\mathcal{N}=4$ $SU(N)$ Yang–Mills theory ($\mathcal{N}=2^*$) in $R^{3,1}$ consists of a vector multiplet V, an adjoint chiral superfield Φ related by $\mathcal{N}=2$ supersymmetry to the gauge field, and two additional adjoint chiral multiplets Q and $\tilde{Q}$ which form the $\mathcal{N}=2$ hypermultiplet. In addition to the usual gauge-invariant kinetic terms for these fields, the theory has additional interactions and hypermultiplet mass term summarized in the superpotential[1]

$$W=\frac{2\sqrt{2}}{g_{\rm YM}^2}\,{\rm Tr}\big([Q,\tilde{Q}]\Phi\big)+\frac{m}{g_{\rm YM}^2}\big({\rm Tr}\,Q^2+{\rm Tr}\,\tilde{Q}^2\big). \tag{1}$$

When $m=0$ the gauge theory is superconformal with $g_{\rm YM}$ characterizing an exactly marginal deformation. The theory has classical $3(N-1)$-complex-dimensional moduli space. This moduli space is protected by supersymmetry against (non)perturbative quantum corrections. With $m\neq 0$, the $\mathcal{N}=4$ supersymmetry is softly broken to $\mathcal{N}=2$. This mass deformation lifts $\{Q,\tilde{Q}\}$ hypermultiplet moduli directions, leaving the $(N-1)$-complex-dimensional Coulomb branch of the $\mathcal{N}=2$ $SU(N)$ Yang–Mills theory, parameterized by expectation values of the adjoint scalar

$$\Phi={\rm diag}(a_1,a_2,\ldots,a_N),\qquad \sum_i a_i=0, \tag{2}$$

in the Cartan subalgebra of the gauge group. For generic values of the moduli a_i the gauge symmetry is broken to that of the Cartan subalgebra $U(1)^{N-1}$, up to the permutation of individual $U(1)$ factors. Additionally, the superpotential (1) induces the RG flow of the gauge coupling. While from the gauge theory perspective it is straightforward to study this $\mathcal{N}=2^*$ gauge theory at any point on the Coulomb branch [7], the PW supergravity flow [6] corresponds to a particular Coulomb branch vacuum. More specifically, matching the probe computation in gauge theory and the dual PW supergravity flow it was argued in [8] that the appropriate Coulomb branch vacuum corresponds to a linear distribution of the VEVs (2) as

$$a_i\in[-a_0,a_0],\qquad a_0^2=\frac{m^2g_{\rm YM}^2N}{\pi}, \tag{3}$$

with (continuous in the large-N limit) linear number density

$$\rho(a)=\frac{2}{m^2g_{\rm YM}^2}\sqrt{a_0^2-a^2},\qquad \int_{-a_0}^{a_0}da\,\rho(a)=N. \tag{4}$$

Unfortunately, the extension of the $N=2^*$ gauge/gravity correspondence of [6,8,9] for vacua other than (4) is not known.

In [8,9] the dynamics of the gauge theory on the D3-brane probe in the PW background was studied in details. It was shown in [8] that the probe has one-complex-dimensional moduli space, with bulk induced metric precisely equal to the metric on the appropriate one-complex-dimensional submanifold of the $SU(N+1)$ $\mathcal{N}=2^*$ Donagi–Witten theory Coulomb branch. This one-dimensional submanifold is parameterized by the expectation value u of the $U(1)$ complex scalar on the Coulomb branch of the theory where $SU(N+1)\to U(1)\times SU(N)_{\rm PW}$, and the PW subscript denotes that the $SU(N)$ factor is in the Pilch–Warner vacuum (4). As u coincides with any of the a_i of the PW vacuum, the moduli space metric diverges, signaling the appearance of the additional massless states. Identical divergence is observed [8,9] for the probe D3-brane at the *enhancon* singularity of the PW background. Away from the singularity locus, $u=a\in[-a_0,a_0]$, the gauge theory computation of the probe moduli space metric is 1-loop exact. This is due to the suppression of instanton corrections in the large-N limit [8,10] of $N=2$ gauge theories.

Consider now the $R^{3,1}\to R\times S^3$ or $R^{3,1}\to dS_4$ deformations of the $N=2^*$ gauge theory. Both deformations introduce a new scale, let us call it μ, to the model—the S^3 scale in the former case and the Hubble parameter in the latter. Depending on the ratio μ/m we expect an interesting interplay between the strongly coupled $N=2^*$ IR dynamics and the IR curvature induced cutoff. For one reason, we expect that for the sufficiently high μ the number density distribution $\rho(a)$ should be just a δ-function at zero. In what follows we present and indication for this phase

[1] The classical Kähler potential is normalized $(2/g_{\rm YM}^2)\times{\rm Tr}[\bar{\Phi}\Phi+\bar{Q}Q+\bar{\tilde{Q}}\tilde{Q}]$.

transition while postponing the detailed analysis for the future.

2.2. PW RG flow

The gauge theory RG flow induced by the superpotential (1) corresponds to five-dimensional gauged SUGRA flow induced by scalars $\alpha \equiv \ln\rho$ and χ. The effective 5d action is

$$S = \int d\xi^5 \sqrt{-g}\left(\frac{1}{4}R - 3(\partial\alpha)^2 - (\partial\chi)^2 - \mathcal{P}\right), \tag{5}$$

where the potential $\mathcal{P}$ is[2]

$$\mathcal{P} = \frac{1}{48}\left(\frac{\partial W}{\partial\alpha}\right)^2 + \frac{1}{16}\left(\frac{\partial W}{\partial\chi}\right)^2 - \frac{1}{3}W^2, \tag{6}$$

with the superpotential

$$W = -\frac{1}{\rho^2} - \frac{1}{2}\rho^4\cosh(2\chi). \tag{7}$$

The PW geometry [6] has the flow metric

$$ds_5^2 = e^{2A}\left(-dt^2 + d\bar{x}^2\right) + dr^2. \tag{8}$$

The scalar equations of motion and the Einstein equations can be reduced to the first order equations

$$\frac{d\alpha}{dr} = \frac{1}{12}\frac{\partial W}{\partial\alpha},\qquad \frac{d\chi}{dr} = \frac{1}{4}\frac{\partial W}{\partial\chi},\qquad \frac{dA}{dr} = -\frac{1}{3}W. \tag{9}$$

2.2.1. Asymptotics of the PW flow

Given the explicit solution of the flow equations (9) in [6] is it easy to extract the UV/IR asymptotics. In the ultraviolet, $r \to +\infty$, we find

$$\text{UV:}\quad \rho \to 1_-,\quad \chi \to 0_+,\quad A \to \frac{1}{2}r, \tag{10}$$

while in the infrared, $r \to 0$

$$\text{IR:}\quad \rho \to 0_+,\quad \chi \to +\infty,\quad A \to -\frac{8}{3}\chi. \tag{11}$$

[2] We set the 5d gauged SUGRA coupling to one. This corresponds to setting S^5 radius $L = 2$.

2.3. Deformations of the PW flow

Unlike the PW flow, the deformed flows break the supersymmetry and are given by second order equations. From (5) we have Einstein equations

$$\frac{1}{4}R_{\mu\nu} = 3\partial_\mu\alpha\partial_\nu\alpha + \partial_\mu\chi\partial_\nu\chi + \frac{1}{3}g_{\mu\nu}\mathcal{P}, \tag{12}$$

plus the scalar equations

$$\begin{aligned} 0 &= \frac{6}{\sqrt{-g}}\partial_\mu\left(g^{\mu\nu}\sqrt{-g}\,\partial_\mu\alpha\right) - \frac{\partial\mathcal{P}}{\partial\alpha},\\ 0 &= \frac{2}{\sqrt{-g}}\partial_\mu\left(g^{\mu\nu}\sqrt{-g}\,\partial_\mu\chi\right) - \frac{\partial\mathcal{P}}{\partial\chi}. \end{aligned} \tag{13}$$

We consider two deformations of the flow metric (8):

$$\begin{aligned} &\text{(a)}\quad ds_5^2 = e^{2A}\left(-dt^2 + e^{2B}\,dS_3^2\right) + dr^2,\\ &\text{(b)}\quad ds_5^2 = e^{2A}\left(-dt^2 + \cosh^2 t\,dS_3^2\right) + dr^2. \end{aligned} \tag{14}$$

In the first case from (12), (13) we find

$$\begin{aligned} 0 &= \alpha'' + (4A' + 3B')\alpha' - \frac{1}{6}\frac{\partial\mathcal{P}}{\partial\alpha},\\ 0 &= \chi'' + (4A' + 3B')\chi' - \frac{1}{2}\frac{\partial\mathcal{P}}{\partial\chi},\\ 0 &= B'' + 4A'B' + 3(B')^2 - 2e^{-2A-2B},\\ &\frac{1}{4}A'' + (A')^2 + \frac{3}{4}A'B' = -\frac{1}{3}\mathcal{P},\\ &-A'' - (A')^2 - \frac{3}{2}A'B' - \frac{3}{4}B'' - \frac{3}{4}(B')^2\\ &\quad = 3(\alpha')^2 + (\chi')^2 + \frac{1}{3}\mathcal{P}, \end{aligned} \tag{15}$$

while in case (b) we find

$$\begin{aligned} 0 &= \alpha'' + 4A'\alpha' - \frac{1}{6}\frac{\partial\mathcal{P}}{\partial\alpha},\\ 0 &= \chi'' + 4A'\chi' - \frac{1}{2}\frac{\partial\mathcal{P}}{\partial\chi},\\ &\frac{1}{4}A'' + (A')^2 - \frac{3}{4}e^{-2A} = -\frac{1}{3}\mathcal{P},\\ &-A'' - (A')^2 = 3(\alpha')^2 + (\chi')^2 + \frac{1}{3}\mathcal{P}. \end{aligned} \tag{16}$$

It is easy to check that above equations are consistent. Thus for the deformed flows we could use the same scalars as in the PW case.

2.3.1. *Asymptotics of the S^3 deformation*

The flow equations are given by (15). The nonsingular in the IR flows are represented by a two parameter $\{\rho_0 > 0, \chi_0\}$ Taylor series expansion[3]

$$e^A = 1 + \left(\sum_{i=1}^{\infty} \alpha_i r^{2i}\right),$$
$$e^B = r\left(1 + \sum_{i=1}^{\infty} b_i r^{2i}\right),$$
$$\rho = \rho_0 + \left(\sum_{i=1}^{\infty} \rho_i r^{2i}\right),$$
$$\chi = \chi_0 + \left(\sum_{i=1}^{\infty} \chi_i r^{2i}\right), \tag{17}$$

with the first terms being

$$a_1 = \frac{1}{24}\rho_0^{-4} + \frac{1}{12}\rho_0^2\cosh(2\chi_0) - \frac{1}{96}\rho_0^8\sinh^2(2\chi_0),$$
$$b_1 = -\frac{1}{36}\rho_0^{-4} - \frac{1}{18}\rho_0^2\cosh(2\chi_0) + \frac{1}{144}\rho_0^8\sinh^2(2\chi_0),$$
$$\rho_1 = \frac{1}{48}\rho_0^{-3} - \frac{1}{48}\rho_0^3\cosh(2\chi_0) + \frac{1}{96}\rho_0^9\sinh^2(2\chi_0),$$
$$\chi_1 = -\frac{1}{16}\rho_0^2\sinh(2\chi_0) + \frac{1}{128}\rho_0^8\sinh(4\chi_0). \tag{18}$$

We expect that for an appropriate choice of $\{\rho_0, \chi_0\}$ we recover the UV asymptotics (10). It is tempting to identify the 2 dimensionless parameters of the regular in the IR flow with the ratio of m/μ of the gauge theory (the χ_0 parameter), and the ρ_0 parameter as a characteristic of the brane distribution (similar to the enhancon scale a_0 in (4)) in the IR. Notice, that unlike PW flow, where $\chi \to +\infty$ in the IR, here it is consistent to choose[4] $\chi_0 = 0$. In fact $\chi(r) \equiv 0$ is a solution to (15).[5]

The nonsingular flows that asymptote to (10) would have a well defined (finite) mass, being a function of $\{\rho_0, \chi_0\}$, characterizing phases of the model.[6]

2.3.2. *Asymptotics of the dS_4 deformation*

The flow equations are given by (16). The nonsingular in the IR flows are represented by a two parameter $\{\rho_0 > 0, \chi_0\}$ Taylor series expansion

$$e^A = r\left(1 + \sum_{i=1}^{\infty} a_i r^{2i}\right),$$
$$\rho = \rho_0 + \left(\sum_{i=1}^{\infty} \rho_i r^{2i}\right),$$
$$\chi = \chi_0 + \left(\sum_{i=1}^{\infty} \chi_i r^{2i}\right), \tag{19}$$

with the first terms being

$$a_1 = \frac{1}{72}\rho_0^{-4} + \frac{1}{36}\rho_0^2\cosh(2\chi_0) - \frac{1}{288}\rho_0^8\sinh^2(2\chi_0),$$
$$\rho_1 = \frac{1}{60}\rho_0^{-3} - \frac{1}{60}\rho_0^3\cosh(2\chi_0) + \frac{1}{120}\rho_0^9\sinh^2(2\chi_0),$$
$$\chi_1 = -\frac{1}{20}\rho_0^2\sinh(2\chi_0) + \frac{1}{160}\rho_0^8\sinh(4\chi_0). \tag{20}$$

As in the case of the S^3 deformation it is also consistent here to choose $\chi(r) \equiv 0$.

3. The ten-dimensional solutions

3.1. *Type IIB SUGRA equations of motion*

We use mostly positive convention for the signature $(- + \cdots +)$ and $\epsilon_{1\cdots 10} = +1$. The type IIB equations consist of [11]:

- The Einstein equations:

$$R_{MN} = T^{(1)}_{MN} + T^{(3)}_{MN} + T^{(5)}_{MN}, \tag{21}$$

[3] Without loss of generality we set $A|_{r=0} = 0$. This corresponds to rescaling the time coordinate in (14).

[4] We would like to interpret $\chi_0 = 0$ flow as a supergravity dual to the $\mathcal{N} = 2^*$ flow induced by the $\mathcal{N} = 4$ scalar expectation values. Typically, scalar expectation value does not give rise to an RG flow. Since these scalars are conformal (and thus couple to the S^3 curvature), given them an expectation value would induce a flow.

[5] Also, $\chi(r) \equiv 0$ and $\rho(r) \equiv 1$ is a trivial solution corresponding to the global AdS_5.

[6] The details of the phase structure will be discussed elsewhere.

where the energy momentum tensors of the dilaton/axion field, $\mathcal{B}$, the three index antisymmetric tensor field, $F_{(3)}$, and the self-dual five-index tensor field, $F_{(5)}$, are given by

$$T^{(1)}_{MN} = P_M P_N{}^* + P_N P_M{}^*, \tag{22}$$

$$T^{(3)}_{MN} = \frac{1}{8}\biggl(G^{PQ}{}_M G^*_{PQN} + G^{*PQ}{}_M G_{PQN} - \frac{1}{6} g_{MN} G^{PQR} G^*_{PQR}\biggr), \tag{23}$$

$$T^{(5)}_{MN} = \frac{1}{6} F^{PQRS}{}_M F_{PQRSN}. \tag{24}$$

In the unitary gauge $\mathcal{B}$ is a complex scalar field and

$$P_M = f^2 \partial_M \mathcal{B}, \qquad Q_M = f^2 \operatorname{Im}\bigl(\mathcal{B}\partial_M \mathcal{B}^*\bigr), \tag{25}$$

with

$$f = \frac{1}{(1 - \mathcal{B}\mathcal{B}^*)^{1/2}}, \tag{26}$$

while the antisymmetric tensor field $G_{(3)}$ is given by

$$G_{(3)} = f\bigl(F_{(3)} - \mathcal{B}F^*_{(3)}\bigr). \tag{27}$$

- The Maxwell equations:

$$\bigl(\nabla^P - iQ^P\bigr)G_{MNP} = P^P G^*_{MNP} - \frac{2}{3} i F_{MNPQR} G^{PQR}. \tag{28}$$

- The dilaton equation:

$$\bigl(\nabla^M - 2iQ^M\bigr)P_M = -\frac{1}{24} G^{PQR} G_{PQR}. \tag{29}$$

- The self-dual equation:

$$F_{(5)} = \star F_{(5)}. \tag{30}$$

In addition, $F_{(3)}$ and $F_{(5)}$ satisfy Bianchi identities which follow from the definition of those field strengths in terms of their potentials:

$$F_{(3)} = dA_{(2)},$$
$$F_{(5)} = dA_{(4)} - \frac{1}{8} \operatorname{Im}\bigl(A_{(2)} \wedge F^*_{(3)}\bigr). \tag{31}$$

For the 10d uplift of the RG flows in the 5d gauged SUGRA the metric ansatz and the dilaton is basically determined by group theoretical properties of the $d = 5$ $\mathcal{N} = 8$ scalars, and thus must be the same for both the deformed and original PW flows. Specifically, we assume [6] that the 10d Einstein frame metric is

$$ds^2_{10} = \Omega^2 ds^2_5 + 4\frac{(cX_1X_2)^{1/4}}{\rho^3} \times \biggl(c^{-1}d\theta^2 + \rho^6 \cos^2\theta\biggl(\frac{\sigma_1^2}{cX_2} + \frac{\sigma_2^2 + \sigma_3^2}{X_1}\biggr) + \sin^2\theta \frac{d\phi^2}{X_2}\biggr), \tag{32}$$

where ds^2_5 is either the original PW flow metric (8) or its deformations (14), $c \equiv \cosh(2\chi)$. The warp factor is given by

$$\Omega^2 = \frac{(cX_1X_2)^{1/4}}{\rho}, \tag{33}$$

and the two functions X_i are defined by

$$X_1(r,\theta) = \cos^2\theta + \rho(r)^6 \cosh\bigl(2\chi(r)\bigr)\sin^2\theta,$$
$$X_2(r,\theta) = \cosh\bigl(2\chi(r)\bigr)\cos^2\theta + \rho(r)^6 \sin^2\theta. \tag{34}$$

As usual, σ_i are the $SU(2)$ left-invariant forms normalized so that $d\sigma_i = 2\sigma_j \wedge \sigma_k$. For the dilaton/axion we have

$$f = \frac{1}{2}\biggl(\biggl(\frac{cX_1}{X_2}\biggr)^{1/4} + \biggl(\frac{cX_1}{X_2}\biggr)^{-1/4}\biggr),$$
$$f\mathcal{B} = \frac{1}{2}\biggl(\biggl(\frac{cX_1}{X_2}\biggr)^{1/4} - \biggl(\frac{cX_1}{X_2}\biggr)^{-1/4}\biggr)e^{2i\phi}. \tag{35}$$

The consistent truncation ansatz does not specify the (3-) 5-form fluxes. As in [6] we assume the most general ansatz allowed by the global symmetries of the background

$$A_{(2)} = e^{i\phi}\bigl(a_1(r,\theta)\,d\theta \wedge \sigma_1 + a_2(r,\theta)\sigma_2 \wedge \sigma_3 + a_3(r,\theta)\sigma_1 \wedge d\phi + a_4(r,\theta)\,d\theta \wedge d\phi\bigr), \tag{36}$$

where $a_i(r,\theta)$ are arbitrary complex functions. For the 5-form flux we assume

(a) $F_5 = \mathcal{F} + \star\mathcal{F}$,
$\mathcal{F} = dt \wedge \mathrm{vol}_{S^3} \wedge d\omega$,

(b) $F_5 = \mathcal{F} + \star\mathcal{F}$,
$\mathcal{F} = \cosh^3 t\, dt \wedge \mathrm{vol}_{S^3} \wedge d\omega$, (37)

where $\omega(r,\theta)$ is an arbitrary function.

We will do all the computation in the natural orthonormal frame given by

$$
\begin{aligned}
&e^1 \propto dt, \quad e^2 \propto dr, \quad e^3 \propto \tilde{\sigma}_1, \quad e^4 \propto \tilde{\sigma}_2,\\
&e^5 \propto \tilde{\sigma}_3, \quad e^6 \propto d\theta, \quad e^7 \propto \sigma_1, \quad e^8 \propto \sigma_2,\\
&e^9 \propto \sigma_3, \quad e^{10} \propto d\phi,
\end{aligned} \tag{38}
$$

where $\tilde{\sigma}_i$ are again $SU(2)$ left-invariant one forms, such that the round S^3 metric of unit radius is $(dS^3)^2 = \sum \tilde{\sigma}_i^2$.

As in the PW case, examination of the Einstein equations reveals that 2-form potential functions a_i have the following properties: $a_4 \equiv 0$, a_1, a_2 are pure imaginary, and a_3 is real.

3.2. Lift of S^3 deformation

Explicitly computing Ricci tensor with above ansatz, we find nonvanishing components $R_{11}, R_{22}, R_{33} = R_{44} = R_{55}, R_{66}, R_{77}, R_{88} = R_{99}, R_{1010}, R_{26} = R_{62}$. Given the 5d flow equations (15), we find relations

$$
\begin{aligned}
&R_{77} + R_{88} = 2R_{11},\\
&R_{11} + R_{33} = 0.
\end{aligned} \tag{39}
$$

The 3-form energy–momentum tensor has nontrivial components $T^{(3)}_{11} = -T^{(3)}_{33} = -T^{(3)}_{44} = -T^{(3)}_{55}, T^{(3)}_{22}, T^{(3)}_{66}, T^{(3)}_{77}, T^{(3)}_{88} = T^{(3)}_{99}, T^{(3)}_{1010}, T^{(3)}_{26} = T^{(3)}_{62}$. The nonvanishing components of the dilaton/axion energy–momentum tensor are $T^{(1)}_{22}, T^{(1)}_{66}, T^{(1)}_{1010}, T^{(1)}_{26} = T^{(1)}_{62}$. Finally, the 5-form energy–momentum tensor has nonvanishing components

$$
\begin{aligned}
&T^{(5)}_{11} = -T^{(5)}_{33} = -T^{(5)}_{44} = -T^{(5)}_{55} = T^{(5)}_{77} = T^{(5)}_{88}\\
&\quad = T^{(5)}_{99} = T^{(5)}_{1010} = \mathcal{A}_1^2 + \mathcal{A}_2^2,\\
&T^{(5)}_{22} = -T^{(5)}_{66} = \mathcal{A}_2^2 - \mathcal{A}_1^2,\\
&T^{(5)}_{26} = T^{(5)}_{62} = 2\mathcal{A}_1\mathcal{A}_2,
\end{aligned} \tag{40}
$$

where

$$
\mathcal{A}_1 \propto \frac{\partial\omega}{\partial r}, \qquad \mathcal{A}_2 \propto \frac{\partial\omega}{\partial\theta}. \tag{41}
$$

Besides Einstein equations, we have nontrivial 5-form Bianchi identity, dilaton/axion equation (29), and 4 equations from the Maxwell equation (28) for components $\{MN\} = \{27, 67, 710, 89\}$.

As in [6] we find the following consistency checks on the metric and dilaton/axion ansatz:[7]

$$
\begin{aligned}
&T^{(3)}_{1010} - T^{(3)}_{11} = \frac{e^{-2i\phi}}{24} G_{MNP} G^{MNP},\\
&R_{1010} - R_{11} = 2|P_{10}|^2 - e^{-2i\phi}\left(\nabla^M - 2iQ^M\right)P_M.
\end{aligned} \tag{42}
$$

Next combination is

$$
R_{1010} - R_{77} - 2|P_{10}|^2 = T^{(3)}_{1010} - T^{(3)}_{77}. \tag{43}
$$

As in [6], we find that (43) (and the linearized solution of all equations in the UV) is satisfied provided[8]

$$
\begin{aligned}
&a_1 = -i4\tanh(2\chi)\cos\theta,\\
&a_2 = i4\frac{\rho^6\sinh(2\chi)}{X_1}\sin\theta\cos^2\theta,\\
&a_3 = -4\frac{\sinh(2\chi)}{X_2}\sin\theta\cos^2\theta.
\end{aligned} \tag{44}
$$

Finally, from the $\{MN\} = \{11, 22\}$ Einstein equations we find

$$
\begin{aligned}
\frac{\partial\omega}{\partial\theta} &= -\frac{3}{2}e^{4A+3B}(\ln\rho)'\sin 2\theta,\\
\frac{\partial\omega}{\partial r} &= \frac{1}{8}e^{4A+3B}\frac{1}{\rho^4}\left(-\rho^{12}\sinh^2(2\chi)\sin^2\theta\right.\\
&\quad + 2\rho^6\cosh(2\chi)\left(1+\sin^2\theta\right)\\
&\quad \left. + 2\cos^2\theta\right).
\end{aligned} \tag{45}
$$

We explicitly verified that supplementing the metric and the dilaton/axion ansatz of the previous section with (44), (45), and the 5d flow equations (15), all the equations of 10d type IIB supergravity are satisfied.

3.3. Lift of dS_4 deformation

In this case the analysis are similar to those in the previous section. Thus we present only the results. First, we find the same complex functions a_i, specifying the 2-form potential (36)

$$
\begin{aligned}
&a_1 = -i4\tanh(2\chi)\cos\theta,\\
&a_2 = i4\frac{\rho^6\sinh(2\chi)}{X_1}\sin\theta\cos^2\theta,\\
&a_3 = -4\frac{\sinh(2\chi)}{X_2}\sin\theta\cos^2\theta.
\end{aligned} \tag{46}
$$

[7] There is a typo in the second equation in (42) in [6] (Eq. (4.3)).

[8] Note that there is a sign typo for a_3 in the corresponding equations in [6], (Eq. (4.8)).

Second, the ω in the 5-form potential (37) is

$$\frac{\partial\omega}{\partial\theta} = -\frac{3}{2}e^{4A}(\ln\rho)'\sin 2\theta,$$
$$\frac{\partial\omega}{\partial r} = \frac{1}{8}e^{4A}\frac{1}{\rho^4}\big(-\rho^{12}\sinh^2(2\chi)\sin^2\theta + 2\rho^6\cosh(2\chi)\big(1+\sin^2\theta\big) + 2\cos^2\theta\big). \tag{47}$$

4. Conclusion

In this Letter we observed that certain 5d gauged supergravity flows on the background $R^{3,1} \times R_+$ can be deformed to flows on backgrounds $S^3 \times R \times R_+$ or $dS_4 \times R_+$ with the *same* 5d scalars. If the 10-dimensional lift of the original backgrounds is known, this implies that deformed flows can be uplifted to ten dimensions as well. We explicitly demonstrated this for the $\mathcal{N} = 2^*$ PW flow, constructing for the first time massive RG flow with asymptotically global AdS_5 geometry. We hope that study of these backgrounds would help develop gauge/gravity dictionary for gauge theories in curved space–time, including dS_4 deformations which might be relevant for understanding strings in backgrounds with cosmological horizons [4,5].

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