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Soft BCK/BCI-algebras

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ABSTRACT

Molodtsov [D. Molodtsov, Soft set theory — First results, Comput. Math. Appl. 37 (1999) 19–31] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. In this paper we apply the notion of soft sets by Molodtsov to the theory of BCK/BCI-algebras. The notion of soft BCK/BCI-algebras and soft subalgebras are introduced, and their basic properties are derived.

1. Introduction

To solve complicated problems in economics, engineering, and environment, we can't successfully use classical methods because of various uncertainties typical for those problems. There are three theories: the theory of probability, the theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties can't be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as the probability theory, the theory of (intuitionistic) fuzzy sets, the theory of vague sets, the theory of interval mathematics, and the theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [1]. Molodtsov [1] and Maji et al. [2] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov [1] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [2] described the application of soft set theory to a decision making problem. Maji et al. [3] also studied several operations on the theory of soft sets. Chen et al. [4] presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. The algebraic structure of set theories dealing with uncertainties has been studied by some authors. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh [5]. The author (together with colleagues) applied the fuzzy set theory to BCK-algebras [6,7], BCC-algebras [8], B-algebras [9], hyper BCK-algebras [10], MTL-algebras [11], hemirings [12], implicative algebras [13], lattice implication algebras [14], and incline algebras [15]. In this paper we apply the notion of soft sets by Molodtsov to the theory of BCK/BCI-algebras. We introduce the notion of soft BCK/BCI-algebras and soft subalgebras, and then derive their basic properties.

2. Basic results on BCK/BCI-algebras

A BCK/BCI-algebra is an important class of logical algebras introduced by K. Iséki and was extensively investigated by several researchers.

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An algebra $(X; *, 0)$ of type $(2, 0)$ is called a *BCI-algebra* if it satisfies the following conditions:

(I) $(\forall x, y, z \in X)(((x * y) * (x * z)) * (z * y) = 0)$,
(II) $(\forall x, y \in X)((x * (x * y)) * y = 0)$,
(III) $(\forall x \in X)(x * x = 0)$,
(IV) $(\forall x, y \in X)(x * y = 0, y * x = 0 \Rightarrow x = y)$.

If a BCI-algebra X satisfies the following identity:

(V) $(\forall x \in X)(0 * x = 0)$,

then X is called a *BCK-algebra*. Any BCK-algebra X satisfies the following axioms:

(a1) $(\forall x \in X)(x * 0 = x)$,
(a2) $(\forall x, y, z \in X)(x \leq y \Rightarrow x * z \leq y * z, z * y \leq z * x)$,
(a3) $(\forall x, y, z \in X)((x * y) * z = (x * z) * y)$,
(a4) $(\forall x, y, z \in X)((x * z) * (y * z) \leq x * y)$,

where $x \leq y$ if and only if $x * y = 0$. A BCK-algebra X is s said to be *commutative* if $x \wedge y = y \wedge x$ for all $x, y \in X$ where $x \wedge y = y * (y * x)$. A commutative BCK-algebra will be written by cBCK-algebra for short. A nonempty subset S of a BCK/BCI-algebra X is called a *BCK/BCI-subalgebra* of X if $x * y \in S$ for all $x, y \in S$. A mapping $f : X \to Y$ of BCK/BCI-algebras is called a *homomorphism* if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$. For a homomorphism $f : X \to Y$ of BCK/BCI-algebras, the *kernel* of f, denoted by $\ker(f)$, is defined to be the set

$$\ker(f) = \{x \in X \mid f(x) = 0\}.$$

Let X be a BCK/BCI-algebra. A fuzzy set $\mu : X \to [0, 1]$ is called a *fuzzy subalgebra* of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in X$. We refer the reader to the paper [17] and book [18] for further information regarding BCK/BCI-algebras.

3. Basic results on soft sets

Molodtsov [1] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $\mathscr{P}(U)$ denotes the power set of U and $A \subset E$.

Definition 3.1 (*[1]*). A pair (F, A) is called a *soft set* over U, where F is a mapping given by

$$F : A \to \mathscr{P}(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A). Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [1].

Definition 3.2 (*[3]*). Let (F, A) and (G, B) be two soft sets over a common universe U. The *intersection* of (F, A) and (G, B) is defined to be the soft set (H, C) satisfying the following conditions:

(i) $C = A \cap B$,
(ii) $(\forall e \in C)\,(H(e) = F(e)$ or $G(e)$, (as both are same set)).

In this case, we write $(F, A)\widetilde{\cap}(G, B) = (H, C)$.

Definition 3.3 (*[3]*). Let (F, A) and (G, B) be two soft sets over a common universe U. The *union* of (F, A) and (G, B) is defined to be the soft set (H, C) satisfying the following conditions:

(i) $C = A \cup B$,
(ii) for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B, \\ G(e) & \text{if } e \in B \setminus A, \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

In this case, we write $(F, A)\widetilde{\cup}(G, B) = (H, C)$.

Definition 3.4 (*[3]*). If (F, A) and (G, B) are two soft sets over a common universe U, then "(F, A) *AND* (G, B)" denoted by $(F, A)\widetilde{\wedge}(G, B)$ is defined by $(F, A)\widetilde{\wedge}(G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$ for all $(\alpha, \beta) \in A \times B$.

Definition 3.5 (*[3]*). If (F, A) and (G, B) are two soft sets over a common universe U, then "(F, A) *OR* (G, B)" denoted by $(F, A)\widetilde{\vee}(G, B)$ is defined by $(F, A)\widetilde{\vee}(G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cup G(\beta)$ for all $(\alpha, \beta) \in A \times B$.

Definition 3.6 (*[3]*). For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a *soft subset* of (G, B), denoted by $(F, A)\widetilde{\subset}(G, B)$, if it satisfies:

(i) $A \subset B$,
(ii) For every $\varepsilon \in A$, $F(\varepsilon)$ and $G(\varepsilon)$ are identical approximations.

4. Soft BCK/BCI-algebras

In what follows, let X and A be a BCK/BCI-algebra and a nonempty set, respectively, and R will refer to an arbitrary binary relation between an element of A and an element of X, that is, R is a subset of $A \times X$ unless otherwise specified. A set-valued function $F : A \to \mathscr{P}(X)$ can be defined as $F(x) = \{y \in X \mid xRy\}$ for all $x \in A$. The pair (F, A) is then a soft set over X. For any element x of a BCI-algebra X, we define the order of x, denoted by $o(x)$, as

$$o(x) = \min\{n \in \mathbb{N} \mid 0 * x^n = 0\}.$$

Definition 4.1. Let (F, A) be a soft set over X. Then (F, A) is called a *soft BCK/BCI-algebra* over X if $F(x)$ is a BCK/BCI-subalgebra of X for all $x \in A$.

Let us illustrate this definition using the following examples.

Example 4.2. Let $X = \{0, a, b, c, d\}$ be a BCK-algebra with the following Cayley table:

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	a	a
b	b	b	0	b	b
c	c	c	c	0	c
d	d	d	d	d	0

Let (F, A) be a soft set over X, where $A = X$ and $F : A \to \mathscr{P}(X)$ is a set-valued function defined by

$$F(x) = \{y \in X \mid xRy \Leftrightarrow y \in x^{-1}I\}$$

for all $x \in A$ where $I = \{0, a\}$ and $x^{-1}I = \{y \in X \mid x \wedge y \in I\}$. Then $F(0) = F(a) = X$, $F(b) = \{0, a, c, d\}$, $F(c) = \{0, a, b, d\}$, and $F(d) = \{0, a, b, c\}$ are BCK-subalgebras of X. Therefore (F, A) is a soft BCK-algebra over X.

Example 4.3. Consider a BCI-algebra $X = \{0, a, b, c\}$ with the following Cayley table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Let $A = X$ and let $F : A \to \mathscr{P}(X)$ be a set-valued function defined as follows:

$$F(x) = \{y \in X \mid xRy \Leftrightarrow y = x^n, n \in \mathbb{N}\}$$

for all $x \in A$ where $x^n = x*x*\cdots*x$ in which x appears n-times. Then $F(0) = \{0\}$, $F(a) = \{0, a\}$, $F(b) = \{0, b\}$ and $F(c) = \{0, c\}$ which are BCI-subalgebras of X. Hence (F, A) is a soft BCI-algebra. If we define a set-valued function $H : A \to \mathscr{P}(X)$ by

$$H(x) = \{y \in X \mid xRy \Leftrightarrow o(x) = o(y)\}$$

for all $x \in A$, then $H(0) = \{0\}$ is a BCI-subalgebra of X, but $H(a) = H(b) = H(c) = \{a, b, c\}$ is not a BCI-subalgebra of X. This shows that there exists a set-valued function $H : A \to \mathscr{P}(X)$ such that the soft set (H, A) is not a soft BCI-algebra over X.

Example 4.4. Let $X = \{0, a, b, c, d, e, f, g\}$ and consider the following Cayley table:

$*$	0	a	b	c	d	e	f	g
0	0	0	0	0	d	d	d	d
a	a	0	0	0	e	d	d	d
b	b	b	0	0	f	f	d	d
c	c	b	a	0	g	f	e	d
d	d	d	d	d	0	0	0	0
e	e	d	d	d	a	0	0	0
f	f	f	d	d	b	b	0	0
g	g	f	e	d	c	b	a	0

Then, $(X; *, 0)$ is a BCI-algebra (see [16]). Let (F, A) be a soft set over X, where $A = X$ and $F : A \to \mathscr{P}(X)$ is a set-valued function defined as follows:

$$F(x) = \{0\} \cup \{y \in X \mid xRy \Leftrightarrow o(x) = o(y)\}$$

for all $x \in A$. Then $F(0) = F(a) = F(b) = F(c) = \{0, a, b, c\}$ is a BCI-subalgebra of X, but $F(d) = F(e) = F(f) = F(g) = \{0, d, e, f, g\}$ is not a BCI-subalgebras of X. Hence (F, A) is not a soft BCI-algebra over X. If we take $B = \{a, b, c\} \subset X$ and define a set-valued function $G : B \to \mathscr{P}(X)$ by

$$G(x) = \{y \in X \mid xRy \Leftrightarrow o(x) = o(y)\}$$

for all $x \in B$, then (G, B) is a soft BCI-algebra over X since $G(a) = G(b) = G(c) = \{0, a, b, c\}$ is a BCI-subalgebra of X.

Let A be a fuzzy BCK/BCI-subalgebra of X with membership function μ_A. Let us consider the family of α-level sets for the function μ_A given by

$$F(\alpha) = \{x \in X \mid \mu_A(x) \geq \alpha\}, \alpha \in [0, 1].$$

Then, $F(\alpha)$ is a BCK/BCI-subalgebra of X. If we know the family F, we can find the functions $\mu_A(x)$ by means of the following formula:

$$\mu_A(x) = \sup\{\alpha \in [0, 1] \mid x \in F(\alpha)\}.$$

Thus, every fuzzy BCK/BCI-subalgebra A may be considered as the soft BCK/BCI-algebra $(F, [0, 1])$.

Theorem 4.5. *Let (F, A) be a soft BCK/BCI-algebra over X. If B is a subset of A, then $(F|_B, B)$ is a soft BCK/BCI-algebra over X.*

Proof. Straightforward. □

The following example shows that there exists a soft set (F, A) over X such that

(i) (F, A) is not a soft BCI-algebra over X.
(ii) there exists a subset B of A such that $(F|_B, B)$ is a soft BCI-algebra over X.

Example 4.6. Let (F, A) be a soft set over X given in Example 4.4. Note that (F, A) is not a soft BCI-algebra over X. But if we take $B = \{a, b, c\} \subset A$, then $(F|_B, B)$ is a soft BCI-algebra over X.

Theorem 4.7. *Let (F, A) and (G, B) be two soft BCK/BCI-algebras over X. If $A \cap B \neq \emptyset$, then the intersection $(F, A)\widetilde{\cap}(G, B)$ is a soft BCK/BCI-algebra over X.*

Proof. Using Definition 3.2, we can write $(F, A)\widetilde{\cap}(G, B) = (H, C)$, where $C = A \cap B$ and $H(x) = F(x)$ or $G(x)$ for all $x \in C$. Note that $H : C \to \mathscr{P}(X)$ is a mapping, and therefore (H, C) is a soft set over X. Since (F, A) and (G, B) are soft BCK/BCI-algebras over X, it follows that $H(x) = F(x)$ is a BCK/BCI-subalgebra of X, or $H(x) = G(x)$ is a BCK/BCI-subalgebra of X for all $x \in C$. Hence $(H, C) = (F, A)\widetilde{\cap}(G, B)$ is a soft BCK/BCI-algebra over X. □

Corollary 4.8. *Let (F, A) and (G, A) be two soft BCK/BCI-algebras over X. Then, their intersection $(F, A)\widetilde{\cap}(G, A)$ is a soft BCK/BCI-algebra over X.*

Proof. Straightforward. □

Theorem 4.9. *Let (F, A) and (G, A) be two soft BCK/BCI-algebras over X. If A and B are disjoint, then the union $(F, A)\widetilde{\cup}(G, A)$ is a soft BCK/BCI-algebra over X.*

Proof. Using Definition 3.3, we can write $(F, A)\widetilde{\cup}(G, B) = (H, C)$, where $C = A \cup B$ and for every $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B, \\ G(e) & \text{if } e \in B \setminus A, \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

Since $A \cap B = \emptyset$, either $x \in A \setminus B$ or $x \in B \setminus A$ for all $x \in C$. If $x \in A \setminus B$, then $H(x) = F(x)$ is a BCK/BCI-subalgebra of X since (F, A) is a soft BCK/BCI-algebra over X. If $x \in B \setminus A$, then $H(x) = G(x)$ is a BCK/BCI-subalgebra of X since (G, B) is a soft BCK/BCI-algebra over X. Hence $(H, C) = (F, A)\widetilde{\cup}(G, A)$ is a soft BCK/BCI-algebra over X. □

Theorem 4.10. *If (F, A) and (G, B) are soft BCK/BCI-algebras over X, then $(F, A)\widetilde{\wedge}(G, B)$ is a soft BCK/BCI-algebra over X.*

Proof. By means of Definition 3.4, we know that

$$(F, A)\widetilde{\wedge}(G, B) = (H, A \times B),$$

where $H(x, y) = F(x) \cap G(y)$ for all $(x, y) \in A \times B$. Since $F(x)$ and $G(y)$ are BCK/BCI-subalgebras of X, the intersection $F(x) \cap G(y)$ is also a BCK/BCI-subalgebra of X. Hence $H(x, y)$ is a BCK/BCI-subalgebra of X for all $(x, y) \in A \times B$, and therefore $(F, A)\widetilde{\wedge}(G, B) = (H, A \times B)$ is a soft BCK/BCI-algebra over X. □

Definition 4.11. A soft BCK/BCI-algebra (F, A) over X is said to be *trivial* (resp., *whole*) if $F(x) = \{0\}$ (resp., $F(x) = X$) for all $x \in A$.

Example 4.12. Consider the BCI-algebra $X = \{0, a, b, c\}$ in Example 4.3. For $A = X$, let $F : A \to \mathscr{P}(X)$ be a set-valued function defined by

$$F(x) = \{0\} \cup \{y \in X \mid xRy \Leftrightarrow o(x) = o(y)\}$$

for all $x \in A$. Then, $F(x) = X$ for all $x \in A$, and so (F, A) is a whole soft BCI-algebra over X.

Let $f : X \to Y$ be a mapping of BCK/BCI-algebras. For a soft set (F, A) over X, $(f(F), A)$ is a soft set over Y where $f(F) : A \to \mathscr{P}(Y)$ is defined by $f(F)(x) = f(F(x))$ for all $x \in A$.

Lemma 4.13. *Let $f : X \to Y$ be a homomorphism of BCK/BCI-algebras. If (F, A) is a soft BCK/BCI-algebra over X, then $(f(F), A)$ is a soft BCK/BCI-algebra over Y.*

Proof. For every $x \in A$, we have $f(F)(x) = f(F(x))$ is a BCK/BCI-subalgebra of Y since $F(x)$ is a BCK/BCI-subalgebra of X and its homomorphic image is also a BCK/BCI-subalgebra of Y. Hence $(f(F), A)$ is a soft BCK/BCI-algebra over Y. □

Theorem 4.14. *Let $f : X \to Y$ be a homomorphism of BCK/BCI-algebras and let (F, A) be a soft BCK/BCI-algebra over X.*
(i) *If $F(x) = \ker(f)$ for all $x \in A$, then $(f(F), A)$ is the trivial soft BCK/BCI-algebra over Y.*
(ii) *If f is onto and (F, A) is whole, then $(f(F), A)$ is the whole soft BCK/BCI-algebra over Y.*

Proof. (i) Assume that $F(x) = \ker(f)$ for all $x \in A$. Then, $f(F)(x) = f(F(x)) = \{0_Y\}$ for all $x \in A$. Hence $(f(F), A)$ is the trivial soft BCK/BCI-algebra over Y by Lemma 4.13 and Definition 4.11.

(ii) Suppose that f is onto and (F, A) is whole. Then, $F(x) = X$ for all $x \in A$, and so $f(F)(x) = f(F(x)) = f(X) = Y$ for all $x \in A$. It follows from Lemma 4.13 and Definition 4.11 that $(f(F), A)$ is the whole soft BCK/BCI-algebra over Y. □

Definition 4.15. Let (F, A) and (G, B) be two soft BCK/BCI-algebras over X. Then (F, A) is called a *soft subalgebra* of (G, B), denoted by $(F, A)\widetilde{<}(G, B)$, if it satisfies:
(i) $A \subset B$,
(ii) $F(x)$ is a BCK/BCI-subalgebra of $G(x)$ for all $x \in A$.

Example 4.16. Let (F, A) be a soft BCK-algebra over X which is given in Example 4.2. Let $B = \{a, c, d\}$ be a subset of A and let $G : B \to \mathscr{P}(X)$ be a set-valued function defined by

$$G(x) = \{y \in X \mid xRy \Leftrightarrow y \in x^{-1}I\}$$

for all $x \in B$, where $I = \{0, a\}$ and $x^{-1}I = \{y \in X \mid x \wedge y \in I\}$. Then $G(a) = X$, $G(c) = \{0, a, b, d\}$ and $G(d) = \{0, a, b, c\}$ are BCK-subalgebras of $F(a)$, $F(c)$ and $F(d)$, respectively. Hence (G, B) is a soft subalgebra of (F, A).

Theorem 4.17. *Let (F, A) and (G, A) be two soft BCK/BCI-algebras over X.*
(i) *If $F(x) \subset G(x)$ for all $x \in A$, then $(F, A)\widetilde{<}(G, A)$.*
(ii) *If $B = \{0\}$ and (H, B), (F, X) are soft BCK/BCI-algebras over X, then $(H, B)\widetilde{<}(F, X)$.*

Proof. Straightforward. □

Theorem 4.18. *Let (F, A) be a soft BCK/BCI-algebra over X and let (G_1, B_1) and (G_2, B_2) be soft subalgebras of (F, A). Then*
(i) $(G_1, B_1)\widetilde{\cap}(G_2, B_2)\widetilde{<}(F, A)$.
(ii) $B_1 \cap B_2 = \emptyset \Rightarrow (G_1, B_1)\widetilde{\cup}(G_2, B_2)\widetilde{<}(F, A)$.

Proof. (i) Using Definition 3.2, we can write

$$(G_1, B_1)\widetilde{\cap}(G_2, B_2) = (G, B),$$

where $B = B_1 \cap B_2$ and $G(x) = G_1(x)$ or $G_2(x)$ for all $x \in B$. Obviously, $B \subset A$. Let $x \in B$. Then $x \in B_1$ and $x \in B_2$. If $x \in B_1$, then $G(x) = G_1(x)$ is a BCK/BCI-subalgebra of $F(x)$ since $(G_1, B_1)\widetilde{<}(F, A)$. If $x \in B_2$, then $G(x) = G_2(x)$ is a BCK/BCI-subalgebra of $F(x)$ since $(G_2, B_2)\widetilde{<}(F, A)$. Hence $(G_1, B_1)\widetilde{\cap}(G_2, B_2) = (G, B)\widetilde{<}(F, A)$.

(ii) Assume that $B_1 \cap B_2 = \emptyset$. We can write $(G_1, B_1)\widetilde{\cup}(G_2, B_2) = (G, B)$ where $B = B_1 \cup B_2$ and

$$G(x) = \begin{cases} G_1(x) & \text{if } x \in B_1 \setminus B_2, \\ G_2(x) & \text{if } x \in B_2 \setminus B_1, \\ G_1(x) \cup G_2(x) & \text{if } x \in B_1 \cap B_2 \end{cases}$$

for all $x \in B$. Since $(G_i, B_i)\widetilde{<}(F, A)$ for $i = 1, 2$, $B = B_1 \cup B_2 \subset A$ and $G_i(x)$ is a BCK/BCI-subalgebra of $F(x)$ for all $x \in B_i$, $i = 1, 2$. Since $B_1 \cap B_2 = \emptyset$, $G(x)$ is a BCK/BCI-subalgebra of $F(x)$ for all $x \in B$. Therefore $(G_1, B_1)\widetilde{\cup}(G_2, B_2) = (G, B)\widetilde{<}(F, A)$. □

Theorem 4.19. *Let $f : X \to Y$ be a homomorphism of BCK/BCI-algebras and let (F, A) and (G, B) be soft BCK/BCI-algebras over X. Then*

$$(F, A)\widetilde{<}(G, B) \Rightarrow (f(F), A)\widetilde{<}(f(G), B).$$

Proof. Assume that $(F, A)\widetilde{<}(G, B)$. Let $x \in A$. Then $A \subset B$ and $F(x)$ is a BCK/BCI-subalgebra of $G(x)$. Since f is a homomorphism, $f(F)(x) = f(F(x))$ is a BCK/BCI-subalgebra of $f(G(x)) = f(G)(x)$ and, therefore, $(f(F), A)\widetilde{<}(f(G), B)$. □

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