



# Interpretation of angular distributions of Z-boson production at colliders



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## ARTICLE INFO

### Article history:

Received 4 January 2016

Received in revised form 2 May 2016

Accepted 11 May 2016

Available online 16 May 2016

Editor: D.F. Geesaman

### Keywords:

Drell–Yan process

Lam–Tung relation

QCD

## ABSTRACT

High precision data of dilepton angular distributions in  $\gamma^*/Z$  production were reported recently by the CMS Collaboration covering a broad range of the dilepton transverse momentum,  $q_T$ , up to  $\sim 300$  GeV. Pronounced  $q_T$  dependencies of the  $\lambda$  and  $\nu$  parameters, characterizing the  $\cos^2\theta$  and  $\cos 2\phi$  angular distributions, were found. Violation of the Lam–Tung relation was also clearly observed. We show that the  $q_T$  dependence of  $\lambda$  allows a determination of the relative contributions of the  $q\bar{q}$  annihilation versus the  $qG$  Compton process. The violation of the Lam–Tung relation is attributed to the presence of a non-zero component of the  $q - \bar{q}$  axis in the direction normal to the “hadron plane” formed by the colliding hadrons. The magnitude of the violation of the Lam–Tung relation is shown to reflect the amount of this “non-coplanarity”. The observed  $q_T$  dependencies of  $\lambda$  and  $\nu$  from the CMS and the earlier CDF data can be well described using this approach.

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The Drell–Yan process [1], in which a lepton pair is produced in a hadron–hadron collision, is one of the most extensively studied reactions. This process together with Deep Inelastic Scattering (DIS) are the main tools for extracting the parton distributions in hadrons [2]. However, some characteristics of the lepton decay angular distributions in the Drell–Yan process are still not well understood. In particular, the Lam–Tung relation [3], which is expected to be largely valid in the presence of QCD corrections, was found to be significantly violated in pion-induced Drell–Yan data collected at CERN [4] and Fermilab [5]. Very recently, the CMS Collaboration reported a precision measurement of angular distribution in  $Z$  production at  $\sqrt{s} = 8$  TeV, again showing a significant violation of the Lam–Tung relation [6].

A general expression for the Drell–Yan angular distribution is [3]

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi, \quad (1)$$

where  $\theta$  and  $\phi$  denote the polar and azimuthal angle, respectively, of the  $l^-$  in the dilepton rest frame. In the “naive” Drell–Yan model, where the transverse momenta of the partons and QCD processes

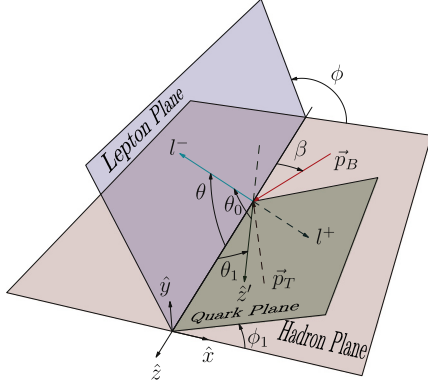
involving gluons are ignored,  $\lambda = 1$  and  $\mu = \nu = 0$ . When QCD effects [7] and intrinsic transverse momentum [8] are included,  $\lambda \neq 1$  and  $\mu, \nu \neq 0$  are allowed. Nevertheless,  $\lambda$  and  $\nu$  are expected to largely satisfy the Lam–Tung relation [3],  $1 - \lambda = 2\nu$ . This relation, obtained as a result of the spin-1/2 nature of the quarks, is analogous to the Callan–Gross relation [9] in DIS. However, unlike the Callan–Gross relation, the Lam–Tung relation is predicted to be insensitive to QCD corrections [10].

The Drell–Yan angular distributions were first measured in fixed-target experiments with pion beams by the CERN NA10 [4] and the Fermilab E615 Collaborations [5]. A sizable  $\nu$ , increasing with the dilepton transverse momentum  $q_T$  was observed by NA10 and E615. Perturbative QCD calculations predict much smaller values of  $\nu$  [7]. A large violation of the Lam–Tung relation was also found in the E615 data [5], suggesting the presence of effects other than perturbative QCD. Several non-perturbative effects [11–14] were suggested to explain the data. Boer suggested [15] that the observed behavior of  $\nu$  can be explained by the existence of a transverse-momentum dependent function [16]. This interpretation was later shown to be consistent with a fixed-target Drell–Yan experiment using a proton beam [17,18].

A measurement of the angular distributions of electrons in the  $p\bar{p} \rightarrow e^+e^- + X$  reaction at  $\sqrt{s} = 1.96$  TeV in the  $Z$  mass region ( $66 < M_{ee} < 116$  GeV/ $c^2$ ) with  $q_T$  up to 90 GeV was reported by the CDF Collaboration [19]. The CDF data were found to be in good

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**Fig. 1.** Definition of the Collins-Soper coordinates, the hadron plane, the lepton plane, the quark plane, and the various angles discussed in the text.

agreement with the Lam-Tung relation, in contrast to the findings in fixed-target experiments. Very recently, the CMS Collaboration reported a high-statistics measurement [6] of angular distributions of  $\gamma^*/Z$  production in  $p + p$  collisions at  $\sqrt{s} = 8$  TeV with  $q_T$  up to 300 GeV, clearly observing the violation of the Lam-Tung relation [6]. The different conclusions reached by the CDF and the CMS Collaborations regarding the Lam-Tung relation in  $\gamma^*/Z$  production are surprising and require further study. Moreover, the much larger values of  $q_T$  covered by the CDF and CMS experiments imply that the cross sections are dominated by QCD processes involving hard gluon emissions [20]. This is different from the fixed-target Drell-Yan experiments at low  $q_T$ , where the leading-order  $q - \bar{q}$  annihilation and non-perturbative effects dominate. The collider data could offer important insights on the impact of perturbative QCD effects on the validity of the Lam-Tung relation.

In this paper, we present an intuitive interpretation for the CMS and CDF results on the  $q_T$  dependencies of  $\lambda$  and  $\nu$ , as well as the origin for the violation of the Lam-Tung relation. We show that the emission of more than one gluon in higher-order ( $\geq \alpha_s^2$ ) QCD processes would lead to a non-coplanarity between the  $q - \bar{q}$  axis and the beam/target hadron plane in the  $\gamma^*/Z$  rest frame, resulting in a violation of the Lam-Tung relation. Using this geometric picture, the pronounced  $q_T$  dependencies of  $\lambda$  and  $\nu$  observed by the CMS and CDF Collaborations can be well described.

The angular distributions of the leptons are typically expressed in the rest frame of  $\gamma^*/Z$ , where the  $l^-$  and  $l^+$  have equal momenta with opposite directions. Clearly, the  $q$  and  $\bar{q}$  forming the  $\gamma^*/Z$  are also co-linear in the rest frame. Various choices of the coordinate system in the rest frame have been considered. In the Collins-Soper frame [21], the  $\hat{x}$  and  $\hat{z}$  axes lie in the hadron plane formed by the two colliding hadrons and the  $\hat{z}$  axis bisects the momentum vectors of the two hadrons (see Fig. 1). We define the momentum unit vector of the quark as  $\hat{z}'$ , which has polar and azimuthal angles  $\theta_1$  and  $\phi_1$ , as shown in Fig. 1. The corresponding angles of the lepton  $l^-$  ( $e^-$  or  $\mu^-$ ) from the  $\gamma^*/Z$  decay are labeled as  $\theta$  and  $\phi$ , as in Eq. (1). Note that for any given values of  $\theta$  and  $\phi$ ,  $\theta_1$  and  $\phi_1$  can vary over a range of values.

In the dilepton rest frame the angular distribution of  $l^-$  must be azimuthally symmetric with respect to the  $\hat{z}'$  axis with the following polar angular dependence [22]

$$\frac{d\sigma}{d\Omega} \propto 1 + a \cos \theta_0 + \cos^2 \theta_0. \quad (2)$$

The forward-backward asymmetry coefficient,  $a$ , comes from the parity-violating coupling to the  $Z$  boson, and  $\theta_0$  is the angle between the  $l^-$  momentum vector and  $\hat{z}'$ . One must convert Eq. (2) into an expression in terms of the physically measurable quantities  $\theta$  and  $\phi$ . The expression given by CMS is

$$\begin{aligned} \frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{A_0}{2}(1 - 3 \cos^2 \theta) + A_1 \sin 2\theta \cos \phi \\ & + \frac{A_2}{2} \sin^2 \theta \cos 2\phi + A_3 \sin \theta \cos \phi + A_4 \cos \theta \\ & + A_5 \sin^2 \theta \sin 2\phi + A_6 \sin 2\theta \sin \phi \\ & + A_7 \sin \theta \sin \phi. \end{aligned} \quad (3)$$

To go from Eq. (2) to Eq. (3), we note that  $\cos \theta_0$  satisfies the following relation:

$$\cos \theta_0 = \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos(\phi - \phi_1). \quad (4)$$

Substituting Eq. (4) into Eq. (2), we obtain

$$\begin{aligned} \frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{\sin^2 \theta_1}{2}(1 - 3 \cos^2 \theta) \\ & + \left(\frac{1}{2} \sin 2\theta_1 \cos \phi_1\right) \sin 2\theta \cos \phi \\ & + \left(\frac{1}{2} \sin^2 \theta_1 \cos 2\phi_1\right) \sin^2 \theta \cos 2\phi \\ & + (a \sin \theta_1 \cos \phi_1) \sin \theta \cos \phi + (a \cos \theta_1) \cos \theta \\ & + \left(\frac{1}{2} \sin^2 \theta_1 \sin 2\phi_1\right) \sin^2 \theta \sin 2\phi \\ & + \left(\frac{1}{2} \sin 2\theta_1 \sin \phi_1\right) \sin 2\theta \sin \phi \\ & + (a \sin \theta_1 \sin \phi_1) \sin \theta \sin \phi. \end{aligned} \quad (5)$$

From Eq. (3) and Eq. (5) one can express  $A_0$  to  $A_7$  in terms of  $\theta_1$ ,  $\phi_1$  and  $a$  as follows:

$$\begin{aligned} A_0 &= \langle \sin^2 \theta_1 \rangle & A_1 &= \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle \\ A_2 &= \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle & A_3 &= a \langle \sin \theta_1 \cos \phi_1 \rangle \\ A_4 &= a \langle \cos \theta_1 \rangle & A_5 &= \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle \\ A_6 &= \frac{1}{2} \langle \sin 2\theta_1 \sin \phi_1 \rangle & A_7 &= a \langle \sin \theta_1 \sin \phi_1 \rangle. \end{aligned} \quad (6)$$

Equation (6) is a generalization of an earlier work [23] which considered the special case of  $\phi_1 = 0$  and  $a = 0$ . The  $\langle \dots \rangle$  in Eq. (6) is a reminder that the measured values of  $A_n$  are averaged over the event sample. A comparison of Eq. (1) and Eq. (3) gives

$$\lambda = \frac{2 - 3A_0}{2 + A_0}; \quad \mu = \frac{2A_1}{2 + A_0}; \quad \nu = \frac{2A_2}{2 + A_0}. \quad (7)$$

Equation (7) shows that the Lam-Tung relation,  $1 - \lambda = 2\nu$ , becomes  $A_0 = A_2$ .

From Eq. (6) and Eq. (7) several remarks regarding the nature of the  $\gamma^*/Z$  decay angular distribution can be made:

- In the “naive” Drell-Yan the  $q - \bar{q}$  axis coincides with the  $\hat{z}$  axis of the Collins-Soper frame, hence  $\theta_1 = 0$  and  $\lambda = 1$ . The deviation of  $\lambda$  from the “naive” Drell-Yan prediction of unity is due to non-zero  $\theta_1$ , which reflects the mis-alignment between the  $q - \bar{q}$  axis and the  $\hat{z}$  axis of the Collins-Soper frame [23,22]. It is important to note that  $\lambda$  (or  $A_0$ ) does not depend on  $\phi_1$ , which is a measure of the non-coplanarity between the  $q - \bar{q}$  axis and the hadron plane. In contrast,  $\mu$  and  $\nu$  (or  $A_1$  and  $A_2$ ) depend on both  $\theta_1$  and  $\phi_1$ .
- Eq. (6) also shows that the Lam-Tung relation,  $A_0 = A_2$ , is valid when  $\phi_1 = 0$ , i.e., for the co-planar case. Violation of the Lam-Tung relation is caused by the presence of the  $\cos 2\phi_1$  term in  $A_2$  (or  $\nu$ ), and not due to the  $A_0$  (or  $\lambda$ ) term. Moreover, the non-coplanarity factor,  $\cos 2\phi_1$ , ensures that  $A_0 \geq A_2$ , or  $1 - \lambda - 2\nu \geq 0$ .

- c) The parity-violating parameter  $a$  is not present for the coefficients  $A_0, A_1, A_2, A_5$  and  $A_6$ . In particular, the parameter  $a$  has no effect on the Lam–Tung relation,  $A_0 = A_2$ .
- d) The forward–backward asymmetry,  $A_4$ , is the only term which does not vanish when  $\theta_1$  is zero.  $A_4$  is reduced by a factor  $\cos\theta_1$  compared to the value of  $a$ . The mis-alignment between the  $q - \bar{q}$  axis and the  $\hat{z}$  axis of the Collins–Soper frame will dilute the forward–backward asymmetry. Moreover,  $A_4$  is independent of the angle  $\phi_1$ , thus un-affected by the non-coplanarity between the  $q - \bar{q}$  axis and the hadron plane.
- e) The coefficients  $A_5, A_6, A_7$  are all odd functions of  $\phi_1$ . From symmetry consideration, the  $\gamma^*/Z$  events must have symmetric  $\phi_1$  distributions. Hence  $\langle \sin\phi_1 \rangle$  and  $\langle \sin 2\phi_1 \rangle$  would vanish in the limit of large statistics. Therefore, the values of these three coefficients, summed over a sufficiently large data sample, should approach zero. This is consistent with the observation by the CMS Collaboration [6].

In perturbative QCD at the order of  $\alpha_s$ , ignoring the intrinsic transverse momenta of the colliding partons, the  $q\bar{q} \rightarrow \gamma^*/ZG$  annihilation process gives [24–26]

$$\langle \sin^2\theta_1 \rangle = \sin^2\theta_1 = q_T^2 / (Q^2 + q_T^2) \quad (8)$$

in the Collins–Soper frame, where  $q_T$  and  $Q$  are the transverse momentum and mass, respectively, of the dilepton. One notes that  $\theta_1$  given in Eq. (8) is identical to the angle  $\beta$  between  $\vec{P}_B$  (or  $\vec{P}_T$ ) and the  $\hat{z}$  axis in the Collins–Soper frame (see Fig. 1). This result can be readily understood as follows. Emission of a gluon from one of the colliding partons would not affect the momentum of the other parton, which moves along the  $\vec{P}_B$  or  $\vec{P}_T$  direction (see Fig. 1). Hence the  $q - \bar{q}$  collision axis ( $\hat{z}'$  in Fig. 1) is along either the  $\vec{P}_B$  or  $\vec{P}_T$  direction, and Eq. (8) is obtained. For the  $qG \rightarrow \gamma^*/Zq$  Compton process, it was shown [4,28,29] that  $\langle \sin^2\theta_1 \rangle$  is approximately described by

$$\langle \sin^2\theta_1 \rangle = 5q_T^2 / (Q^2 + 5q_T^2). \quad (9)$$

Unlike Eq. (8), which is an exact relation, Eq. (9) is an approximation, since the exact value depends on the details of the parton distribution functions involved in the  $qG$  Compton process. As shown in Ref. [28], this approximate expression is expected to be valid over a broad range of kinematics. This can be qualitatively understood [20], since  $A_0$  (or equivalently,  $\sin^2\theta_1$ ) can be expressed as the ratio of two cross sections,  $A_0 = 2d\sigma^L/d\sigma^{U+L}$ , where  $d\sigma^{U+L}$  is the unpolarized cross section and  $d\sigma^L$  is the cross section for longitudinally polarized virtual photon. While each cross section depends on the parton distributions, the ratio is largely insensitive to them.

Using Eq. (7), Eq. (8) and Eq. (9) imply

$$\begin{aligned} \lambda &= \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2} & \nu &= \frac{2q_T^2}{2Q^2 + 3q_T^2} & (q\bar{q}) \\ \lambda &= \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2} & \nu &= \frac{10q_T^2}{2Q^2 + 15q_T^2} & (qG). \end{aligned} \quad (10)$$

We note that for both processes,  $\lambda = 1$  and  $\theta_1 = 0$  at  $q_T = 0$ , while  $\lambda \rightarrow -1/3$  and  $\theta_1 \rightarrow 90^\circ$  as  $q_T \rightarrow \infty$ . Moreover, Eq. (10) shows that the Lam–Tung relation,  $1 - \lambda = 2\nu$ , is satisfied for both the  $q\bar{q}$  and  $qG$  processes at order  $\alpha_s$ .

Fig. 2(a) shows the CMS results for  $\lambda$  versus  $q_T$  for two rapidity regions,  $y \leq 1.0$  and  $y \geq 1.0$ . We use Eq. (7) to convert the CMS measurement of  $A_0$  into  $\lambda$ , since the original Lam–Tung relation was expressed in terms of  $\lambda$  and  $\nu$ . Both the statistical and systematic uncertainties are taken into account. The dashed and dash-dotted curves in Fig. 2(a) correspond to the calculation using

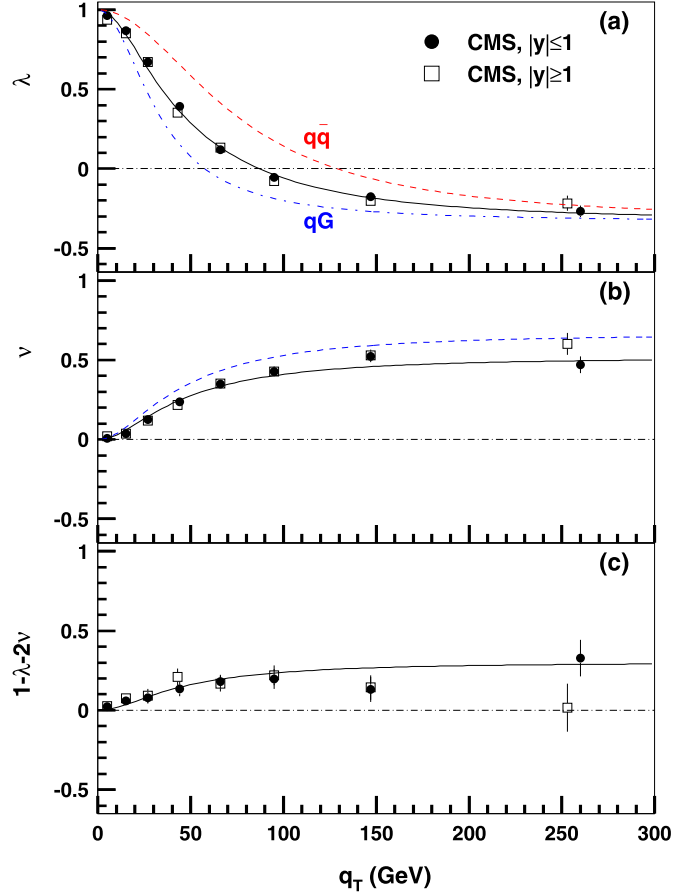
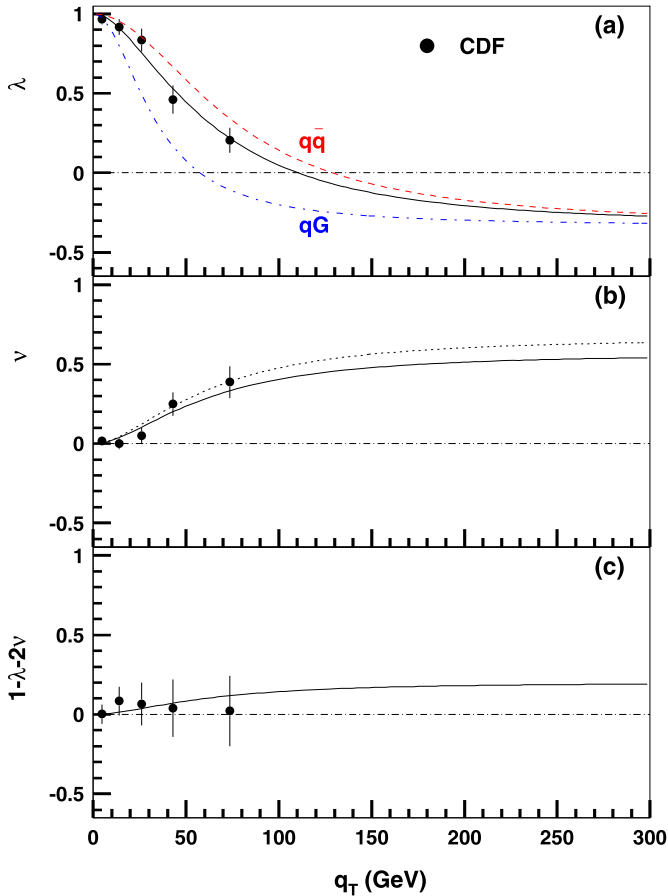


Fig. 2. Comparison between the CMS data [6] on  $\gamma^*/Z$  production at two rapidity regions with calculations for (a)  $\lambda$  vs.  $q_T$ , (b)  $\nu$  vs.  $q_T$  (c)  $1 - \lambda - 2\nu$  vs.  $q_T$ . Curves correspond to calculations described in the text.

Eq. (10) for the  $q\bar{q}$  annihilation and the  $qG$  Compton processes, respectively. Both the  $q\bar{q}$  and  $qG$  processes are expected to contribute to the  $pp \rightarrow \gamma^*/ZX$  reaction, and the observed  $q_T$  dependence of  $\lambda$  must reflect the combined effect of these two contributions. Adopting a simple assumption that the fraction of these two processes is  $q_T$  independent, a best-fit to the CMS data is obtained with a mixture of  $58.5 \pm 1.6\%$   $qG$  and  $41.5 \pm 1.6\%$   $q\bar{q}$  processes. The solid curve in Fig. 2(a) shows that the data at both rapidity regions can be well described by this mixture of the  $qG$  and  $q\bar{q}$  processes. In  $pp$  collisions the  $qG$  process is expected to be more important than the  $q\bar{q}$  process [27], in agreement with the best-fit result. While the amount of  $qG$  and  $q\bar{q}$  mixture can in principle depend on the rapidity,  $y$ , the CMS data indicate a very weak, if any,  $y$  dependence. The good description of  $\lambda$  shown in Fig. 2(a) also suggests that higher-order QCD processes are relatively unimportant.

We next consider the CMS data on the  $\nu$  parameter. As shown in Eqs. (6) and (7),  $\nu$  depends not only on  $\theta_1$ , but also on  $\phi_1$ . In leading order  $\alpha_s$  where only a single undetected parton is present in the final state, the  $q - \bar{q}$  axis must be in the hadron plane, implying  $\phi_1 = 0$  and the Lam–Tung relation is satisfied. We first compare the CMS data, shown in Fig. 2(b), with the calculation for  $\nu$  using Eq. (10), which is obtained at the leading order  $\alpha_s$ . The dashed curve uses the same mixture of 58.5%  $qG$  and 41.5%  $q\bar{q}$  components as deduced from the  $\lambda$  data. The data are at a variance with this calculation, suggesting the presence of higher-order QCD processes leading to a non-zero value of  $\phi_1$ . We performed a fit to the  $\nu$  data allowing  $A_2/A_0$  to deviate from unity. The best-fit value is



**Fig. 3.** Comparison between the CDF data [19] on  $\gamma^*/Z$  production with calculations for (a)  $\lambda$  vs.  $q_T$ , (b)  $\nu$  vs.  $q_T$  (c)  $1 - \lambda - 2\nu$  vs.  $q_T$ . Curves correspond to calculations described in the text.

$A_2/A_0 = 0.77 \pm 0.02$ . The solid curve in Fig. 2(b), corresponding to the best-fit, is in better agreement with the data. The deviation of  $A_2/A_0$  from unity is due to non-zero values of  $\phi_1$  signifying the presence of non-coplanar processes. Fig. 2(c) shows that the  $q_T$  dependence of  $1 - \lambda - 2\nu$ , a measure of the violation of the Lam–Tung relation, is well described by the calculation using  $A_2/A_0 = 0.77$ .

The violation of the Lam–Tung relation reflects the non-coplanarity between the  $q - \bar{q}$  axis and the hadron plane. This can be caused by higher-order QCD processes, where multiple partons are present in the final state in addition to the detected  $\gamma^*/Z$ . To illustrate this, one considers a specific quark–antiquark annihilation diagram at order  $\alpha_s^2$  in which both the quark and antiquark emit a gluon before they annihilate. The hadron plane in this case is related to the vector sum of the two emitted gluons, and the  $q - \bar{q}$  axis is in general not in the hadron plane. This would lead to a non-zero  $\phi_1$  and a violation of the Lam–Tung relation. Similar consideration would also explain why the intrinsic transverse momenta of the colliding quark and antiquark in the “naive” Drell–Yan could also lead to the violation of the Lam–Tung relation, since the vector sum of the two uncorrelated transverse momenta would lead in general to a non-zero value of  $\phi_1$ .

There remains the question why the CDF  $\bar{p}p$   $Z$ -production data are consistent with the Lam–Tung relation [19]. Fig. 3(a) shows  $\lambda$  versus  $q_T$  in  $\bar{p}p$  collision at 1.96 TeV from CDF. The  $q_T$  range covered by the CDF measurement is not as broad as the CMS, and the statistical accuracy is somewhat limited. Nevertheless, a striking  $q_T$  dependence of  $\lambda$  is observed. The dashed and dash-dotted curves are calculations using Eq. (10) for the  $q\bar{q}$  annihilation and the  $qG$  Compton processes, respectively. The solid curve in Fig. 3(a) shows

that the CDF data can be well described with a mixture of 72.5%  $q\bar{q}$  and 27.5%  $qG$  processes. This is consistent with the expectation that the  $q\bar{q}$  annihilation has the dominant contribution to the  $\bar{p}p \rightarrow \gamma^*/Z$  reaction. It is also consistent with the perturbative QCD calculations showing that the  $qG$  process contributes  $\approx 30\%$  to the production of  $Z$  bosons at the Tevatron energy [19]. The CDF data on the  $\nu$  parameter, shown in Fig. 3(b), are first compared with the calculation (dotted curve) using Eq. (10) with a mixture of 72.5%  $q\bar{q}$  and 27.5%  $qG$  deduced from the  $\lambda$  data. The solid curve in Fig. 3(b) results from a fit allowing  $A_2/A_0$  to deviate from unity. The best-fit value is  $A_2/A_0 = 0.85 \pm 0.17$ . The relatively large uncertainties of the data prevent an accurate determination of the degree of non-coplanarity. Nevertheless, the data do allow a non-zero value of  $\phi_1$ , implying that the Lam–Tung relation could be violated. The quantity  $1 - \lambda - 2\nu$ , shown in Fig. 3(c), is compared with the solid curve obtained using  $A_2/A_0 = 0.85$ . The CDF data is consistent with the solid curve, and the presence of some violation of the Lam–Tung relation can not be excluded by the CDF data.

In conclusion, we have presented an intuitive explanation for the observed  $q_T$  dependencies of  $\lambda$  and  $\nu$  for the CMS and CDF  $\gamma^*/Z$  data. The violation of the Lam–Tung relation can be attributed to the non-coplanarity of the  $q - \bar{q}$  axis and the hadron plane, which occur for QCD processes involving more than one gluon. The present analysis could be further extended to the other coefficients,  $A_1, A_3$  and  $A_4$  [30]. It could also be extended to the case of fixed-target Drell–Yan experiments, where the non-coplanarity at low  $q_T$  can be caused by the intrinsic transverse momenta of the colliding partons in the initial states [30]. The effects of non-coplanarity on other inequality relations, as discussed in Ref. [31], are also being studied.

## Acknowledgements

We gratefully acknowledge helpful discussion with Aram Kotzian, Hsiang-Nan Li, and Stephane Platchkov. This work was supported in part by the U.S. National Science Foundation (NSF PHY 15-06416) and the National Science Council of the Republic of China.

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