An extended chirp scaling algorithm for spaceborne sliding spotlight synthetic aperture radar imaging

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Abstract A system impulse response with low sidelobes is critical in synthetic aperture radar (SAR) images because sidelobes contribute to noise and interfere with nearby scatterers. However, the conventional tricks of sidelobe suppression are unable to be exactly applied to the case of spaceborne sliding spotlight SAR due to great azimuth shifts in both time and frequency domains. In this paper, an extended chirp scaling algorithm is presented for spaceborne sliding spotlight SAR data imaging. The proposed algorithm firstly uses the spectral analysis (SPECAN) technique to avoid the azimuth spectrum folding effect and then employs the chirp scaling (CS) algorithm to achieve data focusing, i.e., the so-called two-step approach. To suppress the sidelobe level, an efficient strategy for the azimuth spectral weighting which only involves matrix multiplications and short fast Fourier transformations (FFTs) is proposed, which is a post-process executed on the focused SAR image and particularly simple to be implemented. The SAR image processed by the proposed extended CS algorithm is very precise and perfectly phase-preserving. In the end, computer simulation results verify the analysis and confirm the validity of the proposed algorithm.

1. Introduction

The main characteristic of a synthetic aperture radar (SAR) sensor operating in the sliding spotlight mode is that the velocity of antenna footprint is lower than the platform velocity. It indicates that the azimuth scene size is larger than that in the pure spotlight mode and smaller than that in the classic stripmap mode. On the contrary, the azimuth resolution is lower than that in the spotlight mode and higher than that in the stripmap mode. Similar to the pure spotlight mode using a real rotation point at the scene center, the sliding spotlight mode uses a virtual rotation point which is further away from the radar than the scene being illuminated.

It is remarkable that two operational spotlight modes of the German TerraSAR-X are both designed in a sliding geometry with 1 m and 2 m azimuth resolutions at 5 km and 10 km along
In this paper, we present an alternative algorithm for SAR imaging that eliminates the problem of subaperture-division. We firstly use the SPECAN technique to avoid the azimuth spectrum folding effect and the chirp scaling (CS) algorithm to achieve data focusing, i.e., the so-called two-step approach. Then we propose a novel strategy for the azimuth spectral weighting, which is executed on the focused SAR image. The focused azimuth signal is divided into segments in the image domain, and then each segment is transformed into the frequency domain to perform the spectral weighting. Finally, the whole weighted signal is obtained by the recombination of the weighted segments. Obviously, it is a post-process of the original SAR imaging process which only involves matrix multiplications and FFTs, so it is not only precise and simple to be implemented but also very efficient; furthermore, it is a perfectly phase-preserving process.

This paper is organized as follows. In Section 2, we give the principle of the proposed algorithm for spaceborne sliding spotlight SAR imaging. Section 3 is dedicated to analyze the two key points of the proposed azimuth spectral weighting strategy. Section 4 provides simulation results which verify the analysis and confirm the validity of the proposed algorithm. Section 5 concludes this paper.

2. Principle of the proposed algorithm

The geometry of the standard sliding spotlight mode is shown in Fig. 1, where \( x \) and \( r \) are the velocity and range axis, \( V \) is the platform velocity, \( V_t \) is the antenna footprint velocity, and \( R \) and \( R_{\text{rot}} \) are the broadside slant ranges to the real scene center and to the rotation point, respectively. The system is operating in the broadside mode, and the illumination starts at position \( x_{\text{start}} \) and ends at position \( x_{\text{stop}} \). During the data acquisition time, the radar antenna, which transmits pulses of a linear FM chirp signal with a chirp rate of \( \gamma \), is steered to the virtual rotation point.

The characteristic of the Doppler histories is shown in Fig. 2, where \( f_a \) and \( f_s \) are azimuth time and frequency, respectively. From Fig. 2, two main inconveniences arise in the sliding spotlight mode. First is the azimuth spectrum folding effect due to the PRF being much smaller than the total azimuth bandwidth. Second is the azimuth sidelobe suppression. As is well known, convolution in the time domain is equivalent to

![Fig. 1 Geometry of the standard sliding spotlight mode.](Image)

![Fig. 2 Doppler histories of the sliding spotlight mode.](Image)
point wise product in the frequency domain. Hence, sidelobe suppression can be applied either in the time domain or in the frequencey domain. For the stripmap mode, we can take the azimuth spectral weighting using a window function to multiply the data in the frequency domain because the Doppler histories of the targets in the scene are identical, and we can achieve the azimuth sidelobe suppression of the pure spotlight mode in the azimuth time domain due to the same azimuth time history. However, the operations do not work well for the azimuth sidelobe suppression of the sliding spotlight mode since both the azimuth time and the Doppler histories of targets at different azimuth positions are not identical (as shown in Fig. 2).

To overcome the first problem, we use the available SPECAN technique which is well discussed in Ref.6, and in order to achieve the azimuth sidelobe suppression, an efficient spectral weighting strategy is proposed. We take the viewpoint at the sliding spotlight SAR image where a scatterer can be very approximately represented by the peak pixel and its nearby pixels as long as the azimuth signal is focused adequately. Therefore, a small sliding window can be taken with the step of one pixel to make each scatterer be in the middle of the window, and then the sidelobe suppression can be performed for each scatterer. However, it is extremely inefficient. Here, we propose an efficient strategy for the azimuth sidelobe suppression, which is executed on the focused SAR image.

Fig. 3 gives the trick of the proposed weighting strategy. As shown in Fig. 3, in the image domain, the focused azimuth signal of \(N_a\) samples is divided into \(M\) segments with each size of \(N_{a\text{,sub}}\). The spectral shifts between the point targets in the same segment are assumed to be so small that they can safely be ignored. After transforming each segment into the frequency domain, we can apply normal spectral weighting methods to each segment by multiplying a spectral weighting function. Note that, for short FFT operations, the non-overlapping division inevitably leads to a segment-edge problem, i.e., the scatterers situated nearby or right between any two segments cannot be right weighted. In order to resolve the problem, two-time divisions are performed as (1) and (2) shown in Fig. 3. There exists a displacement of \(N_{a\text{,sub}}/2\) samples between the first and second divisions, i.e., 50% overlap, and in fact, the weighting result with a smaller overlap (e.g., 13.5%) of the segment length is quite acceptable. After the weighting operation in the frequency domain, short IFFT operations are then performed to transform each segment back into the image domain. As the weighting operation does not change the positions of each scatterer, we can easily acquire the weighted signal by the recombination of the weighted segments, as Fig. 3 shows, and the segments only reserve the middle \(N_{a\text{,sub}}/2\) samples, except that the first and last segments of the first division reserve the former \(3N_{a\text{,sub}}/4\) and the latter \(3N_{a\text{,sub}}/4\) samples, respectively, while the other parts are all discarded.

Based on the aforementioned analysis, an extended chirp scaling algorithm is presented for sliding spotlight SAR imaging as Fig. 4 shows. In Fig. 4, \(H_1\) is the de-ramping phase function, \(H_2\) is phase compensation function, \(H_3\) is phase cancellation function, \(H_4\) is chirp scaling function, \(H_5\) and \(H_6\) are range and azimuth matched filter functions, \(W_a\) is azimuth weighting function.

The steps are explained as follows:

**Step 1.** De-ramping: The received demodulated raw data is de-ramped via the multiplication of a phase function as

\[
H_1 = \exp \left[ j2\pi \frac{(IA\Delta x')^2}{N_{ref}} \right] (i = -N_p/2, -N_p/2 + 1, \ldots, N_p/2 - 1)
\]

where \(N_a\) and \(\Delta x' = \frac{V}{f_{PRF}}\) are the raw data azimuth pixel number and dimension, \(f_{PRF}\) is the value of PRF. \(r\) is the carrier wavelength and \(r_{ref}\) is a reference slant range.

**Step 2.** Azimuth FFT: As the analysis in Ref.7, the following equation is satisfied,

\[
\frac{x_{ref}}{2\Delta x'} = N_a\Delta x' \geq L_a
\]

Where \(L_a\) is the along-track scene extension and \(\Delta x''\) is the focused image azimuth pixel dimension. \(N_a\) is a natural number which allows the use of efficient FFT codes. As \(N_a\) is larger than \(N_p\), a limited zero padding of the raw data is required before performing the FFT operation. A factor of \(\xi\) is defined as

\[
\xi = \frac{\Delta x''}{\Delta x'} = \left( \frac{f_{PRF}}{V} \right)^2 \frac{x_{ref}}{2N_a}
\]

**Step 3.** Phase compensation: The compensation function is

\[
H_2 = \exp \left( \frac{j\pi N_a f_s}{2f_{RF}^2} \right)
\]

where \(f_s = \frac{n}{N_{RF}} f_{PRF}\) with \(n = -N_a/2, -N_a/2 + 1, \ldots, N_a/2 - 1\) is the azimuth frequency.

**Step 4.** Azimuth FFT: Steps 1–3 are the key point of the two-step approach (i.e., the azimuth convolution between the raw data and the quadratic phase function), which eliminates the azimuth spectral folding effect when transforming the raw data into the azimuth frequency domain. At this point, data is transformed into the nominal azimuth-frequency/rang-time domain.

**Step 5.** Phase cancellation: To directly implement the CS algorithm, we should multiply the data with a phase function of \(H_3\) in the nominal azimuth-frequency/rang-time domain to cancel the phase caused by the convolution.

\[
H_3 = \exp \left\{ j2\pi \frac{(nN_a + N_p/2)^2}{N_{ref}} \right\} (n = -N_a/2, -N_a/2 + 1, \ldots, N_a/2 - 1)
\]
**Step 6.** Chirp scaling: This step is to equalize the range curvature of scatterers at all ranges by multiplying the data with a chirp scaling function as

\[ H_4 = \exp \left( j \pi \gamma_c a_i \left( \frac{i}{F_s} - \frac{2R_0a_i}{c} \right)^2 \right) \]

\( i = -N_r/2, -N_r/2 + 1, \ldots, N_r/2 - 1 \)  

(6)

where \( n_r \) is the raw data range pixel number, \( F_s \) is the A/D sampling frequency, and \( R_0 \) is the closest slant range between the scatterer and the flight path. \( c \) is the light speed. This operation causes a residual phase which will be cancelled later in step 10.

**Step 7.** Range FFT: The data is transformed into the two-dimensional frequency domain at this point.

**Step 8.** Range matched filter: It includes range compression and range cell migration correction by multiplying a phase function as

\[ H_5 = W_r(f_r) \exp \left( j \pi \gamma_c c_c \left( \frac{f_r^2}{c_c} \right) \right) \exp \left( \frac{4\pi R_0a_i}{c} f_r \right) \]

where \( f_r \) represents the range frequency. Note that range side-lobe suppression can also be performed by a weighting function \( W_r(f_r) \).

**Step 9.** Range IFFT: Transform the data back into the azimuth-frequency/rang-time domain.

**Step 10.** Azimuth matched filter: At this point, the azimuth compression is performed, and the residual phase caused by the chirp scaling operation is also cancelled by multiplying a phase function as

\[ H_6 = \exp \left\{ j \frac{4\pi}{z} \frac{R_b}{c} \sqrt{1 - \left( \frac{\tilde{f}_a}{2V} \right)^2} \left( 1 - \frac{\tilde{f}_a}{2V} \right) \right\} \]

\[ \cdot \exp \left\{ -j \frac{4\pi}{c^2} \gamma_c c_c (1 + a_i) (R_b - R_0)^2 \right\} \]

(10)

**Step 11.** Azimuth IFFT: The focused image can be obtained after performing this operation.

**Step 12.** Sub-segment formation: Each azimuth signal is divided into sub-segments at this step, and some overlap, e.g., 13.5%, is needed in order to avoid the segment-edge problem.

**Step 13.** Azimuth short FFT: Each sub-segment is transformed into the frequency domain to perform the spectral weighting operation.

**Step 14.** Azimuth weighting: Each sub-segment is weighted by an azimuth weighting function \( W_a(f_a) \), which will be discussed in detail in Section 3.
Step 15. Azimuth short IFFT: Every weighted sub-segment is transformed back into the time domain and recombined to form the integrated weighted azimuth signal.

3. Two key points of the spectral weighting strategy

Two key points of the spectral weighting strategy are the determination of the size of each segment and the design of the sliding window functions. These are discussed in detail as follows.

3.1. Determination of the size of each segment

From Eq. (3), we can get the azimuth pixel dimension of the focused image as

\[ \Delta x' = \frac{\lambda_{ref} f_{PRF}}{2VN_a} \]  (11)

We can also obtain the sample spacing in the frequency domain as

\[ \Delta f = \frac{f_{PRF}}{N_{a\text{sub}}} \]  (12)

The shift amount of spectrum per meter in azimuth is calculated as

\[ \Delta b = \frac{B_s - B_p}{L_a} \]  (13)

where \( B_s \) and \( B_p \) are the Doppler bandwidth of the whole scene and the Doppler bandwidth of a single point target, respectively. Then the size of each segment can be obtained as

\[ N_{a\text{sub}} = \frac{N_f}{\Delta f \Delta x'} \]  (14)

where \( \mu \) is a factor defined as the maximum shift in units of frequency sample spacing among the point targets in each segment. Note that we can control the weighting precision by setting the value of \( \mu \). It is taken as 1 in the following simulation.

Substituting Eqs. (11)–(13) into Eq. (14) yields

\[ N_{a\text{sub}} = \frac{M f_{PRF}}{\Delta b \Delta x'} \]  (15)

Finally, the size of each segment can be determined as

\[ N_{a\text{sub}} = 2 \left( \left\lfloor \frac{f_{PRF}}{\Delta b \Delta x'} \right\rfloor \right) \]  (16)

where \( \left\lfloor \cdot \right\rfloor \) rounds \( \cdot \) to the nearest integer smaller than or equal to \( \cdot \).

3.2. Design of the sliding window function

The pixel number \( N_{a,p} \) of the Doppler bandwidth of a single point target in the sliding spotlight scene is

\[ N_{a,p} = 2 \left\lfloor \frac{N_{a\text{sub}}}{2M} \right\rfloor \]  (17)

where \( \left\lfloor \cdot \right\rfloor \) rounds \( \cdot \) to the nearest integer smaller than or equal to \( \cdot \).

The shift amount between any two neighboring segments is

\[ \Delta s = N_{a\text{sub}} \Delta x' \Delta b / \Delta f \]  (18)

Consequently, the window function can be constructed as

\[
W_a(n, i_d) = \begin{cases} \text{Kaiser}(N_{a,p}, 2.5)n = \frac{N_{a\text{sub}} - N_{a,p}}{2} - [i_d \cdot \Delta s] + 1, & \\
\frac{N_{a\text{sub}} - N_{a,p}}{2} - [i_d \cdot \Delta s] + 2, & \\
\frac{N_{a\text{sub}} + N_{a,p}}{2} + [i_d \cdot \Delta s] & \text{Others}
\end{cases}
\]

where \( M \) is the total number of segments, \( \lfloor \cdot \rfloor \) is the nearest integer to \( \cdot \). Here we use the Kaiser \((L, \beta)\) window (other windows such as Hamming or Hanning windows can also certainly be used), wherein \( L \) is the length of the Kaiser window and \( \beta \) is a factor that affects the sidelobe attenuation of the Fourier transform of the window.

![Fig. 5](image_url) Locations of three and nine point targets.

<table>
<thead>
<tr>
<th>Table 1 Simulation parameters.</th>
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<tbody>
<tr>
<td>Parameter</td>
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<tr>
<td>Scene center slant range</td>
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<tr>
<td>Scene extension (range × azimuth)</td>
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<tr>
<td>Resolution</td>
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<tr>
<td>Rotation center slant range</td>
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<tr>
<td>PRF</td>
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</table>
4. Simulation results

Table 1 lists the sliding spotlight SAR parameters in the simulations. Fig. 5 shows the locations of three and nine point targets in the simulation scene, where \( P_1, P_2, \ldots, P_9 \) are point targets.

4.1. Verification of the azimuth spectral weighting strategy

In order to well demonstrate the aforementioned analysis, we simulate the 1D azimuth signal of three point scatterers. As Fig. 5(a) shows, the three points are at the same arbitrary range position, but different azimuth positions (two at the azimuth edges and one at the center).

The left graph in Fig. 6 shows the azimuth spectra of the three scatterers, and we separate each scatterer’s spectrum as shown in the right graph for a better illustration. From Fig. 6, we can clearly see that the spectra of the different scatterers at the extension of the whole along-track scene have great shift amount which is aforementioned in Section 2.

As Fig. 7 shows, using the Kaiser window, \( L \) covers the Doppler bandwidth of the whole azimuth scene, but only the central scatterer \( P_2 \) can be well weighted when \( \beta \) is about 7, while the others are all in bad conditions. Fig. 8 plays another

![Fig. 6 Azimuth spectra of three scatterer signals.](image)

![Fig. 7 Sidelobe control over the whole azimuth Doppler bandwidth.](image)
Fig. 8  Sidelobe control over the central scatterer’s azimuth Doppler bandwidth.

Fig. 9  Weighting results of the proposed strategy.
trick that only covers the central scatterer’s Doppler bandwidth. Similar to the first case, the results are not satisfying either.

The simulation results of the proposed spectral weighting strategy are shown in Fig. 9. We take the central segment’s weighting for an example. In the top left corner of the figure, the sliding window just covers the spectrum of the segment’s central point scatterer totally (i.e., the aforementioned $P_2$ scatterer). From Fig. 9, we can see that the first sidelobes of the three scatterers are reduced to low levels below $-20$ dB with a slightly wider main lobe which meets the system requirement, e.g., the resolution, PSLR, and ISLR of $P_1$ before weighting are $0.82$ m, $-13.27$ dB, and $-9.68$ dB, respectively, and after sidelobe suppression, they turn out to be $0.95$ m, $-20.61$ dB, and $-18.12$ dB. Obviously, the sidelobes are suppressed accurately and the results are satisfying. Furthermore, in order to investigate the phase-preserving accuracy (for specialized applications such as InSAR and GMTI) of the proposed method, the geometrical slant ranges between simulation points and corresponding antenna phase centers are computed, and the SAR imaging phase values of focused points are measured by two-dimensional FFT interpolation, that is, a local window (in the experiment the size of the window is set to be $32 \times 32$ pixels) which contains a focused point and neighboring pixels is selected to be interpolated, and the phase of the intensity peak point is considered to be the SAR imaging point phase. By comparison between the measured and geometrically computed phase values, it can be concluded that the proposed method is a phase-preserving process, since the simulation turns out that the weighting operation only causes a discrepancy of $0.05^\circ$.

4.2. Simulated data set imaging results

Using the parameters listed in Table 1, the raw data of nine point targets (as Fig. 5(b) shows) are simulated to validate the proposed procedure. Fig. 10 shows the contour plots of the processing results without weighting while Fig. 11 shows the weighted results. The simulation results have assessed that the proposed algorithm is precise and the phase accuracy is within $1^\circ$, thus demonstrating the phase-preserving capability of the presented algorithm.

In addition, extended target simulation is carried out to demonstrate the performance of the proposed method. The simulation scene is composed of a flat area with a size of $200 \text{ m} \times 200 \text{ m}$ (azimuth x range) and three buildings, the parameters (i.e., starting position along track, ending position along track, starting position across track, ending position across track), and the SAR imaging phase values of focused points are measured by two-dimensional FFT interpolation.
across track, building height) of which are (−60, −30, 10, 10, 45), (0, 90, −50, −30, 30), and (0, 90, −10, 30, 60), respectively. The top view of the whole simulation scene is shown in Fig. 12(a) and the positions are shown in Fig. 12(b) considering the geometrical layover and shadow with an incident angle of 40°. Fig. 13 is the simulated SAR raw echo. Fig. 14(a) is the SAR image by the proposed method and Fig. 14(b) is the corresponding gray intensity image.

From both of the point and extended target simulations, it can be seen that the proposed method has the ability to achieve fine performance of SAR imaging.
5. Conclusions

This paper proposes an alternative extended chirp scaling algorithm to process spaceborne sliding spotlight SAR data based on a two-step approach. We firstly use SPECAN technique to avoid the azimuth spectrum folding effect and CS algorithm to achieve data focusing. In order to resolve the azimuth sidelobe suppression problem, a novel azimuth spectral weighting strategy is proposed which is executed on the focused image. It is precise, and due to a post-process which only involves matrix multiplications and short FFTs, it is also efficient and particularly simple to be implemented. Compared with other imaging method, the algorithm presented in this paper may suffer a little more computation except when the raw data azimuth pixel number is very approximate to the focused image azimuth pixel number. However, the proposed method is free of the limitation of sub-aperture division and does work even when the PRF approaches to the instantaneous azimuth bandwidth while the former one may fail. Experiments carried out on simulated data confidently demonstrate the validity of the proposed algorithm.

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References


Fig. 14 SAR imaging results.


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