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Simulation modeling and analysis of the hop count distribution in cognitive radio ad-hoc networks with shadow fading

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ABSTRACT

The number of hops between source node and destination node is a key parameter in studying multi-hop wireless networks. Although hop count in wireless ad-hoc networks (AHNs) has been studied in the literature, no works on investigating the hop count characteristics in cognitive environments have been carried out. In this paper, we model cognitive radio ad-hoc networks (CRAHNs) as geometric random graphs and then propose a framework for studying the hop count distribution and correlated connectivity of communication path between two arbitrary nodes in CRAHNs with shadow fading. The framework consists of an algorithm and a methodology. Specifically, from the perspective of geometric random graph, the algorithm finds all possible paths between two arbitrary nodes and returns the hop count of the shortest path between them by using the global location information of nodes, i.e. primary users – PUs and secondary users – SUs, and the active states of PUs as input data. Meanwhile, through huge number of random network topology trials, the methodology returns the hop count distribution and connection status of communication path between two arbitrary nodes in CRAHNs with shadow fading. From the evaluating scenarios in this paper, important features of hop count distribution and connectivity and their correlating relationship in CRAHNs with shadow fading are revealed and compared with those in AHNs and in CRAHNs without shadow fading.

1. Introduction

A wireless ad-hoc network is a collection of wireless nodes which dynamically form a network without relying on any infrastructure [1]. The network is formed as soon as one of wireless nodes wants to communicate with one or many nodes. Routing paths in wireless ad-hoc network are often multi-hop paths because destination node is usually out of the transmission range of source node. Thus, before reaching the destination, data packets travel through some intermediate nodes between source node and destination node. The hop count specifies the number of hops on the path between source node and destination node. The study of the hop count of multi-hop path in wireless ad-hoc networks is very important because

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it can provide an evaluation of the network performance such as: (i) estimation of packet delivery ratio, (ii) estimation of end-to-end-delay if per hop delay is known, (iii) estimation of network traffic if the number of simultaneous communication flows is given, (iv) determining the flooding cost and search latency of route stabling process in routing protocols, (v) studying of connectivity and capacity of multi-hop path.

In the literature, the hop count information has been exploited in many applications in wireless AHNs. Particularly, in [2], the effect of slot reuse in MAC protocols is studied. Slot reuse scheme increases channel utilization and thus reduces the size of superframe required for every sensor node to send its data to a sink safely. However, a slot scheduling scheme may produce wasted slots and incur transmission interference. The authors calculate the superframe size based on the information of node depth distribution in tree topology and conclude that the removal of slot reuse tends to make the proposed I-MAC protocol more robust against node mobility in small network. In [3], the authors consider distance estimation based on hop counts, i.e. the minimum number of relay nodes need for communication, and analyze the effects of mobility on this estimation technique. It is indicated that mobility positively influences the error rate, counteracts the error induced by low density. Nevertheless, a high mobility may increase the error, turn a natural overestimation into an underestimation of the respective distance. A tree link state routing protocol (TLSR) in which topology management function and routing function cooperates closely, is proposed in [4]. More specifically, topology management protocol builds topological information in terms of hop count at the Internet Gateway (IG), and a routing protocol exploits the topology information, tackling the inherent problem of excessive control overhead which appears in link state routing protocols.

Due to important influences of hop count to the performance of ad-hoc networks, researchers get attracted to analyzing the hop count of multi-hop path in conventional ad-hoc networks (AHNs) by using both mathematical analysis approach and simulation approach. Specifically, the probabilistic analysis in [5] captures the bounds on hop count from a given Euclidean distance between two nodes and vice versa. This analysis and its potential applications are fully given in [6]. The authors in [7] derive the average progress per hop. Then, from the derived per-hop progress and the path connectivity probability, they express the probability distribution of the expected hop count in multi-hop wireless networks. A theoretical study of the expected number of hops between two random nodes in multi-hop ad hoc networks where the distance between them follows uniform distribution or non-uniform distribution (i.e. due to the random waypoint mobility model) are presented in [8,9] and in [10], respectively. The probability related to the number of hop between two nodes subject to both shadowing and small-scale fading is provided in [11]. However, these aforementioned works consider the hop count of the path between two nodes locating at a specific distance far away from each other, which may result in major differences in the hop count compared with that of the path when considering two arbitrary nodes locating randomly in the network.

Recently, cognitive radio ad-hoc networks (CRAHNs) [12] have already received much attraction from researchers. In CRAHNs, the location and operation of primary users (PUs) follow spatial and temporal distributions, respectively. The secondary users (SUs) opportunistically utilize the spectrum holes unoccupied by the PUs so that the efficiency of spectrum usage is significantly improved. Consequently, in a secondary network, communication links depends not only on the distance between them but also on the availability of the communication channel. When PUs appear, the SUs have to evacuate the borrowed licensed band and move to other available ones. Some SUs may fail to obtain any available channel and have to stop their transmissions until available channels emerge. This feature reduces the actual number of SUs involving during the path discovery phase; as a result, communication paths among SUs may be longer or cannot be established. Similar to other kinds of wireless networks, connectivity is also a fundamental property of CRAHNs. Studies on connectivity of CRAHNs can be classified according to wireless channel model, i.e. non-fading [13–15] and fading [16], or antenna model equipped for SUs, i.e. omnidirectional antenna [13–16] and directional antenna [17].

In this paper, we are interested in investigating the hop count distribution of path in CRAHNs with shadow fading. Additionally, the correlative relationship between the hop count and connectivity of CRAHNs is also considered and compared with AHNs. The motivations and contributions of our work in this paper are presented in the following section.

2. Motivations and contributions

Fig. 1 describes the motivations behind our work in this paper, i.e. the coexistence of PUs in CRAHNs may influence the hop count and connectivity of the path between two SUs. As we can see in Fig. 1, when no active PUs influence all

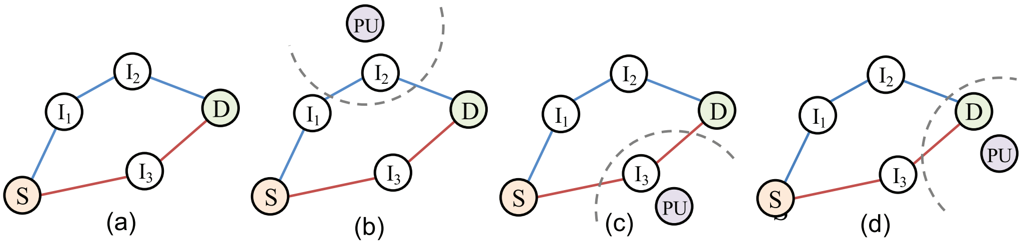


Fig. 1. Motivation: the impact of cognitive environment on the hop count and connectivity of the path between two secondary users selected as source node (S) and destination node (D).

wireless links as in Fig. 1(a) or influence wireless links of I_2 as in Fig. 1(b), the shortest path from source node (S) to destination node (D) is still $S \to I_3 \to D$ whose hop count is 2. When an active PU locates near I_3 as in Fig. 1(c), I_3 is not allowed to communicate. Thus, the new shortest path is $S \to I_1 \to I_2 \to D$ whose hop count is 3. In the worst case, when an active PU locates near S or D as in Fig. 1(d), no paths between S and D can be established. In summary, because of cognitive environment, the hop count and connectivity of multi-hop path in CRAHNs may have major differences compared with conventional AHNs. We will study these features from the viewpoint of stochastic geometry and random graph [18,19] via huge instances of network topologies. Specifically, we are interested in the questions: How do the density, average active rate of PU, the characteristics of wireless channel quantitatively affect the hop count distribution of communication path between two arbitrary SUs in CRAHNs? What is the relationship between path length and path connectivity? How different is the hop count distribution in CRAHNs with shadow fading compared with that in AHNs and in CRAHNs without shadow fading? Since the deep knowledge of hop count properties in cognitive environment is vitally important for network designers to evaluate the network performance of CRAHNs, in this paper, we propose a framework to answer the above questions through various evaluating scenarios. The following is a summary of the main contributions in this paper.

- In contrast to previous works which consider the hop count between two nodes staying far away at a specific distance in AHNs, we study the distribution of hop count between two arbitrary nodes locating randomly in CRAHNs.
- We model CRAHNs as geometric random graphs and propose a framework to achieve the probability mass function of hop count distribution in CRAHNs by using the global location information of PUs and SUs, together with the active state of PU and channel condition in each network topology.
- The characteristics of hop count distribution under the impact of SU's density, PU's average active rate, and shadow fading degree are examined. In addition, the correlation between path length and path connectivity are also considered.
- The distribution of hop count and connectivity of path in CRAHNs under shadow fading wireless channel is compared to those in AHNs with shadow fading and in CRAHNs without fading.

The rest of this paper is organized as follows. Section 3 describes the system model of CRAHNs used in this paper. In Section 4, we present our proposed framework to investigate the distribution of hop count between two arbitrary SUs in CRAHNs. Section 5 shows the results and discussions of the hop count distribution and connectivity of communication path in CRAHNs and compared results with AHNs under several evaluating scenarios. Finally, Section 6 concludes the paper.

3. System model

3.1. Spatial node distribution

In this paper, we consider a wireless multihop CRAHN where a secondary ad-hoc network coexists with a primary ad-hoc network on a licensed frequency band. The SUs can use this frequency band only when their transmissions do not interfere with PUs. Similar to previous works in the literature [13–17,20,21], wireless nodes, i.e. SU and PU, are identically and independently distributed (i.i.d.) in a square area $a \times a$ according to a homogeneous Poisson point process [22] with a known node density $\rho = N/a^2$. The homogeneous Poisson point process is defined by the following property:

– The number of nodes M in a specific finite subarea A follows a Poisson distribution, i.e.

$$P(m \text{ nodes in } A) = P(M = m) = \frac{\eta^m}{m!} e^{-\eta}, \tag{1}$$

with an expected value $E[M] = \eta = \rho A$.

The entire CRAHN is modeled as geometric random graph where the presence or absence of a link between any two nodes depends on deterministic location dependent factor and stochastic factor coming from the availability and the quality of licensed channel, and the node selection criterion of routing algorithm. Detailed descriptions of primary network, secondary network, wireless channel, and routing algorithm used in this paper are as follows.

3.2. Primary network

The primary network consists of N_P primary users distributed in network area $a \times a$ according to homogeneous Poisson point process of density $\rho_P = N_P/a^2$. The operation of primary user on licensed spectrum band is associated with an independent and identical ON–OFF state where the number of times that PUs occupy licensed spectrum in a unit of time follows Poisson distribution with average rate λ_P. The primary network is denoted as $G(\rho_P, \lambda_P)$.

3.3. Secondary network

The secondary network consists of N_S secondary users, also distributed in network area $a \times a$ according to homogeneous Poisson point process of density $\rho_S = N_S/a^2$, and denoted by $G(\rho_S)$. Without the impact of primary network, $G(\rho_S)$ models a standalone networks as in AHNs. However, in CRAHNs, the coexistence of PUs creates temporal-spatial spectrum agility. Thus, some network related properties, e.g. the hop count distribution and connectivity between two arbitrary SUs, is different compared with that in AHNs. These features will be investigated in detail in this paper.

3.4. Wireless channel

Generally, to characterize the wireless channel between two nodes, a simple model called *geometric disk model* is used [13–15,20]. According to this model, two nodes are link together, if the distance between them is less than a certain threshold distance r_0. Such a purely geometric disk model is only sufficient when deterministic, distance-dependent channels are considered. However, it is well-known that the wireless channels can be modeled in a more realistic manner, i.e. we should consider the randomness of channel conditions induced by shadowing effects that are caused by obstacles in communication environments.

To describe the shadow fading wireless channel used in this paper, we consider two nodes i and j locating at a relative distance $l(i, j)$. Nodes i transmits signal with power $p_t(i)$ that is received by node j with power $p_r(j)$. Consequently, the signal attenuation between these two nodes is given by

$$\beta(i, j) = \frac{p_t(i)}{p_r(j)}, \tag{2}$$

and can be expressed in dB as

$$\beta(i, j) = 10\log_{10}\left(\frac{p_t(i)}{p_r(j)}\right)(dB). \tag{3}$$

In a shadow fading environment, $\beta(i, j)$ is comprised of two components: a deterministic component $\beta_1(i, j)$ and a stochastic component $\beta_2(i, j)$. The deterministic component is given by

$$\beta_1(i, j) = \alpha 10\log_{10} l(i, j)\ (dB), \tag{4}$$

where α is the path loss exponent (e.g. $\alpha \approx 2$ in free space, $\alpha \approx 2.7$ to 5 in urban area). The stochastic component $\beta_2(i, j)$ is assumed to follow a log-normal probability density function. Thus, $\beta_2(i, j)$ in dB follows a normal density function [23,24], i.e.,

$$f_{\beta_2}(\beta_2) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\beta_2^2}{2\sigma^2}\right)(dB). \tag{5}$$

Generally, the value of standard deviation σ ranges up to 10 dB. The total attenuation in dB is the summation of two above components

$$\beta(i, j) = \beta_1(i, j) + \beta_2(i, j)\ (dB). \tag{6}$$

From the perspective of signal propagation, node j receives the signal from i properly if $p_r(j)$ is larger than or equal to a certain threshold power $p_{r,\, th}(j)$, which can be referred as receiver sensitivity. In other words, if signal attenuation $\beta(i, j)$ is less or equal to a threshold β_{th}, it is said that node i can establish a wireless link to node j. We assume that channels are symmetric and all nodes have the same p_t and $p_{r,\, th}$. Those assumptions will be used to evaluate the hop count distribution and corresponding connectivity of CRAHNs in our proposed simulation analysis framework.

When $\sigma = 0$ the log-normal shadowing model reduces to the geometric disk model with radius $r_0 = 10^{\frac{\beta_{th}}{10\alpha}}$. Without loss of generality, for better illustration, we use wireless nodes with geometric disk model when illustrating CRAHNs and AHNs.

3.5. Routing algorithm

In addition to the impact of fading on the wireless channel between two nodes, the impact of higher layer also affects network performance. In this paper, we consider the cross-layer issue by investigating the performance of CRAHNs using the greedy forwarding (GF) routing algorithm [25], a typical distributed routing algorithm. The GF routing belongs to the category of geographic routing algorithms and is a widely used routing algorithm for wireless multihop networks. According to GF, each node makes routing decision independently of other nodes by using the location information of its own, its neighboring nodes, and the destination. GF has shown its advantages in wireless multihop network because of its distributed feature, simple routing algorithm with low control overhead. For generality of the results, we consider a basic greedy forwarding algorithm, as in [26], that operates following two rules: 1) every node tries to forward data packet to the neighbor which is closest to the destination. 2) if a node cannot find a next-hop neighbor that is closer to the destination than itself, the path from it to destination does not exist.

4. Our proposed simulation analysis framework

In this section, we propose a simulation analysis framework to study the hop count statistics and its correlating relationship with connectivity. Fig. 2 shows the basic concept of our proposed simulation analysis framework. The framework comprises of an algorithm which gives the hop count of the shortest path between two arbitrary SUs and a methodology which exploits the algorithm to achieve the probability mass function (pmf) of the hop count distribution in CRAHNs. We will present in detail two components of our proposed framework in the following subsections.

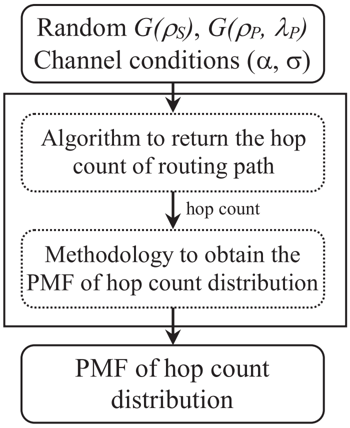


Fig. 2. The basic concept of our proposed simulation analysis framework.

4.1. The proposed algorithm to obtain the hop count between two arbitrary nodes

The main idea of the proposed algorithm is to find all possible multi-hop paths and return the hop count of the shortest path between two arbitrary SUs in CRAHNs. The algorithm takes the information of SU's location in secondary network $G(\rho_S)$, PU's location and active state in primary network $G(\rho_P, \lambda_P)$, and wireless channel conditions (α, σ). Compared with our algorithm, Dijkstra's shortest path algorithm requires a connected network as an input [27]. However, in CRAHNs, due to the random nature of node location and wireless channel quality, a node may not have any connection with other nodes. Thus, a connected network may not be available. However, our proposed algorithm can analyze the connection of two nodes in arbitrary network and returns the hop count of the path between them if they are connected.

Fig. 3 shows the pseudo code of our proposed algorithm. We will explain step-by-step the process of the algorithm as follows.

- **Step 1**: From the information of secondary network $G(\rho_S)$, primary network $G(\rho_P, \lambda_P)$, wireless channel conditions (α, σ), our proposed algorithm will check the connection between source node and destination node from the perspective of GF routing protocol. The algorithm returns *hop count = Inf* if the operation source node and destination node are blocked by any neighboring active PUs (refer to Lines 3–5).
- **Step 2**: The algorithm returns *hop count = 1* if source node and destination node are connected with each other, i.e. signal attenuation at destination node is less or equal to a threshold, $\beta \leq \beta_{th}$ (refer to Lines 6-7).
- **Step 3**: Otherwise, the algorithm finds all the possible paths from source node to destination node and gives the hop count of the shortest path. First, the available child nodes of source node (i.e. nodes which have wireless communication links with source node and not be affected by active PUs) are found. If source node has no child nodes, it means the source-destination path cannot be established and *hop count = Inf*. If not, the child nodes of source node are added into the *gonna_chk_nodes* list that will be checked for their child nodes later. Moreover, their information (*node id, hop count* = 1) is stored in *hopcount_cache*. Then, for each node in *gonna_chk_nodes* list, its hop count and child nodes are obtained. If the hop count of a child node has not been known, the latest hop count of this node is added to *hopcount_cache* and its node id is added to *gonna_chk_nodes* (refer to Lines 20–22). If the hop count of a child node is already in *hopcount_cache* but the latest hop count is less than previous one, the hop count of this child node is updated and child node id is also added to *gonna_chk_nodes* (refer to Lines 23–26). Node which already checked for its child nodes is removed from *gonna_chk_nodes* list. The processes in **Step 3** are repeated as long as the *gonna_chk_nodes* is not empty. If destination node can be reached, the algorithm will return the smallest number of hops from source node to destination node. If not, the algorithm returns *hop count = Inf* because no multi-hop routing paths between source node and destination node can be established.

It should be noticed that our proposed algorithm can also be used for studying the hop count between two arbitrary nodes in AHNs when eliminating the impact of primary network $G(\rho_P, \lambda_P)$ by setting $N_P = 0$ or $\lambda_P = 0$.

We implement our algorithm by using MATLAB and illustrate in Fig. 4 the results of four typical connecting conditions of source SU and destination SU, together with the corresponding hop count of the shortest path between them. There are 100 SUs and 5 PUs in a network area of 1000 m × 1000 m. The average active rate of PU is 0.5. Path loss exponent, $\alpha = 3$, signal attenuation threshold, $\beta_{th} = 65$ dB. Fig. 4(a), (b), and (c) presents the cases when routing path from source node to destination node cannot be established because source node (red dot), or destination node (green dot), or both are in the transmission range of active PUs (black dots), respectively. Fig. 4(d) presents the case when routing paths from source node to destination node can be established. However, due to the existence of active PUs, the routing paths have to detour. The hop count of the shortest path is 6.

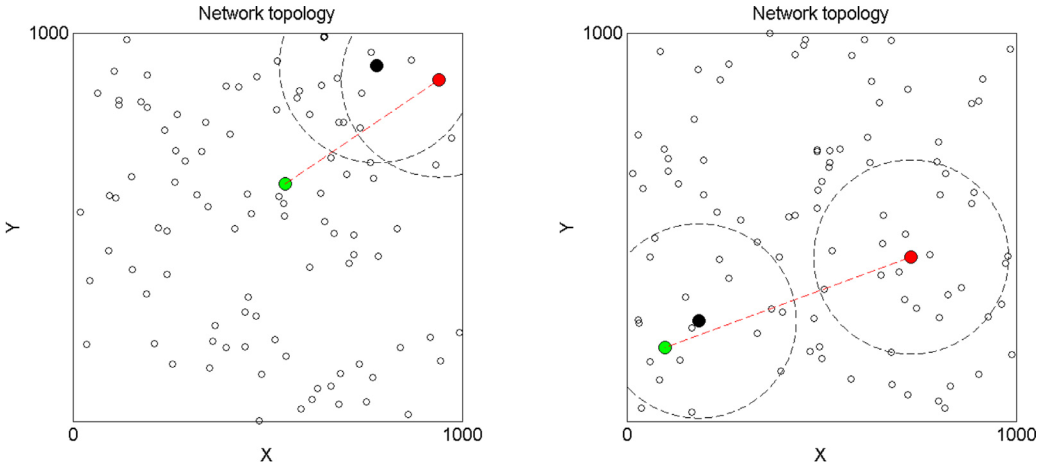
```
HopCountCRAHN($G(\rho_S)$,$G(\rho_P,\lambda_P)$,$\alpha$,$\sigma$)
    for each active PU
        if $\beta$(source, active PU) $\le \beta_{th}$ or $\beta$(destination, active PU) $\le \beta_{th}$ then
            hop count = Inf;
            stop = 1; break;

    if $\beta$(source, destination) $\le \beta_{th}$ and stop $\neq$ 1
        hop count = 1;

    if $\beta$(source, destination) $> \beta_{th}$ and stop $\neq$ 1 then
        Find child nodes of src;
        if child nodes of source = $\emptyset$ then
            hop count = Inf;
        else
            Add child nodes of source to gonna_chk_nodes list;
            Store node ids and hop count = 1 of these nodes to hopcount_cache table;
            while (gonna_chk_nodes $\neq \emptyset$)
                for each chk_node in gonna_chk_nodes
                    Get the hop count of chk_node from hopcount_cache, assign to hc_chk_node;
                    Find child nodes of chk_node, assign to child_nodes list;
                    for each chd_node in child_nodes
                        if chd_node $\notin$ hopcount_cache then
                            Add latest hop count of chd_node, i.e. hc_chk_node + 1, to hopcount_cache;
                            Add chd_node to gonna_chk_nodes;
                        else
                            if latest hop count of chd_node < its previous one in hopcount_cache then
                                Update hop count of chd_node in hopcount_cache with latest hop count;
                                Add chd_node to gonna_chk_nodes;
                Remove chk_node from gonna_chk_nodes;
                if destination $\in$ hopcount_cache then
                    hop count = hop count of destination from source;
            if destination $\notin$ hopcount_cache then
                hop count = Inf;
return (hop count)
```

Fig. 3. The pseudo code of our proposed algorithm to check the connectivity of two arbitrary SUs and return the hop count between them.

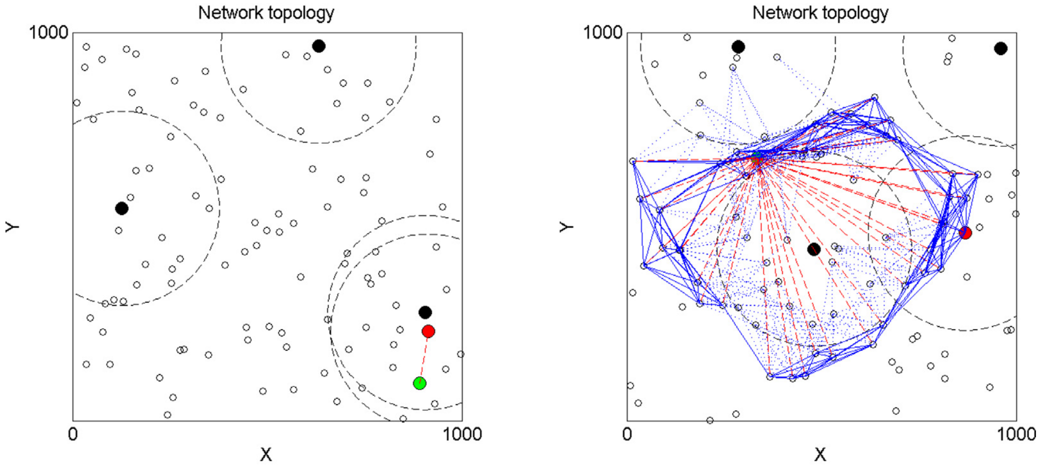
To evaluate the hop count between two arbitrary nodes in AHNs, we run our algorithm with $N_P=0$ or $\lambda_P=0$. The difference in hop count between two nodes in AHNs and CRAHNs with the same configuration of nodes' location is depicted in Fig. 5. As we can see in Fig. 5, hop count of the shortest path from source node to destination node in AHNs is 4. However, in CRAHNs, the presence of two active PUs prohibits some intermediate SUs from communicating. Thus, the hop count of the shortest path is 8.

4.2. The proposed methodology to obtain the PMF of hop count distribution

The main goal of the proposed methodology is to obtain the probability mass function (pmf) of hop count distribution between two arbitrary SUs in CRAHNs by using our proposed algorithm to collect the hop count of the shortest path between these two SUs with a huge number of random network topology trials. The flow chart of our proposed methodology is depicted in Fig. 6. The process of this methodology can be summarized as follows.



(a) Blocked source node (b) Blocked destination node



(c) Blocked source and destination nodes (d) Detoured multi-hop path

Fig. 4. Illustration of connecting conditions of two arbitrary SUs in CRAHNs and corresponding hop count of the path between them; $a = 1000$ m, $N_S = 100$, $N_P = 5$, $\lambda_P = 0.5$, $\alpha = 3$, $\sigma = 0$, $\beta_{th} = 65$ dB.

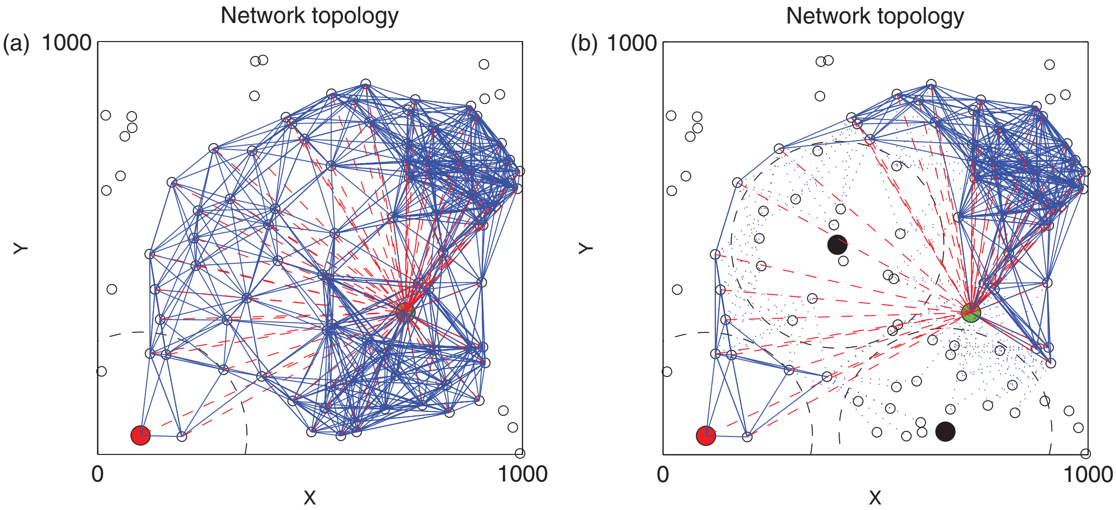


Fig. 5. Differences in the hop count between two nodes in AHNs and CRAHNs. (a) Hop count = 4 (AHNs), (b) Hop count = 8 (CRAHNs).

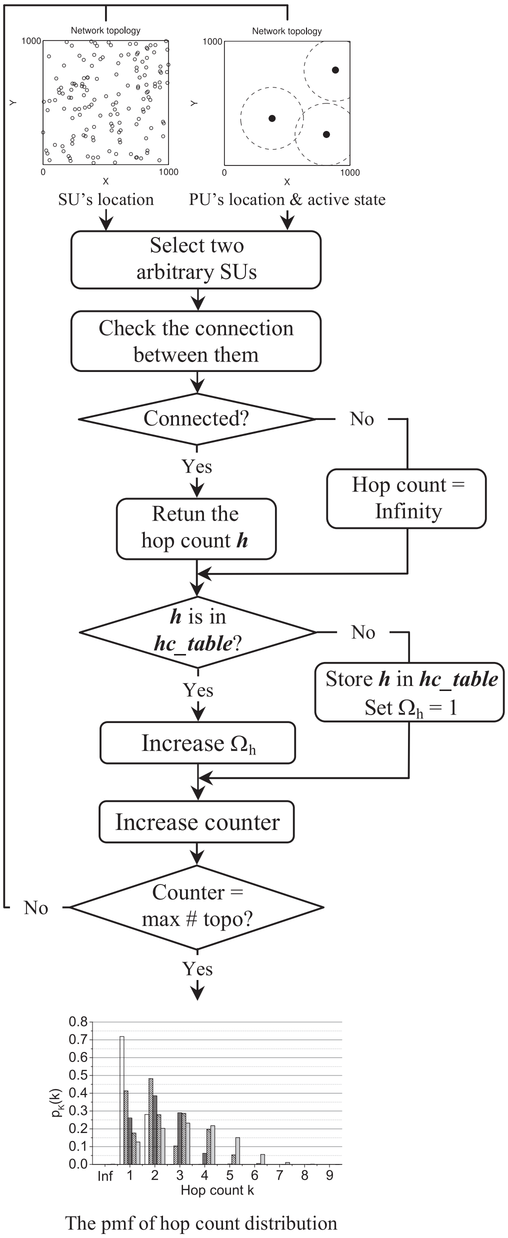


Fig. 6. The flowchart of our proposed simulation methodology used to obtain the hop count distribution (pmf) of multi-hop path between two arbitrary SUs in CRAHNs.

- **Step 1**: A square network area with size of $a \times a$ is created. Next, N_S secondary users and N_P primary users are placed in this square network area by using uniform distribution. The operation of each PU follows Poisson distribution with average active rate λ_P. The communication links among nodes, i.e. for both PUs and SUs, are determined by shadow fading wireless channel model.
- **Step 2**: Two random SUs in the network are selected as source node and destination node. Then, our proposed algorithm is used to check the connection between them. It returns the hop count of the shortest path if there exists at least one path between these two SUs.
- **Step 3**: A *hc_table* is used to collect the information of (i) the value of hop count h and (ii) the number of h-hop paths, Ω_h. More specifically, if the hop count value returned by the algorithm does not exist in *hc_table*, it will be added and Ω_h is set to 1. Otherwise, Ω_h is increased.

The above processes are repeated Ω times for each investigated network parameter. For each time, a new distribution of wireless nodes is created and new source node and destination node are randomly selected. The hop count h between these two nodes and the number of h-hop paths are recorded to derive the final probability mass function (pmf) of hop count distribution.

Obviously, the pmf of hop count distribution of multi-hop path between two arbitrary SUs is the union of the probabilities of all possible h-hop shortest paths appear in the network, which are calculated as the ratio of the number of network topologies with h-hop shortest path, Ω_h, to the total network topology trails, Ω. Thus, in brief, this pmf of hop count distribution can be expressed as

$$\bigcup_{h=1}^{H_{\max}} \frac{\#\text{ topologies with } h\text{-hop shortest paths}}{\#\text{ network topology trials}} = \bigcup_{h=1}^{H_{\max}} \frac{\Omega_h}{\Omega}. \tag{7}$$

5. Experimental results and discussions

In this section, we implement our proposed framework on MATLAB and present experiment results to give insights into the characteristics of hop count distribution in CRAHNs with shadow fading and compare with those in AHNs and CRAHNs without fading. Nodes are deployed in a 500 m × 500 m following homogeneous Poisson process. The path between two arbitratry nodes selected as source SU and destination SU is determined by the basic greeding forwarding algorithm. Hop count distribution of this path is studied with different settings of network parameters such as SU's density, PU's average active rate, and shadow fading degree. In all evaluating scenarios, to reflect the influence of node mobility and spectrum mobility in CRAHNs, we generate 50,000 network topology trails with random locations of SUs and PUs, random operations of PUs on licensed band in each topology, and then record the hop count of the path between two randomly selected SUs in each trail to plot results in simulation graphs. When only connection status of the path between these two nodes is checked, we obtain path connectivity, P_{path}, which is calculated as the statistical average percentage of connected path, i.e.

$$P_{path} = \frac{\#\text{ topologies with connected paths}}{\#\text{ network topology trials}}. \tag{8}$$

This Monte Carlo simulation approach is also used in [17,21]. We should be reminded that random walk mobility models are characterized by a uniform stationary node spatial distribution, independent of how the initial location of a node is selected [28]. Moreover, it is stated in [29] that a homogeneous Poisson point process provides a good approximation for a uniform distribution of nodes in the network area.

5.1. The impact of SU's density

Fig. 7 shows the hop count distribution, average hop count, and corresponding connectivity of the path between two arbitrary nodes in CRAHNs with shadow fading ($\sigma = 6$) as functions of SU's density. The number of SUs, N_S, varies from 20

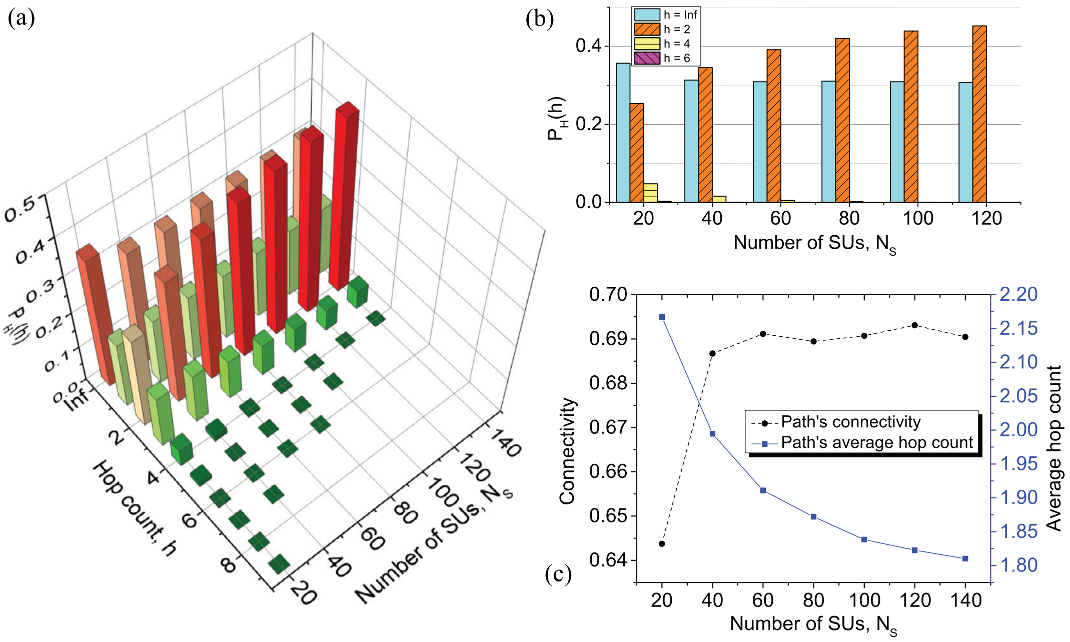


Fig. 7. Hop count distribution, average hop count, and connectivity of the path between two arbitrary nodes in CRAHNs with shadow fading ($\sigma = 6$) versus different SU's density, N_S; $a = 500$ m, $N_P = 3$, $\lambda_P = 0.3$, $\alpha = 3$, $\beta_{th} = 65$ dB.

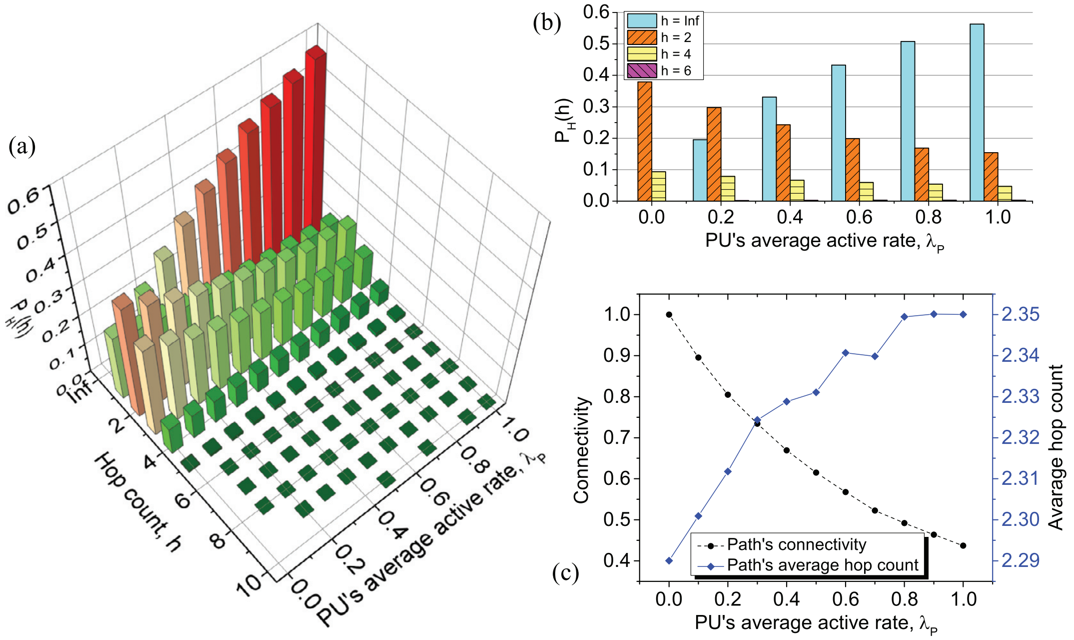


Fig. 8. Hop count distribution, average hop count, and connectivity of the path between two arbitrary nodes in CRAHNs versus different average active rates of PU, λ_P; $a = 500$ m, $N_S = 100$, $N_P = 3$, $\alpha = 3$, $\sigma = 2$, $\beta_{th} = 65$ dB.

to 140. Other parameters are the number of PUs, $N_P = 3$, PU's average active rate, $\lambda_P = 0.3$, path loss exponent, $\alpha = 3$. From Fig. 7(a), the probability that routing paths whose hop count $h \geq 5$ appear in the network is negligible. Especially, no paths whose hop count $h = 6$, 7, and 8 when the number of SUs, $N_S \geq 100$, 60, and 40, respectively. It can also be observed from Fig. 7(b) and (c) that when SU's density is very low, i.e. $N_S = 20$, the probablity of failure in establishing path, $P_H(h = Inf)$, is 0.356 which results in the lowest corresponding path connectivity of 0.644 and the highest average hop count of 2.167. It is because at low node density, a SU can rarely find neighbors or have neighbors which are not toward the destination node, resulting in longer paths. When $N_S = 40$, path connectivity rapidly increases. However, when N_S increases further, path connectivity among SU remains unchanged around 0.69 due to the fact that the spatial-temporal licensed spectrum occupation of PUs, on average, creates a "prohibited area" in CRAHNs. Thus, the remaining network resource available for routing among SUs is reduced. As a result of this effect, even when more SUs are put into the networks, path connectivity cannot be increased further. On the other hand, the average hop count of the path continues reduces to the lowest value of 1.81 when $N_S = 140$ because of the increase in the probability that short path, e.g. $h = 2$, appears in the network. In summary, average hop count and connectivity of the path show major different characteristics under the impact of SU's density.

5.2. The impact of PU's average active rate

Fig. 8 illustrates the impact of PU's average active rate on the hop count distribution, average hop count, and corresponding connectivity of the path. The average active rate of SU, λ_P, is increased from 0 to 1. It should be noticed that in the case of $\lambda_P = 0$, i.e. PUs are not active, the considered CRAHN becomes AHN. An interesting feature can be seen from Fig. 8(a) and (b) that, different from the influence of SU's density, increase in PU's average active rate does not "force" routing paths to be longer. Instead, it reduces the probability that h-hop path appears in the network, $P_H(h)$, with different amounts, e.g. $P_H(h = 2)$ decreases from 0.38 to 0.15 and $P_H(h = 4)$ decreases from 0.09 to 0.05 while increases the probability of failure in establishing path, i.e. $P_H(h = Inf)$ rises significantly, from 0.19 to 0.56. In the case of AHN, i.e. $\lambda_P = 0$, routing path can always be established. The probability of direct communication is $P_H(h = 1) = 0.22$, the hop count values of multi-hop paths are often 2 and 3 with the probabibility of 0.38 and 0.31, respectively. Next, the relationship of average hop count and connectivity of routing path is shown in Fig. 8(c). We can observe that path connectivity is rapidly degraded from 1 to 0.43 as PU's average active rate varies from 0 to 1. In contrast, average hop count just slightly increases from 2.29 to 2.35 (+2.6%). This observation infers that in CRAHNs increasing in PU's average active rate may result in dominant unsuccessful path establishment rather than longer path because there are more possibilities that all paths from source to destination are blocked by PUs due to significant decrease in the available network resource for routing among SUs.

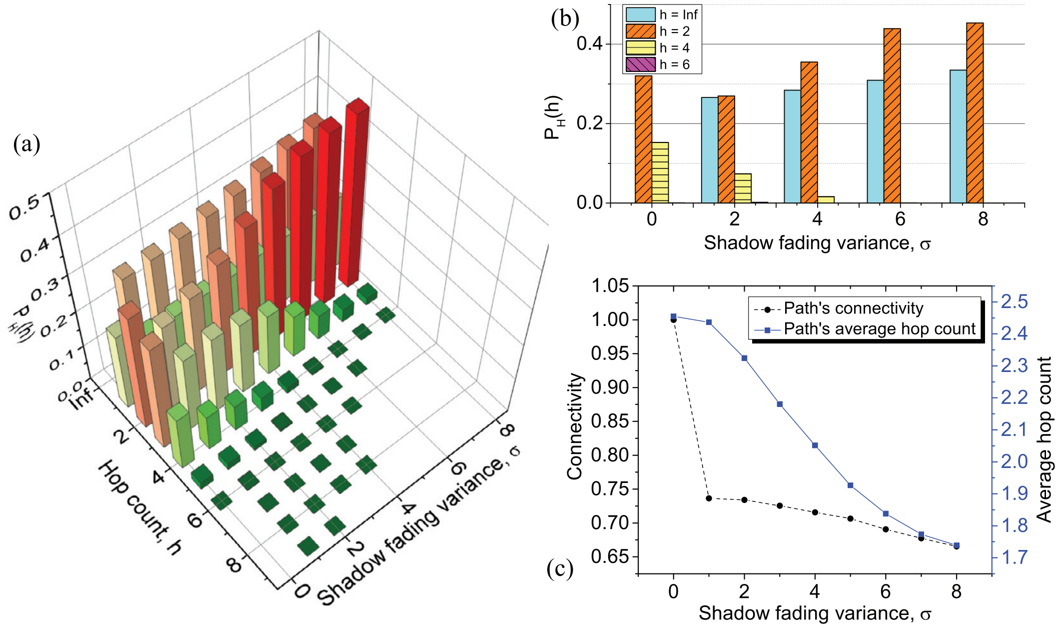


Fig. 9. Hop count distribution, average hop count, and connectivity of the path between two arbitrary nodes in CRAHNs versus different shadow fading degrees, σ; $a = 500$ m, $N_S = 100$, $N_P = 3$, $\lambda_P = 0.3$, $\alpha = 3$, $\beta_{th} = 65$ dB.

5.3. The impact of shadow fading degree

The influence of shadow fading is examined in this scenario. Fig. 9 plots the hop count distribution, average hop count, and corresponding connectivity of the path versus shadow fading variance, σ. To evaluate the impact of different levels of shadow fading, σ is varied from 0 to 8, corresponding to wireless channel with non-fading and high fading. From Fig. 9, we can see some interesting features as follows. In the case of non-fading, path loss exponent $\alpha = 3$, signal attenuation threshold $\beta_{th} = 65$ dB, the transmission area of every node in CRAHN becomes a circular disk with radius $r_0 = 10^{\frac{65}{10\times 3}} = 146.78$ m. The density of SU is $\rho_S = 100/500^2 = 4\times 10^{-4}$ node/m^2. Although PUs exists in the network, with such transmission range and SU's density, there always exist SUs which are not interfered by active PUs and have neighboring SUs. Consequently, path connectivity between two random nodes is 1. However, when $\sigma = 1$, path connectivity drops sharply to 0.74 then gradually reduces as σ further increases. The reason is that higher degree of fading not only adds more SU-SU links but also creates more SU-PU links among SUs and PUs which are located outside the distance r_0. Although high fading variance "helps" to reduce the hop count of the path, at the same time it significantly degrades path connectivity. To sum up, fading has negative impact on the performance of multihop path in CRAHNs.

6. Conclusion

This paper is motivated by the fact that the spatial-temporal existence of PUs and shadow fading of wireless channels have the significant impact on the hop count and connectivity between two arbitrary SUs in CRAHNs. We introduce a framework composing of an algorithm and a simulation-based methodology to obtain the hop count distribution between two arbitrary nodes in CRAHNs and AHNs based on global information, i.e. bird's eye-view, of SU and PU's locations, PU's active state, and the quality of wireless channel. Geometrical location aware greedy routing protocol is also taken into consideration. The impact of several network parameters of CRAHNs such as the SU's density, PU's average active rate, and shadow fading degree on the hop count distribution of communication path between two randomly selected SUs has been studied through different evaluating scenarios. We also compare the hop count distribution and connectivity in CRAHNs under shadow fading environment with those in AHNs, i.e. when PU's average active rate $\lambda_P = 0$, and in CRAHNs without shadow fading, i.e. shadow fading variance $\sigma = 0$. From the experiment results, some important conclusions can be summarized as follows:

- When SU's density gets higher, path connectivity only increases upto a specific maximum value while the average hop count of path continues decreases.
- Increasing PU's average active rate results in dominant unsuccessful path establishment rather than longer paths.
- Shadow fading does not give positive effect in CRAHNs. Although shadow fading helps to reduces average path length, it remarkably degrades path connectivity.

The results of hop count distribution and connectivity obtained in this paper provide guidelines for network designers to evaluate the characteristics of communication path in CRAHNs in terms of path length and path connectivity.

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