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CP-violating rate difference relations for $B \to PP$ and $B \to PV$ in broken $SU(3)$

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Abstract

Within the Standard Model there exist certain relations between CP-violating rate differences in B decays in the $SU(3)$ limit. We study $SU(3)$ breaking corrections to these relations in the case of charmless, hadronic, two body B decays using the improved factorization model of [Nucl. Phys. B 606 (2001) 245]. We consider the cases $B \to PP$ and $B \to PV$ for both B_d and B_s mesons. We present an estimate for $A_{\mathrm{CP}}(\pi^-\pi^+)$ in terms of $A_{\mathrm{CP}}(K^-\pi^+)$.

B decays are a subject of very active research at present since they provide useful information on the dynamics of strong and electroweak interactions for testing the Standard Model (SM) and models beyond and are ideally suited for a critical analysis of CP-violation. The mixing induced CP asymmetry in $\overline{B}^0 \to \psi K_s$ versus $B^0 \to \psi K_s$ has already provided an accurate measurement of $\sin 2\beta$ [1,2]. This result is in excellent agreement with the SM. Other mixing studies, such as $\overline{B} \to \pi^-\pi^+$, are underway for determining $\sin 2\alpha$, but require more data to reduce the theoretical uncertainty associated with penguin contributions. Rate asymmetry measurements in the branching ratios of B mesons into mesons involving light quarks are also underway. These shed light on direct CP-violation in the amplitudes. Analyses of these decays to extract fundamental parameters of the SM are more difficult because of theoretical uncertainty in the calculation of hadronic modes. In general, these asymmetries arise from interference of a Cabbibo suppressed tree amplitude with a (possibly enhanced) penguin amplitude. As such, these asymmetries are sensitive to contributions through loops, that could involve physics beyond the SM. Thus, the study of direct CP-violation can be a powerful tool to probe for physics beyond the SM if the theoretical uncertainty can be reduced.

The goal of this Letter is to study the direct CP-violation asymmetry in a class of processes where there has been recent theoretical progress. These processes involve B decays into two light pseudoscalars $P_1 P_2$ or into a

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light pseudoscalar and a light vector meson PV. We identify relations between rate asymmetries which are valid in the $SU(3)$ limit in the Standard Model, and we compute $SU(3)$ breaking corrections to them using the QCD improved factorization model of Ref. [3]. We also discuss additional relations which are valid in the $SU(3)$ limit when annihilation contributions are neglected.

CP-violation in the SM arises solely from the phase in the 3×3 unitary CKM matrix, $V_{\mathrm{CKM}} = (V_{ij})$, and any CP-violating observable is proportional to $\mathrm{Im}(V_{ij} V_{il}^* V_{kj}^* V_{kl})$, with $i \neq k$ and $j \neq l$. This simple property has important implications, as, for example, it leads to relations among CP-violating rate differences, $\Delta_{PP}^{B} = \Gamma(B \to PP) - \overline{\Gamma}(\overline{B} \to \overline{P}\overline{P})$, for different decay modes. For instance, it has been shown that, with $SU(3)$ flavor symmetry, when small annihilation contributions and phase space differences are neglected, naive factorization yields the relation [4]

$$\Delta_{\pi^-\pi^+}^{\overline{B}^0} \approx -\frac{f_\pi^2}{f_K^2} \Delta_{K^-\pi^+}^{\overline{B}^0}. \tag{1}$$

This can be used to test the SM CP-violation, or to predict one rate difference if the other one is known. The above equation leads to a relation for the CP-violating rate asymmetry,

$$A_{\mathrm{CP}}\big(\pi^-\pi^+\big) \approx -\frac{f_\pi^2}{f_K^2} \frac{\mathrm{Br}(K^-\pi^+)}{\mathrm{Br}(\pi^-\pi^+)} A_{\mathrm{CP}}\big(K^-\pi^+\big), \tag{2}$$

where $\mathrm{Br}(PP)$ are the CP averaged branching ratios, $\mathrm{Br}(\pi^-\pi^+) = (5.2 \pm 0.6) \times 10^{-6}$ and $\mathrm{Br}(K^-\pi^+) = (18.6 \pm 1.1) \times 10^{-6}$ [1,2]. Eq. (2) implies the following relation between the corresponding CP asymmetries: $A_{\mathrm{CP}}(\pi^-\pi^+) \approx -2.4 A_{\mathrm{CP}}(K^-\pi^+)$.

Preliminary data on these asymmetries is just emerging, with BaBar reporting $A_{\mathrm{CP}}(\pi^-\pi^+) = -0.30 \pm 0.25 \pm 0.04$, $A_{\mathrm{CP}}(K^-\pi^+) = -0.102 \pm 0.050 \pm 0.016$ [1] and Belle reporting $A_{\mathrm{CP}}(\pi^-\pi^+) = 0.94^{+0.25}_{-0.31} \pm 0.09$ [2]. At the moment the two experiments disagree on the value of $A_{\mathrm{CP}}(\pi^-\pi^+)$ but they still have very large errors.

The most important question for theory is to establish the precision of Eqs. (1) and (2) within the Standard Model, or equivalently to estimate the corrections they receive. One can easily identify two sources of corrections for Eqs. (1) and (2): annihilation contributions, and $SU(3)$ breaking effects. Even though the relation Eq. (1) already includes some $SU(3)$ breaking effects in the factor f_π^2/f_K^2, it is necessary to have better control over these corrections in order to test the Standard Model.

To begin our analysis of the $B \to PP$ modes, we first note that there are several relations among the rate differences in these decays that follow from $SU(3)$ flavor symmetry in the SM. There are also other relations such as Eq. (1) which rely both on $SU(3)$ symmetry and on the neglect of annihilation amplitudes. It is easy to understand the origin of these relations. The decay amplitude for $B \to PP$ can be parameterized as

$$A(B \to PP) = V_{ub} V_{uq}^* T_{PP}^{B} + V_{cb} V_{cq}^* P_{PP}^{B}, \tag{3}$$

and can be decomposed into $SU(3)$ invariant amplitudes according to the representation of the effective Hamiltonian [4]. $SU(3)$ symmetry predicts that the amplitudes for $\overline{B}^0 \to \pi^-\pi^+, K^-\pi^+$ and $\overline{B}_s^0 \to \pi^-K^+, K^-K^+$ are related and this can be proved by writing the decay amplitudes in terms of the $SU(3)$ invariant amplitudes as

$$\begin{aligned} T_{\pi^-\pi^+}^{\overline{B}^0} &= T_{K^-K^+}^{\overline{B}_s^0} = 2A_{\bar{3}}^T + C_{\bar{3}}^T + C_6^T + A_{\overline{15}}^T + 3C_{\overline{15}}^T, \\ T_{K^-\pi^+}^{\overline{B}^0} &= T_{\pi^-K^+}^{\overline{B}_s^0} = C_{\bar{3}}^T + C_6^T - A_{\overline{15}}^T + 3C_{\overline{15}}^T, \end{aligned} \tag{4}$$

where A_i indicate the annihilation contributions. Both model calculations [3], and fits to experimental data [5] indicate that these annihilation amplitudes are small. The penguin amplitudes P_{PP}^{B} can be parameterized in a similar way.

We note that, even though $T(P)_{\pi^-\pi^+}^{\overline{B}^0} = T(P)_{K^-K^+}^{\overline{B}_s^0}$ and $T(P)_{K^-\pi^+}^{\overline{B}^0} = T(P)_{\pi^-K^+}^{\overline{B}_s^0}$ in the $SU(3)$ limit, there are no simple relations between the branching ratios for these decays, because the CKM factors in the T and

P amplitudes are different. However, because $\Delta^B_{PP} \sim \mathrm{Im}(TP^*)\,\mathrm{Im}(V_{ub}V^*_{uq}V^*_{cb}V_{cq})$ and $\mathrm{Im}(V_{ub}V^*_{ud}V^*_{cb}V_{cd}) = -\mathrm{Im}(V_{ub}V^*_{us}V^*_{cb}V_{cs})$ from the unitarity of the CKM matrix, we have the following relations among the CP-violating rate differences:

$$\Delta^{\overline{B}^0}_{\pi^-\pi^+} = -\Delta^{\overline{B}^0_s}_{K^-K^+}, \qquad \Delta^{\overline{B}^0_s}_{\pi^-K^+} = -\Delta^{\overline{B}^0}_{K^-\pi^+}. \tag{5}$$

These relations can be obtained by interchanging the d and s quarks (U-spin symmetry). If annihilation contributions are neglected, all the amplitudes $T^{\overline{B}^0}_{\pi^-\pi^+}$, $T^{\overline{B}^0}_{K^-\pi^+}$, $T^{\overline{B}^0_s}_{\pi^-K^+}$ and $T^{\overline{B}^0_s}_{K^-K^+}$ are approximately equal and one gets additional relations,

$$\Delta^{\overline{B}^0}_{\pi^-\pi^+} = -\Delta^{\overline{B}^0_s}_{K^-K^+} = \Delta^{\overline{B}^0_s}_{\pi^-K^+} = -\Delta^{\overline{B}^0}_{K^-\pi^+}. \tag{6}$$

Similar relations exist as well for decays with neutral mesons in the final state. These relations are more complicated than those of Eq. (6) because there are more d and s quarks to interchange, and consequently it is harder to study the effect of $SU(3)$ breaking in that case. For the remainder of this Letter we concentrate on the relations in Eqs. (5) and (6).

One must be careful, however, about the validity of these relations. In the exact $SU(3)$ limit, the Δ^B_{PP} are also exactly zero because the Standard Model conserves CP when $m_s = m_d$. It is well known that in this case it is possible to remove the phase in the CKM matrix with an appropriate rotation among d and s quarks. In order to have a non-zero Δ^B_{PP}, one cannot have an exact $SU(3)$ symmetry. In order to have CP-violation in the Standard Model no two quarks with the same charge can have the same mass; as long as $m_s \neq m_d$ there will be CP-violation regardless of how small these masses are. The relations in Eq. (5) are thus valid and non-trivial in the limit where $(m_s - m_d)$ is much smaller than the QCD scale, but not zero.

When $SU(3)$ breaking effects are included, the above mentioned relations will be modified and one needs a good understanding of these effects before using the relations to test the Standard Model. Our limited understanding of the strong interaction dynamics at low energies makes this task quite difficult. In what follows we illustrate the $SU(3)$ breaking corrections that arise within the QCD improved factorization model of Ref. [3].

Within this approach, the relevant decay amplitudes for $B \to PP$ are given by [3,6]

$$\begin{aligned}
A\big(\overline{B}^0 \to \pi^-\pi^+\big) &= i\frac{G_F}{\sqrt{2}}\big(m_B^2 - m_\pi^2\big)F_0^{B\to\pi}\big(m_\pi^2\big)f_\pi \\
&\quad \times \big[V_{ub}V^*_{ud}a_1(\pi\pi) + V_{pb}V^*_{pd}\big(a_4^p(\pi\pi) + a_{10}^p(\pi\pi) + r_\chi^\pi\big(a_6^p(\pi\pi) + a_8^p(\pi\pi)\big)\big)\big] \\
&\quad + i\frac{G_F}{\sqrt{2}}f_B f_\pi^2\bigg[V_{ub}V^*_{ud}b_1(\pi\pi) + \big(V_{ub}V^*_{ud} + V_{cb}V^*_{cd}\big)\big(b_3(\pi\pi) + 2b_4(\pi\pi)\big) \\
&\qquad - \frac{1}{2}\big(b_3^{EW}(\pi\pi) - b_4^{EW}(\pi\pi)\big)\bigg], \\
A\big(\overline{B}^0 \to K^-\pi^+\big) &= i\frac{G_F}{\sqrt{2}}\big(m_B^2 - m_\pi^2\big)F_0^{B\to\pi}\big(m_K^2\big)f_K \\
&\quad \times \big[V_{ub}V^*_{us}a_1(K\pi) + V_{pb}V^*_{ps}\big(a_4^p(K\pi) + a_{10}^p(K\pi) + r_\chi^K\big(a_6^p(K\pi) + a_8^p(K\pi)\big)\big)\big] \\
&\quad + i\frac{G_F}{\sqrt{2}}f_B f_\pi f_K\bigg[\big(V_{ub}V^*_{us} + V_{cb}V^*_{cs}\big)\bigg(b_3(K\pi) - \frac{1}{2}b_3^{EW}(K\pi)\bigg)\bigg],
\end{aligned} \tag{7}$$

where p is summed over u and c, $r_\chi^\pi = 2m_\pi^2/m_b(m_u + m_d)$, $r_\chi^K = 2m_K^2/m_b(m_u + m_s)$ and

$$a_1(M_1M_2) = c_1 + \frac{c_2}{N_c}\bigg[1 + \frac{C_F\alpha_s}{4\pi}\bigg(V_{M_1} + \frac{4\pi^2}{N_c}H_{M_1M_2}\bigg)\bigg],$$

$$
\begin{aligned}
a_4^p(M_1M_2) &= c_4 + \frac{c_3}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}\left(V_{M_1} + \frac{4\pi^2}{N_c}H_{M_1M_2}\right)\right] + \frac{C_F\alpha_s}{4\pi N_c}P^p_{M_1,2},\\
a_6^p(M_1M_2) &= c_6 + \frac{c_5}{N_c}\left(1 - 6\frac{C_F\alpha_s}{4\pi}\right) + \frac{C_F\alpha_s}{4\pi N_c}P^p_{M_1,3},\\
a_8^p(M_1M_2) &= c_8 + \frac{c_7}{N_c}\left(1 - 6\frac{C_F\alpha_s}{4\pi}\right) + \frac{\alpha}{9\pi N_c}P^{p,EW}_{M_1,3},\\
a_{10}^p(M_1M_2) &= c_{10} + \frac{c_9}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}\left(V_{M_1} + \frac{4\pi^2}{N_c}H_{M_1M_2}\right)\right] + \frac{\alpha}{9\pi N_c}P^{p,EW}_{M_1,2},
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
b_1(M_1M_2) &= \frac{C_F}{N_c^2}c_1A_1^i(M_1M_2),\\
b_3(M_1M_2) &= \frac{C_F}{N_c^2}\left[c_3A_1^i(M_1M_2) + c_5\left(A_3^i(M_1M_2) + A_3^f(M_1M_2)\right) + N_cc_6A_3^f(M_1M_2)\right],\\
b_4(M_1M_2) &= \frac{C_F}{N_c^2}\left[c_4A_1^i(M_1M_2) + c_6A_2^i(M_1M_2)\right],\\
b_3^{EW}(M_1M_2) &= \frac{C_F}{N_c^2}\left[c_9A_1^i(M_1M_2) + c_7\left(A_3^i(M_1M_2) + A_3^f(M_1M_2)\right) + N_CC_8A_3^f(M_1M_2)\right],\\
b_4^{EW}(M_1M_2) &= \frac{C_F}{N_c^2}\left[c_{10}A_1^i(M_1M_2) + c_8A_2^i(M_1M_2)\right],
\end{aligned} \tag{9}
$$

where $C_F = (N_c^2 - 1)/2N_c$ and $N_c = 3$ is the number of colors and c_i are the Wilson coefficients. The vertex, the hard gluon exchange with the spectator, and the penguin contributions are:

$$
\begin{aligned}
V_M &= 12\ln\frac{m_b}{\mu} - 18 + \int_0^1 dx\, g(x)\Phi_M(x),\\
P^p_{M,2} &= c_1\left[\frac{4}{3}\ln\frac{m_b}{\mu} + \frac{2}{3} - G_M(s_p)\right] + c_3\left[\frac{8}{3}\ln\frac{m_b}{\mu} + \frac{4}{3} - G_M(0) - G_M(1)\right]\\
&\quad + (c_4 + c_6)\left[\frac{4n_f}{3}\ln\frac{m_b}{\mu} - (n_f - 2)G_M(0) - G_M(s_c) - G_M(1)\right] - 2c_{8g}^{\mathrm{eff}}\int_0^1 \frac{dx}{1-x}\Phi_M(x),\\
P^{p,EW}_{M,2} &= (c_1 + N_cc_2)\left[\frac{4}{3}\ln\frac{m_b}{\mu} + \frac{2}{3} - G_M(s_p)\right] - 3c_{7\gamma}^{\mathrm{eff}}\int_0^1 \frac{dx}{1-x}\Phi_M(x),\\
P^p_{M,3} &= c_1\left[\frac{4}{3}\ln\frac{m_b}{\mu} + \frac{2}{3} - \widehat{G}_M(s_p)\right] + c_3\left[\frac{8}{3}\ln\frac{m_b}{\mu} + \frac{4}{3} - \widehat{G}_M(0) - \widehat{G}_M(1)\right]\\
&\quad + (c_4 + c_6)\left[\frac{4n_f}{3}\ln\frac{m_b}{\mu} - (n_f - 2)\widehat{G}_M(0) - \widehat{G}_M(s_c) - \widehat{G}_M(1)\right] - 2c_{8g}^{\mathrm{eff}},\\
P^{p,EW}_{M,3} &= (c_1 + N_cc_2)\left[\frac{4}{3}\ln\frac{m_b}{\mu} + \frac{2}{3} - \widehat{G}_K(s_p)\right] - 3c_{7\gamma}^{\mathrm{eff}},
\end{aligned}
$$

$$H_{M_1M_2} = \frac{f_B f_\pi}{m_B \lambda_B F_0^{B\to\pi}(0)} \left\{ \int_0^1 \frac{dx}{1-x} \Phi_{M_1}(x) \int_0^1 \frac{dy}{1-y} \Phi_{M_2}(y) + \frac{\alpha_s(\mu_s) r_\chi^\pi(\mu_s)}{\alpha_s(\mu_h)} \int_0^1 \frac{dx}{x} \Phi_{M_1}(x) \int_0^1 \frac{dy}{1-y} \Phi_p(y) \right\},$$

$$A_j^i(M_1M_2) = \pi\alpha_s \int_0^1 dx\, dy\, F_j^i(x,y) \quad j = \overline{1,3},$$

$$A_3^f(M_1M_2) = \pi\alpha_s \int_0^1 dx\, dy\, F_3^f(x,y), \tag{10}$$

with

$$F_1^i(x,y) = \left\{ \Phi_{M_1}(x)\Phi_{M_2}(y) \left[\frac{1}{y(1-x\bar{y})} + \frac{1}{y\bar{x}^2} \right] + \frac{4\mu_{M_2}\mu_{M_1}}{m_b^2} \frac{2}{\bar{x}y} \right\},$$

$$F_2^i(x,y) = \left\{ \Phi_{M_1}(x)\Phi_{M_2}(y) \left[\frac{1}{\bar{x}(1-x\bar{y})} + \frac{1}{y^2\bar{x}} \right] + \frac{4\mu_{M_2}\mu_{M_1}}{m_b^2} \frac{2}{\bar{x}y} \right\},$$

$$F_3^i(x,y) = \left\{ \frac{2\mu_{M_2}}{m_b} \Phi_{M_1}(x) \frac{2\bar{y}}{\bar{x}y(1-x\bar{y})} - \frac{2\mu_{M_1}}{m_b} \Phi_{M_2}(y) \frac{2x}{\bar{x}y(1-x\bar{y})} \right\},$$

$$F_3^f(x,y) = \left\{ \frac{2\mu_{M_2}}{m_b} \Phi_{M_1}(x) \frac{2(1+\bar{x})}{\bar{x}^2 y} + \frac{2\mu_{M_1}}{m_b} \Phi_{M_2}(y) \frac{2(1+y)}{\bar{x}y^2} \right\}, \tag{11}$$

where $\bar{x} = 1-x$, $\bar{y} = 1-y$ and the parameter $2\mu_M/m_b$ coincides with r_χ. The functions $g(x)$, $G_M(x)$ and $\widehat{G}_M(x)$ are given by [3]

$$g(x) = 3\left(\frac{1-2x}{1-x} \ln x - i\pi \right) + \left[2\,\mathrm{Li}_2(x) - \ln^2 x + \frac{2\ln x}{1-x} - (3+2i\pi)\ln x - (x \to 1-x) \right],$$

$$G(s,x) = -4 \int_0^1 du\, u(1-u) \ln\left[s - u(1-u)x \right],$$

$$G_M(s) = \int_0^1 dx\, G(s-i\epsilon, 1-x)\Phi_M(x),$$

$$\widehat{G}_M(s) = \int_0^1 dx\, G(s-i\epsilon, 1-x)\Phi_p(x), \tag{12}$$

where $s_i = m_i^2/m_b^2$ are the mass ratios for the quarks involved in the penguin diagrams, while $\Phi_M(x)$ and $\Phi_p(x)$ are the distribution amplitudes of the M meson. The twist-3 distribution amplitude, $\Phi_p(x)$, is equal to 1, to the order considered in the calculation. The distribution amplitude $\Phi_M(x)$ has the following expansion in Gegenbauer polynomials [3,7]

$$\Phi_M(x) = 6x(1-x)\left[1 + \alpha_1 C_1^{(3/2)}(2x-1) + \alpha_2 C_2^{3/2}(2x-1) + \cdots \right], \tag{13}$$

with $C_1^{3/2}(u) = 3u$ and $C_2^{3/2}(u) = (3/2)(5u^2 - 1)$, and is different for π and K. For π, the distribution in x must be even because the u and d quarks have negligible masses and their distributions inside the pion are symmetric. This dictates $\alpha_1^\pi = 0$. The coefficient α_2^π is estimated to be 0.1 ± 0.3. For K, the u (or d) and s quarks inside the kaon are different, leading to an asymmetry in the x distribution. So a non-zero value for α_1^K is needed and it is estimated to be 0.3 ± 0.3, while $\alpha_2^K = 0.1 \pm 0.3$ [3,7].

One also has to consider divergences contained in the hard scattering and annihilation contributions. The divergent part in the hard scattering comes from $X_H = \int_0^1 \Phi_p(x)\, dy/(1-y) \approx \int_0^1 dy/(1-y)$, while the divergent part in the annihilation is of the same form at leading order. These divergences are logarithmic and, in principle, would be absent in a full theory. Here, we follow Ref. [3] to introduce an infrared cut-off at $\Lambda_h = 0.5$ GeV and use

$$X_{H(A)} = \ln \frac{m_B}{\Lambda_h}. \tag{14}$$

The final results are insensitive to the precise value of the cut-off. As for the numerical inputs, we will use the values of the Wilson coefficients at $\mu = m_b$ [3], $\mu_h = \sqrt{\Lambda_h \mu}$, $\Lambda_h = 0.5$ GeV, $\lambda_B = 0.350$ GeV.

The decay amplitudes for $\overline{B}_s^0 \to \pi^- K^+$ and $\overline{B}_s^0 \to K^- K^+$ can be obtained from Eq. (7), by using the appropriate transition form factor $F_0^{B_s \to K}$ and by changing $1/m_B^2 \lambda_B$ to $1/m_{B_s}^2 \lambda_{B_s}$ in $H_{M_1 M_2}$.

Putting everything together, we are now able to estimate the size of different contributions and, as expected, we find that the annihilation contributions are small.

Eq. (1) incorporates $SU(3)$ breaking effects only through the difference in the decay constants f_π and f_K as they appear in naive factorization. To improve on this approximation we need to consider other sources of $SU(3)$ breaking. For example, there are $SU(3)$ breaking mass differences in both the initial B mesons and in the final state mesons. These mass differences induce corrections that are proportional to m_M^2/m_B^2 and are therefore small. The decay amplitudes are proportional to the decay constant f_{M_1} of M_1 and to the transition form factor $F_0^{B \to M_2}$, depending on which B is decaying into which final state. These form factors can also introduce $SU(3)$ breaking effects. For $\overline{B}^0 \to \pi^-\pi^+$ and $\overline{B}^0 \to K^-\pi^+$, all the corrections mentioned above account for the factor f_π^2/f_K^2 in Eq. (1). Additional $SU(3)$ breaking effects can arise from the distribution amplitude $\Phi_M(x)$, which is different for the d and s quarks. The important effect arises from the twist two distribution amplitudes. In our calculation we have used a constant $\Phi_p = 1$ for the twist-3 distribution amplitude so we have neglected $SU(3)$ breaking in this term. However, $SU(3)$ breaking effects in this term should also be taken into consideration at higher order.

To summarize, the $SU(3)$ breaking effects that we do include are the difference in the decay constants and form factors; and the difference in the α_1 and α_2 terms that appear in the twist-2 distribution amplitude. With these effects taken into account, and using $s_u = s_d = s_s = 0$ and $s_c = (1.3/4.2)^2$ in Eq. (10), the relation in Eq. (1) turns into

$$\frac{\Delta_{\pi^-\pi^+}^{\overline{B}^0}}{\Delta_{K^-\pi^+}^{\overline{B}^0}} \approx -\frac{f_\pi^2}{f_K^2}\left[\frac{1 - 0.748\alpha_1^\pi - 0.109\alpha_2^\pi - 0.017 H_{\pi\pi} - 0.004\delta_A^\pi}{1 - 0.748\alpha_1^K - 0.109\alpha_2^K - 0.017 H_{K\pi} + 0.0004\delta_A^K}\right], \tag{15}$$

where $\delta_A^\pi = 1 - 1.34 X_A^\pi - 0.36 (X_A^\pi)^2$ and $\delta_A^K = 0.1 - 0.8 X_A^K + 1.4 (X_A^K)^2$ indicate the annihilation contributions. The numerical coefficients are obtained for the input parameters discussed before with $X_{H(A)}$ real, and using $X_{H(A)} = \ln(m_B/\Lambda)$. With these input parameters, $H_{\pi\pi}$ and $H_{K\pi}$ are in the range between 0.8 to 1. This leads to very small annihilation and hard scattering contributions as can be seen from Eq. (15). If one allows complex values for $X_{H(A)}$, then the corrections can be larger [3], but we do not have a good estimate for these parameters.

The most important $SU(3)$ breaking effect that we have identified (in addition to the difference in decay constants) arises from twist-2 distribution amplitudes. Using the central values $\alpha_1^\pi = 0$, $\alpha_2^\pi = 0.1$ and $\alpha_1^K = 0.3$, $\alpha_2^K = 0.1$, the size of $\Delta_{\pi^-\pi^+}^{\overline{B}^0}/\Delta_{K^-\pi^+}^{\overline{B}^0}$ increases by a factor of 1.3, and is approximately -0.87. By taking into account the full range of values for the α-parameters, the maximum and minimum numerical values of the above ratios are, respectively, -1.4 and -0.6. This can be used to test the Standard Model and the QCD improved factorization. In particular, the sign of the rate difference is not changed by the $SU(3)$ breaking effects we have considered.

Similar calculations can be carried out for $B_s \to PP$ decays. We find that the ratio of differences

$$\frac{\Delta^{\overline{B}^0}_{\pi^-\pi^+}}{\Delta^{\overline{B}^0}_{K^-\pi^+}} \approx \frac{\Delta^{\overline{B}^0_s}_{\pi^-K^+}}{\Delta^{\overline{B}^0_s}_{K^-K^+}} \tag{16}$$

is independent of the twist-2 distribution amplitudes or meson decay constants. This relation is particularly interesting because it can be used to test the SM with less uncertainties. The related CP asymmetries will be expressed in terms of the corresponding branching ratios which are scaled by transition form factors as

$$\begin{aligned} \mathrm{Br}\big(\overline{B}^0 \to \pi^-\pi^+\big) &= C\left(\frac{F_0^{B\to\pi}}{F_0^{B_s\to K}}\right)^2 \mathrm{Br}\big(\overline{B}^0_s \to \pi^-K^+\big)\frac{Ph^B_{\pi\pi}}{Ph^{B_s}_{\pi K}}, \\ \mathrm{Br}\big(\overline{B}^0 \to K^-\pi^+\big) &= C\left(\frac{F_0^{B\to\pi}}{F_0^{B_s\to K}}\right)^2 \mathrm{Br}\big(\overline{B}^0_s \to K^-K^+\big)\frac{Ph^{B_d}_{\pi K}}{Ph^{B_s}_{KK}}, \end{aligned} \tag{17}$$

where $C = (m_B^2\tau_{B_s}/m_{B_s}^2\tau_B)$ and $Ph^B_{P_1P_2} = [(1-(m_{P_1}+m_{P_2})^2/m_B^2)(1-(m_{P_1}-m_{P_2})^2/m_B^2]^{1/2}/2m_B$ is the phase space factor. In order to test the SM, one needs to know the form factors, these can be obtained from other processes or from theoretical calculations. Alternatively, one can use these relations to obtain the ratio of the form factors using experimental data.

There are similar relations for $B \to PV$ decays [8]. By replacing one of the final octet pseudoscalar mesons with a corresponding octet vector meson in the previously discussed cases, one obtains

$$\begin{aligned} &\Delta^{\overline{B}^0}_{\rho^-\pi^+} = -\Delta^{\overline{B}^0_s}_{K^{*-}K^+} \approx \Delta^{\overline{B}^0_s}_{\rho^-K^+} = -\Delta^{\overline{B}^0}_{K^{*-}\pi^+}, \\ &\Delta^{\overline{B}^0}_{\pi^-\rho^+} = -\Delta^{\overline{B}^0_s}_{K^-K^{*+}} \approx \Delta^{\overline{B}^0_s}_{\pi^-K^{*+}} = -\Delta^{\overline{B}^0}_{K^-\rho^+}, \end{aligned} \tag{18}$$

where the approximate sign indicates relations that hold true only when annihilation contributions are neglected. These relations, Eq. (18), are again expected to receive $SU(3)$ breaking corrections. To estimate the $SU(3)$ breaking effects we use the QCD improved factorization model once again.

When M_2 (the meson which picks up the spectator) is a vector meson, as, for example, in $\overline{B}^0 \to \pi^-\rho^+$, $\overline{B}^0_s \to K^-K^{*+}$ and $\overline{B}^0 \to K^-\rho^+$, $\overline{B}^0_s \to \pi^-K^{*+}$, the corresponding decay amplitudes can be obtained by replacing the form factor $F_0^{B\to P}$ with $A_0^{B\to V}$ and r_χ with $-r_\chi$ in Eq. (7), and by using the same expressions for Eq. (9), except for $H_{M_1M_2}$ which has no twist-3 terms. By neglecting the annihilation contributions, the analogue of Eq. (15) is

$$\frac{\Delta^{\overline{B}^0}_{\pi^-\rho^+}}{\Delta^{\overline{B}^0_s}_{K^-K^{*+}}} \approx -\frac{m_B}{m_{B_s}}\frac{f_\pi^2}{f_K^2}\left(\frac{A_0^{B\to\rho}}{A_0^{B_s\to K^*}}\right)^2 \frac{1+110\alpha_1^\pi+15.5\alpha_2^\pi}{1+110\alpha_1^K+15.5\alpha_2^K}, \tag{19}$$

and the same for $\Delta^{\overline{B}^0_s}_{\pi^-K^{*+}}/\Delta^{\overline{B}^0}_{K^-\rho^+}$. We observe the large coefficient of α_1 in both the numerator and denominator of Eq. (19). Since $\alpha_1^\pi = 0$ and $\alpha_1^K = 0.3 \pm 0.3$, the denominator has a vary large uncertainty, making a prediction for this asymmetry impossible within this framework. On the other hand, this provides an opportunity to constrain (or even to determine) α_1^K when the ratio in Eq. (19) is measured.

When M_1 is the vector meson, as in $\overline{B}^0 \to \rho^-\pi^+$, $\overline{B}^0_s \to K^{*-}K^+$ and $\overline{B}^0_s \to \rho^-K^+$, $\overline{B}^0 \to K^{*-}\pi^+$, the decay amplitudes can be obtained by replacing the r_χ factor in Eq. (7) with $r_K^* = \frac{2m_{K^*}}{m_b}\frac{f_{K^*}^\perp}{f_{K^*}} \approx 0.3$ (and similarly for r_ρ), and by removing the penguin terms $P_{2,3}^{p,EW}$ in the expressions for a_6 and a_8 in Eq. (9), because the vector meson is described only by a twist-2 distribution amplitude. With all this we obtain:

$$\frac{\Delta^{\overline{B}^0}_{\rho^-\pi^+}}{\Delta^{\overline{B}^0_s}_{K^{*-}K^+}} \approx -\frac{m_B}{m_{B_s}}\frac{f_\rho^2}{f_{K^*}^2}\left(\frac{F_1^{B\to\pi}}{F_1^{B_s\to K}}\right)^2 \frac{1-1.25\alpha_1^\rho-0.18\alpha_2^\rho}{1-1.25\alpha_1^{K^*}-0.18\alpha_2^{K^*}}. \tag{20}$$

Using the central values of the ranges $\alpha_1^\rho = 0$, $\alpha_2^\rho = 0.16 \pm 0.09$, $\alpha_1^{K^*} = 0.18 \pm 0.05$, $\alpha_2^{K^*} = 0.05 \pm 0.05$ [9] and taking $f_\rho \approx 0.96 f_{K^*}$[2] we find:

$$\frac{\Delta^{\overline{B}^0}_{\rho^-\pi^+}}{\Delta^{\overline{B}^0_s}_{K^{*-}K^+}} \approx -1.15\left(\frac{F_1^{B\to\pi}}{F_1^{B_s\to K}}\right)^2, \qquad \frac{\Delta^{\overline{B}^0_s}_{\rho^-K^+}}{\Delta^{\overline{B}^0}_{K^{*-}\pi^+}} \approx -1.15\left(\frac{F_1^{B_s\to K}}{F_1^{B\to\pi}}\right)^2. \tag{21}$$

Our calculations show that important $SU(3)$ breaking effects arise from the light-cone distributions of mesons in addition to those already present in the decay constants. These effects can only be estimated with large uncertainty because the parameters $\alpha^P_{1,2}$ are not well determined at present. Using the currently allowed ranges we find,

$$A_{\mathrm{CP}}\left(\pi^-\pi^+\right) \approx -\left(3.1^{+1.9}_{-0.9}\right) A_{\mathrm{CP}}\left(K^-\pi^+\right), \tag{22}$$

which can also be used to test the Standard Model and the improved factorization model to some extent.

We have also shown that in the case of $B \to PV$, when the pseudoscalar meson is factored out, $SU(3)$ breaking is large and estimates have very large uncertainty at present. In the case when the vector meson is factored out, as in Eq. (20), the corrections are smaller.

It is important to emphasize, however, that there are relations which are independent of $\alpha^i_{1,2}$ parameters and decay constants. Examples include Eq. (16), a corresponding relation for the ratio of branching ratios $(\mathrm{Br}(\overline{B}^0 \to \pi^-\pi^+)/\mathrm{Br}(\overline{B}^0 \to K^-\pi^+) \approx \mathrm{Br}(\overline{B}_s \to \pi^-K^+)/\mathrm{Br}(\overline{B}_s \to K^-K^+))$, and their analogues in $B \to PV$ decays. These relations are more reliable than Eq. (1) in the sense that they do not receive the main $SU(3)$ breaking corrections that we have investigated. Although this observation relies on the QCD improved factorization model, it may be more robust than model predictions for absolute values of rates because it only involves ratios.

A systematic framework to study $SU(3)$ breaking in B decays is, of course, needed before the relations we have presented can be used in precision tests of the Standard Model. With the estimates we have presented here, the relations are still useful. Large experimental violations of them would hint at possible new physics; at the very least they would provide information on the limitations of the QCD improved factorization model. To test some of the relations that we have discussed, charmless hadronic two body B_s decays must also be measured. This cannot be done by the B-factories at present, but in the near future such B_s decays will be studied at the Tevatron II and at LHCb.

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[2] We extract this ratio $f_\rho/f_K \approx 0.96$ from $\Gamma(\tau^- \to \rho^-\nu)/\Gamma(\tau^- \to K^{*-}\nu)$.

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