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Overdamping phenomena near the critical point in O(N) model

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Abstract

We consider the dynamic critical behavior of the propagating mode for the order parameter fluctuation of the O(N) Ginzburg–Landau theory, involving the canonical momentum as a degree of freedom. We reexamine the renormalization group analysis of the Langevin equation for the propagating mode. We find the fixed point for the propagating mode as well as that for the diffusive one, the former of which is unstable to the latter. This indicates that the propagating mode becomes overdamped near the critical point. We thus can have a sufficient understanding of the phonon mode in the structural phase transition of solids. We also discuss the implication for the chiral phase transition.

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The quantum chromodynamics (QCD) is believed to have a rich phase structure in the temperature (T) and baryon chemical potential (μ_B) plane [1,2]. The study of the dynamic (i.e., nonequilibrium) critical phenomena of the second order QCD phase transitions including the chiral phase transition [3–8], the tricritical point, the critical end point [9–13] and the transitions associated with the color superconductivity [14] is not only of theoretical interest but also of fundamental importance for understanding of the relativistic heavy ion collision experiments and the early universe where the systems make time developments.

One of the characteristic features of the dynamic critical phenomena is the critical slowing down, which means that it takes very long time for the system near a critical point to relax into the equilibrium state. The long relaxation time is attributed to the slow motion of the long-wavelength fluctuations of the so-called slow (or gross) variables. The slow variables are the fundamental degrees of freedom for description of the dynamic properties near critical points. Usually the slow variables are identified with the collection of the order parameter and the conserved quantities of the system.

The long-wavelength fluctuations of the slow variables make up what we call the slow modes. One should note that there are two kinds of slow modes; the propagating mode and the diffusive mode. The propagating mode involves oscillation with dissipation and corresponds to a pole with both the real and imaginary parts of the spectral function for the slow variables. The diffusive mode, on the other hand, is purely dissipative and may be called the relaxational mode, which corresponds to a pole with the imaginary part only.

In general, it is possible to classify critical points into the universality classes. For the static (i.e., equilibrium) case, as is well known, the universality class can be determined solely by the symmetry and dimensions of the system. For the dynamic case, a classification scheme was proposed by Hohenberg and Halperin [15]: The dynamic universality classes are dependent on what kinds of slow variable (the order parameter and conserved quantities) are contained in the system as well as on the symmetry and dimensions. According to the scheme, Hohenberg and Halperin have classified the whole critical points in the condensed matter physics in a lucid and systematic way [15].

The dynamic universality class of the chiral phase transition was first discussed in Ref. [4]. Hohenberg and Halperin's classification scheme tells us that the chiral phase transition belongs to the same dynamic universality class as that of the antiferromagnet. In Refs. [7,8], however, a crucial difference between

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the two systems has been pointed out. The difference is in the slow modes of the order parameter fluctuation. In the disordered phase, the order parameter fluctuation in the antiferromagnet is known to be a diffusive mode, while the order parameter fluctuation in the chiral phase transition gives the meson or particle mode, which is a propagating mode. It has been argued [8] that in order to describe the meson mode appropriately, the canonical momentum conjugate to the order parameter in addition to the order parameter itself and conserved quantities is needed as a slow variable. The necessity incorporating both the order parameter and its canonical momentum may be accepted if we think of the Heisenberg equation for the Klein–Gordon field. We note that the canonical momentum is neither the order parameter nor a conserved quantity. This means that Hohenberg and Halperin's classification scheme is not able to correctly classify the dynamic universality class of the chiral phase transition and is not a complete one at its face value.

In this Letter, we investigate the propagating mode for the order parameter fluctuation in the O(N) Ginzburg–Landau theory that involves the canonical momentum as a degree of freedom. We analyze the Langevin equation for the propagating mode by using the renormalization group technique. For simplicity, we ignore other possible conserved quantities. The system we will consider is the simplest possible model.

In fact, we know a critical point in the condensed matter physics, the soft mode for which is a propagating mode involving the canonical momentum. It is the structural phase transition of solids [16]. The soft mode is a phonon mode, which is a vibrational mode of the lattice. Experimentally, it is observed that the phonon mode becomes overdamped, eventually turning into a diffusive mode near the critical point. This fact may suggest that the meson mode in the chiral transition also becomes overdamped and its dynamic universality class reduces to that of the antiferromagnet near the critical point as discussed in Ref. [4]. This is indeed the case as we will see later. In a theoretical side, the phonon mode was examined in Ref. [15], in which essentially the Langevin equation for the propagating mode was analyzed. However, the analysis will be found not to be adequate in the sense discussed below. Our investigation will elucidate the profound background of the overdamping phenomenon. The purpose of this Letter is to give a reanalysis of the Langevin equation for the propagating mode of the O(N) Ginzburg–Landau model and to examine the role of the canonical momentum in the critical dynamics. The model includes the structural phase transition as an $N = 1$ case and the chiral phase transition as $N = 4$. Our analysis brings us new findings missed in the previous work, which lead to a deeper insight into the structural transition and the chiral transition as well as into the classification scheme of dynamic universality classes.

The Langevin equation that describes the propagating mode for the order parameter fluctuation is given by a natural extension of that for the Brownian particle [17]:

$$\frac{\mathrm{d}\pi_i(\vec{x},t)}{\mathrm{d}t} = -\frac{\delta F[\phi]}{\delta\phi_i(\vec{x},t)} - \Gamma\frac{\mathrm{d}\phi_i(\vec{x},t)}{\mathrm{d}t} + \zeta_i(\vec{x},t), \tag{1}$$

$$\frac{\mathrm{d}\phi_i(\vec{x},t)}{\mathrm{d}t} = \lambda\pi_i(\vec{x},t), \tag{2}$$

$$\langle\zeta_i(\vec{x},t)\zeta_j(\vec{y},t')\rangle = k_\mathrm{B}T2\Gamma\delta_{ij}\delta(\vec{x}-\vec{y})\delta(t-t'). \tag{3}$$

The $\phi_i(\vec{x},t)$ is the N-component ($i = 1, 2, \dots, N$) order parameter and $\pi_i(\vec{x},t)$ is its canonical momentum. The $F[\phi]$ is the usual O(N) symmetric Ginzburg–Landau free energy given by

$$F[\phi] = \frac{1}{2}\int \mathrm{d}^d x\left[\vec{\nabla}\phi_i(\vec{x},t)\cdot\vec{\nabla}\phi_i(\vec{x},t) + r_0\phi^2 + \frac{1}{2}u\phi^4\right], \tag{4}$$

where d is the spatial dimension and r_0 and u are the usual static parameters. The dynamic parameters λ and Γ represent the square of the propagating velocity and the damping constant, respectively. The $\zeta_i(\vec{x},t)$ is the white noise which satisfies Eq. (3).

In the Fourier space, the Langevin equation reads

$$\frac{\mathrm{d}\pi_{i\vec{k}}(t)}{\mathrm{d}t} = -\left[(r_0+k^2)\phi_{i\vec{k}} + \frac{u}{L^d}\sum_{0<k',k''<\Lambda}\phi_{j\vec{k}'}\phi_{j\vec{k}''}\phi_{i\vec{k}-\vec{k}'-\vec{k}''}\right] - \Gamma\frac{\mathrm{d}\phi_{i\vec{k}}}{\mathrm{d}t} + \zeta_{i\vec{k}}(t), \tag{5}$$

$$\frac{\mathrm{d}\phi_{i\vec{k}}(t)}{\mathrm{d}t} = \lambda\pi_{i\vec{k}}(t), \tag{6}$$

$$\langle\zeta_{i\vec{k}}(t)\zeta_{j\vec{k}'}(t')\rangle = k_\mathrm{B}T2\Gamma\delta_{ij}\delta_{\vec{k},-\vec{k}'}\delta(t-t'), \tag{7}$$

where L^d is the system volume. The wavenumber of the fluctuations is cutoff at Λ.

Our Langevin equation for the propagating mode should be in some connection with the Langevin equation for the diffusive mode. In general, when the friction or damping constant is very large, the oscillation becomes overdamped and the oscillatory nature is lost. The Langevin equation for the propagating mode should be reduced to that for the overdamped or diffusive mode for the large damping constant. Actually, this reduction can be proven explicitly if the nonlinear interaction coupling u is absent [18]. Consider the Langevin equation (5)–(7) with $u = 0$. If we take the overdamped limit by imposing the condition $\lambda\Gamma^2 \gg r_0 + k^2$, then the canonical momentum turns out to be the faster degree of freedom and we can integrate it out explicitly to find the Langevin equation for the slower degree of freedom $\phi_{i\vec{k}}(t)$;

$$\frac{\mathrm{d}\phi_{i\vec{k}}(t)}{\mathrm{d}t} = -\gamma\frac{\delta F[\phi_{i\vec{k}}]}{\delta\phi_{i-\vec{k}}} + \zeta'_{i\vec{k}}(t), \tag{8}$$

$$F[\phi_{i\vec{k}}] = \frac{1}{2}\sum_{k<\Lambda}(r_0+k^2)\phi_{i\vec{k}}\phi_{i-\vec{k}}, \tag{9}$$

$$\langle\zeta'_{i\vec{k}}(t)\zeta'_{j\vec{k}'}(t')\rangle = k_\mathrm{B}T2\gamma\delta_{ij}\delta_{\vec{k},-\vec{k}'}\delta(t-t'), \tag{10}$$

where $\zeta'_{i\vec{k}}(t)$ is the renormalized noise term and $\gamma = 1/\Gamma$.[1] This equation is for the diffusive mode and gives nothing but the model A with $u = 0$ in Ref. [15].

Now we apply the renormalization group to the Langevin equation for the propagating mode, not necessarily assuming the overdamping condition. The renormalization group analysis leading to the recursion relation (15)–(18) has already been

[1] In the formalism of the Fokker–Planck equation, this reduction corresponds to that from Kramer's equation to Smoluchowski equation. See Ref. [19].

performed in Ref. [15]. Although the following calculation is not new, we will present the calculation below to make the discussion self-contained.

The renormalization group program consists of the two procedures [18,20]; (i) integrate out the short-wavelength fluctuation with $\Lambda/b < k < \Lambda$, and (ii) make the scale transformation of $\phi_{i\vec{k}} \to b^{1-\eta/2}\phi_{ib\vec{k}}(tb^{-z})$. After the procedures, we have the recursion relation for the parameters $(\lambda, \Gamma, r_0, u)$. We employ the ϵ-expansion assuming that u is of order ϵ, where $\epsilon = 4 - d$. In this Letter, we consider only the lowest order of the expansion. The dynamic response function for the order parameter fluctuation is given in the form of

$$G(\vec{k}, \omega) = \frac{1}{-\frac{1}{\lambda}\omega^2 + r_0 + k^2 - \mathrm{i}\omega\Gamma + \Sigma(\vec{k}, \omega)}, \tag{11}$$

where $\Sigma(\vec{k}, \omega)$ is the 1PI self-energy.

The diagrammatic rules for the perturbation are almost the same as those for the diffusive model (model A in Ref. [15]). Only the difference is the form of the response function. For the propagating mode, it includes the ω^2 term as seen from Eq. (11). For the overdamped limit where $\lambda\Gamma \gg \omega$ (or $\sqrt{\lambda}\Gamma \gg \Lambda$), the ω^2 term disappears and the response function reduces to that of the diffusive mode [15]. We note that for the particle mode, which is a propagating mode and described by the system with $\lambda = 1$, the pole position lies in the time-like region in the ω–k plane. This is contrary to the diffusive mode for which the pole is always in the space-like region.

After the integration of the fluctuation, the new parameters are defined by

$$r_0' = \lim_{k,\omega\to 0} G^{-1}(\vec{k}, \omega), \tag{12}$$

$$\Gamma' = \lim_{k,\omega\to 0} \frac{\partial}{\partial(-\mathrm{i}\omega)} G^{-1}(\vec{k}, \omega), \tag{13}$$

$$\frac{1}{\lambda'} = \lim_{k,\omega\to 0} \frac{\partial^2}{\partial(-\mathrm{i}\omega)^2} G^{-1}(\vec{k}, \omega), \tag{14}$$

where the last equation is absent in the diffusive model and new in the present model. Up to the lowest order in the ϵ-expansion, the self-energy is given by the tadpole diagram, which has no frequency and wavenumber dependence. Thus Γ' and λ' receive no corrections from the fluctuation integration. After the scale transformation, we obtain the recursion relation;

$$\frac{1}{\lambda'} = b^{2-2z}\frac{1}{\lambda}, \tag{15}$$

$$\Gamma' = b^{2-z}\Gamma, \tag{16}$$

$$r_0' = b^2(r_0 + \Delta r), \tag{17}$$

$$u' = b^{4-d-\eta}(u + \Delta u). \tag{18}$$

We note that Eq. (15) is given in Eq. (4.36) in Ref. [15]. The first two equations are for the dynamic parameters while the last two for the static ones. We note that the recursions for the dynamic and static parameters are decoupled to this order. The recursions for the static parameters are the usual ones for the static Ginzburg–Landau theory with $\Delta r = u(N/2 + 1)\int \frac{\mathrm{d}^d q}{(2\pi)^d}(r_0 + q^2)^{-1}$ and $\Delta u = -u^2(N + 8)/2\int \frac{\mathrm{d}^d q}{(2\pi)^d}(r_0 + q^2)^{-2}$, and give rise to the Gaussian and Wilson–Fisher (WF) fixed points as usual. Although there are two dynamic parameters, one of them just fixes the time scale and only some ratio of the two is meaningful. It is useful to define the dynamic parameter as $\rho \equiv \sqrt{\lambda}\Gamma/\Lambda$. From the recursion relation, we find that there arise two fixed points associated with the WF fixed point in the parameter space (ρ, r_0, u).[2] See Fig. 1.

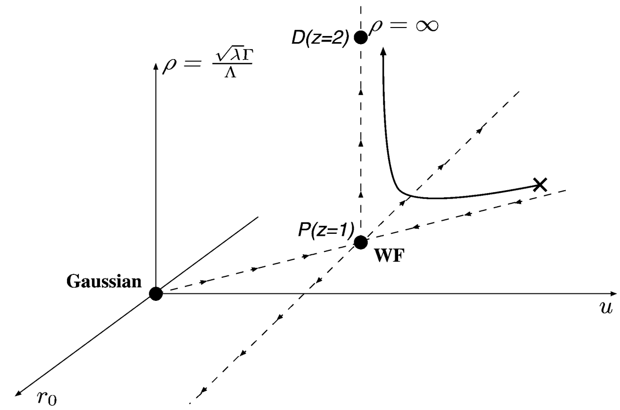

Fig. 1. The two fixed points and the renormalization group flow.

The two fixed points are located at $\rho = 0$ and $\rho = \infty$, which will be denoted as P and D, respectively. The fixed point P has the dynamic critical exponent $z = 1$ [6,8] and represents the purely propagating mode, while the fixed point D has the dynamic exponent $z = 2$ [20] and corresponds to the diffusive mode. The dynamic exponents are obtained in a usual manner [18,20]. The arrows indicate the flow of the renormalization group. We see that the parameter ρ is relevant with respect to the fixed point P and the line extended from the Gaussian fixed point to the WF fixed point gives the critical “surface”. Thus the fixed point P is unstable to the fixed point D. The system represented by the point near the critical surface, such as the cross in Fig. 1, is firstly taken to the vicinity of P but eventually driven away to the fixed point D; namely, there occurs a crossover between the two fixed points.

An illuminative example of a crossover phenomenon between two fixed points is provided us with the static critical phenomena in the magnetic systems [21]. Consider the Heisenberg ferromagnet which has the rotational O(3) symmetry. Its critical property would be controlled by the associated fixed point, i.e., the Heisenberg fixed point. The full rotational symmetry may be broken into the uniaxial one by, say, distortion of the lattice, the effect of which is described by the appropriate anisotropic Hamiltonian with the coupling constant g. It is known [21] that the Heisenberg fixed point is unstable with respect to the parameter g. The stable fixed points are supplied by the Ising or XY fixed points, depending on the sign of g. A point with small g in the parameter space is driven by

[2] Actually, we have two fixed points associated with the Gaussian fixed point as well. The two sets of the two fixed points have the same feature with respect to the ρ-direction because the recursions for the dynamic and static parameters are decoupled. We will concentrate on the two fixed points for the Wilson–Fisher fixed point.

the renormalization group to the Ising or XY fixed point via the vicinity of the Heisenberg fixed point. This means that the system firstly exhibits the critical behavior of the Heisenberg universality class, but the Ising- or XY-like behavior eventually shows up as the critical point is approached.

Our analysis indicates that the propagating mode crosses over into the diffusive mode near the critical point: At first, the propagating mode is softened with the exponent $z = 1$ under the control of the propagating fixed point, and then it becomes overdamped to be governed by the diffusive fixed point with $z = 2$.

Now we have obtained a lucid explanation for the phonon mode in the structural phase transition[3]: the phonon mode changes its behavior from the propagating into the diffusive as a consequence of the crossover between the two fixed points. As we mentioned, the recursion relation (15)–(18) was already derived in Ref. [15]. However it was utilized only to discuss the stability of the diffusive fixed point with respect to the small perturbation in the ρ direction. Namely only the vicinity of the diffusive fixed point was investigated. In the present analysis, we have examined the whole region in the parameter space within the validity of the ϵ-expansion, and found out the unstable propagating fixed point.

The finding of the unstable propagating fixed point brings us new insights into the structural phase transition: We can have a simple understanding of the overdamping phenomenon as a crossover between the two fixed points. The mechanism of how the propagating behavior changes into the diffusive one has become more concrete. It is clear that because of the instability of the fixed point, the propagating mode inevitably becomes overdamped even if we start from anywhere in the parameter space except in the $\rho = 0$ plane. Moreover the universality of the propagating behavior of the phonon mode are confirmed by the fixed point. These ingredients give a full explanation in terms of the renormalization group language, which is sufficient for understanding of the phonon mode in the structural phase transition [22].

We note that it is not until the propagating fixed point is given that its universality is guaranteed. In fact, the renormalization group analysis of the phonon mode has been restricted only to the diffusive behavior near the critical point up to the present. The propagating fixed point we have found gives us a firm basis to discuss the universal nature of the propagating behavior of the phonon mode.

It should be noted that though being unstable, the propagating fixed point gives one dynamic universality class in the sense that fixed points and universality classes make a one-to-one correspondence. The Heisenberg fixed point in the magnetic system actually gives one universality class although it is unstable against the anisotropic interaction. As can be seen from the recursion relation (15)–(18), within the $\rho = 0$ plane including the propagating fixed point, the renormalization group flow closes itself. If the system exists which has the parameters just in the plane, the propagating fixed point is no longer unstable.[4] If the system takes the parameters very close to the plane initially, the overdamped region near the critical point is correspondingly small and the critical behavior is almost governed by the propagating fixed point. Thus the propagating behavior constitutes one dynamic universality class.

Since the propagating mode eventually reduces to the diffusive mode near the critical point, the canonical momentum becomes a rapid degree of freedom and fades out of the member of the slow variables at that point. Thus if one restricts one's interest to the very vicinity of the critical point, Hohenberg and Halperin's classification scheme of dynamic universality classes, which is based on the order parameter and conserved quantities, practically works. However, the canonical momentum is still an important degree of freedom to find the dynamic universality class associated with the propagating fixed point. In this respect, Hohenberg and Halperin's classification scheme may be regarded as incomplete.

We now turn to the chiral phase transition. Our analysis predicts that as the system approaches the critical point, the meson mode turns into the diffusive mode after the softening.[5] Thus the dynamic universality class of the chiral transition certainly reduces to that of the antiferromagnet as argued in Ref. [4]. However the propagating behavior of the meson mode is still important to analyze since it shows universal properties belonging to a fixed point. There have been various approaches, such as the mode coupling theory [7,8], the Nambu–Jona-Lasinio model [3] and the microscopic approach in Ref. [6]. Unfortunately, the overdamping phenomenon was not noticed in these analyses within the adopted approximations. Actually, up to now there have been two kinds of works based on different standpoints: some works treat the meson mode as a diffusive mode while others assume the mode to be propagating. These two standpoints are reconciled now that the crossover between the two kinds of modes has been realized. We should note that the overdamping of the meson mode itself has a significant physical meaning that the sigma mesons and pions are not able to propagate and lose a particle nature near the chiral phase transition.

To summarize, we have found the fixed point for the propagating mode, which is unstable to the fixed point for the diffusive mode. This means that the propagating mode for the O(N) Ginzburg–Landau theory becomes overdamped near the critical point. The analysis gives a satisfactory account of the character change of the phonon mode in the structural phase transition and also predicts the fate of the meson mode near the chiral phase transition. In the future work, we will investigate the higher order calculation in the ϵ-expansion, which will clar-

[3] Although we have considered only the order parameter fluctuation, and the other conserved quantity, i.e., the energy, has not been taken care of in this Letter, we believe that our analysis is sufficient for the qualitative feature of the phonon mode. This would be so because the instability of the propagating fixed point would not be affected even if conserved quantities are taken into account.

[4] The plane within which the renormalization group flow closes itself coincides with the $\rho = 0$ plane in the leading order of the ϵ-expansion. In the higher orders, however, the plane may be lifted from the $\rho = 0$ plane, as suggested by Eq. (4.80) in Ref. [15]. This means that those propagating modes, which lie in the plane and do not become overdamped, can have a finite width.

[5] This means that the pole moves from the time-like to the space-like region.

ify the deviations of the dynamic critical exponents from the mean field values obtained in this work. The $1/N$ expansion is also interesting. Moreover it is necessary to take account of the conserved quantities for the proper description of the structural phase transition, the chiral phase transition and so on. We hope to report the progresses in these directions in the future.

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