Semi-simple unification on $T^6/Z_{12}$ orientifold in the type IIB supergravity

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Abstract

The semi-simple unification model based on $SU(5)_{\text{GUT}} \times U(3)_H$ gauge group is an interesting extension of the minimal $SU(5)_{\text{GUT}}$ grand unification theory (GUT), since it solves the two serious problems in the standard GUT: the triplet–doublet splitting problem and the presence of dangerous dimension five operators for proton decay. Here, the extra $U(3)_H$ gauge interaction plays a crucial role on the GUT breaking. In this Letter, we show that the full multiplet structure of the $U(3)_H$ sector required for the desired GUT breaking is reproduced naturally on $T^6/Z_{12}$ orientifold in the type IIB supergravity with a $D3$–$D7$ system. The $SU(5)_{\text{GUT}}$ vector multiplet lives on $D7$-branes and the $U(3)_H$ sector resides on $D3$-branes. We also show that various interesting features in the original $SU(5)_{\text{GUT}} \times U(3)_H$ model are explained in the present brane-world scenario. A possible extension to the type IIB string theory is also discussed.

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1. Introduction

Supersymmetric grand unification theory (SUSY-GUT) is strongly supported by the success of the gauge-coupling unification [1]. The minimal $SU(5)$ GUT model, however, suffers from two serious problems; one is the triplet–doublet splitting problem and the other is the presence of the dimension five operators [2] causing a too fast proton decay. Semi-simple unification model based on $SU(5)_{\text{GUT}} \times U(3)_H$ gauge group [3,4] is an extension of the minimal $SU(5)_{\text{GUT}}$ model that solves the above two problems keeping the success of the original GUT model. In this model color-triplet Higgs multiplets acquire large masses of order of the GUT scale together with their partners, while weak-doublet Higgs multiplets remain massless [5]. The masslessness of the Higgs doublets is guaranteed by an $R$-symmetry and hence the Higgs doublets get SUSY-breaking scale masses through the Giudice–Masiero mechanism [6].

In a recent article [7] Imamura and the present authors have pointed out that the above semi-simple unification model is quite naturally embedded in the type IIB supergravity with a $D3$–$D7$ brane system. In this higher-dimensional theory various phenomenologically important features of the original semi-simple unification model are well understood by the brane-world structure. For instance, the hierarchy between the GUT scale and the Planck scale and the disparity between gauge coupling constants of $SU(5)_{\text{GUT}}$
and $U(3)_H$ are simultaneously explained. Moreover, a major part of the matter content and interactions that are most relevant to the GUT breaking is also reproduced from the $D3$–$D3$ and the $D3$–$D7$ sector fields.

The purpose of this Letter is to show that problems left unsolved in the previous article [7] that are related to the hypercolor $U(3)_H$ sector (relevant to the GUT breaking dynamics) is completely resolved when one adopts the $T^6/Z_{12}$ orientifold in the type IIB supergravity.

We take a bottom-up approach to construct a model in the ten-dimensional space–time. This strategy is very similar to the construction orientifold background in the ten-dimensional space–anomalies and obtain a consistent field theory on an orbifold fixed point. Therefore, we see that some anomalies appear at orbifold fixed points through the orbifold projection. Then, we take the standard procedure of orbifolding to obtain an SUSY four-dimensional theory. It is very surprising that the complete multiplet in the hypercolor $U(3)_H$ sector required for the successful phenomenology is obtained on the $T^6/Z_{12}$ orientifold. We also find that the above mentioned anomalies appear only at a unique $Z_{12}$ fixed point and they are easily removed by introducing new fields at the fixed point. A possible connection to the type IIB string theory is discussed in the last section.

2. Brief review of the semi-simple unification model in the brane-world

Let us first review briefly on the semi-simple unification model [4]. The gauge group is $SU(5)_{\text{GUT}} \times U(3)_H$. Quark, lepton and Higgs supermultiplets are singlets under the $U(3)_H$ and transform under the $SU(5)_{\text{GUT}}$ as in the standard $SU(5)_{\text{GUT}}$ model. Fields introduced for GUT breaking are given as follows:

$$X^\alpha_\beta (\alpha, \beta = 1, 2, 3) \text{ transforming as } (1, \text{ adj.} = 8 + 1)$$

under the $SU(5)_{\text{GUT}} \times U(3)_H$ gauge group, and $Q^a_i + Q^a_6 (i = 1, \ldots, 5)$ and $\bar{Q}^6_i + \bar{Q}^6_6 (i = 1, \ldots, 5)$ transforming as $(5^* + 1, 3^*)$ and $(5 + 1, 3^*)$. Indices $\alpha$ and $\beta$ are for the $U(3)_H$ and $i$ for the $SU(5)_{\text{GUT}}$.

Superpotential is given by

$$W = \bar{Q}^6_i Q^a_i Q^a_6 - v^2 X^\alpha_\alpha + H \bar{Q}^i_\alpha Q^a_6 + \bar{Q}^6_i Q^{a \alpha}_i H^1 + y_{10} 10 \cdot 10 \cdot H + y_{5^*} 5^* \cdot \bar{Q}^6_i \cdot \bar{H} + \cdots,$$

where $k = 1, \ldots, 6$, and the parameter $v$ is of order of the GUT scale, and $y_{10}$ and $y_{5^*}$ are Yukawa coupling constants of the quarks and leptons. One can see that the above superpotential has $SU(5)_{\text{GUT}} \times U(3)_H$ R-symmetry with a charge assignment given in Table 1, and this symmetry forbids the mass term $W = H \bar{H}$. The bifundamental representation $Q^{a \alpha}_i$ and $\bar{Q}^i_\alpha$ acquire vacuum expectation values, $(\bar{Q}^{a \alpha}_i) = \phi^{a \alpha}_i$ and $(\bar{Q}^i_\alpha) = \phi^{i \alpha}$, because of the first line in Eq. (1), and hence the gauge group $SU(5)_{\text{GUT}} \times U(3)_H$ is broken down to that of the standard model [4].

The mass terms

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<td><strong>Fields</strong></td>
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<td><strong>$Z_4$ R charge</strong></td>
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1 Orientifold $p$ planes are $(p + 1)$-dimensional objects in string theories that have Ramond–Ramond charge opposite to that of the $Dp$-branes [8]. Although their existence is not manifest within the supergravity, we assume their existence because the D-brane charges must be canceled. We call $T^6/(I') \simeq Z_2 (\Omega R_{k9})$ as $T^6/I'$ orientifold in this Letter.

2 Recently, a similar GUT breaking model that uses expectation values of bifundamental fields is discussed in [10].
of the colored Higgs multiplets arise from the second line in Eq. (1) in the GUT-breaking vacuum. The mass terms of the Higgs doublets are still forbidden by the unbroken $Z_4$ $R$-symmetry. One can also see that the $Z_4$ $R$-symmetry forbids the dangerous dimension five proton decay operators $W = 10 \cdot 10 \cdot 10 \cdot 5^*$. There are two remarks here. First of all, fine structure constants of the $U(3)_H \simeq U(1)_H \times SU(3)_H$ must be larger than that of the $SU(5)_{\text{GUT}}$. This is because the gauge coupling constants of the standard model are given by

$$\frac{1}{\alpha_C} = \frac{1}{\alpha_{\text{GUT}}} + \frac{1}{\alpha_{3H}},$$

$$\frac{1}{\alpha_L} = \frac{1}{\alpha_{\text{GUT}}}$$

and

$$\frac{3/5}{\alpha_Y} = \frac{1}{\alpha_{\text{GUT}}} + \frac{2/5}{\alpha_{1H}}$$

at the GUT scale, where $\alpha_C$, $\alpha_L$, $\alpha_Y$, $\alpha_{\text{GUT}}$, $\alpha_{3H}$ and $\alpha_{1H}$ are fine structure constants of the three standard model gauge groups, $SU(5)_{\text{GUT}}$, $SU(3)_H$ and $U(1)_H$, respectively. $1/\alpha_{3H}$ and $1/\alpha_{1H}$ must be within a few % of the $1/\alpha_{\text{GUT}}$ to reproduce the approximate unification of $\alpha_C$, $\alpha_L$ and $5\alpha_Y/3$. Secondly, the cut-off scale of the theory $M_\ast$ must be lower than the Planck scale $M_{\text{Planck}} \simeq 2.4 \times 10^{18}$ GeV. Indeed, the gauge coupling constant of the $U(1)_H$ is already large at the GUT scale, as stated above, and it becomes infinity below the Planck scale because of its asymptotic non-free nature.

This relatively low cut-off scale $M_\ast (\simeq 10^{17}$ GeV) has motivated us [7] to consider a brane-world in a higher dimensions. In this brane-world the Planck scale is merely an effective scale and its relatively large value compared with the cut-off scale is explained by an effect of a slightly large volume of the extra dimensions [11]. Interesting is that the disparity between the gauge coupling constants of the $SU(5)_{\text{GUT}}$ and $U(3)_H$ is also explained if the $SU(5)_{\text{GUT}}$ gauge multiplet propagates in a higher-dimensional space (bulk) and the hypercolor $U(3)_H$ gauge multiplets reside on a “3-brane” [7].

Moreover, there is another reason [7] that supports such a brane-world structure behind the semi-simple unification model. The $U(3)_H$ sector (GUT breaking sector), which consists of a $U(3)_H$ vector multiplet, a $U(3)_H$-adjoint chiral multiplet $X^a_{\overline{\beta}}$ and vector-like chiral multiplets $Q_k$, $Q^{\overline{K}}_k$ ($k = 1, \ldots, 6$), has a multiplet structure of the $N = 2$ SUSY, and the first line of the superpotential Eq. (1) is a part of the “gauge interaction” of the $N = 2$ SUSY gauge theory\(^3\) with Fayet–Iliopoulos (FI) $F$ term [12]. Therefore, it seems quite natural to consider that there exists a higher-dimensional structure as an origin of such an extended SUSY.

In the previous work [7] we have found that the type IIB D3–D7 system on an orientifold geometry is a suitable framework to accommodate the original semi-simple unification model. The $U(3)_H$ gauge symmetry is present on the D3-branes and the $SU(5)_{\text{GUT}}$ on the D7-branes. The volume of the D7-branes transverse to the D3-branes is considered to be larger than the $(1/M_\ast)^4$ to realize the desired features explained above. Furthermore, we have considered that the D3-branes on which the $U(3)_H$ sector resides are not located at a fixed point, and hence in this case the multiplet structure of the $N = 2$ SUSY and the form of the $N = 2$ superpotential in Eq. (1) are naturally accounted for by the $N = 2$ SUSY of the D3–D7 system. Here, the $U(3)_H$-adjoint $X^a_{\overline{\beta}}$ field arises from the D3–D3 sector fields and the bifundamental $Q^a_{\overline{\alpha}}$ fields from the D3–D7 sector fields.

Although the above D3–D7 system on an orientifold is a good framework for the semi-simple unification model, there are several problems left unsolved. First of all, the origin of the hypermultiplets $Q_6$ and $Q^{\overline{6}}$ was not clearly found. Secondly, the position of the $U(3)_H$ D3-branes in $D7$-tangential directions must be fixed by some dynamics, or otherwise Nambu–Goldstone (NG) modes of the brane position remain light and alter the renormalization group (RG) running of the standard model gauge coupling constants. We show in this Letter that all these problems are solved in the $T^6/Z_{12}$ orientifold model. However, the origin of the quarks and leptons is still unclear, and hence we put these fields at a four-dimensional fixed point by hand.

\(^3\) $N = 2$ SUSY insists the Yukawa coupling of $\overline{Q} X Q$ to be given by the $U(3)_H$ gauge coupling constant as $\sqrt{2} g_{3H}$. This property is, however, not necessarily required from phenomenology.
3. Model construction on the $T^6/Z_{12}$ orientifold

As mentioned in the introduction we adopt the type IIB supergravity with a D3–D7 system and assume, on D-branes, gauge symmetries and massless fields obtained from the type IIB string theory, since it is known to provide a consistent higher-dimensional supergravity.

3.1. Whole $U(3)_H$ sector out of orbifold projection

$N = 2$ SUSY is preserved in a D3–D7 system on an orientifold $T^6/(Z_2(R_{4567}) \times Z_2(\Omega R_{89}))$. Here, $Z_2(R_{4567})$ is a $Z_2$-symmetry generated by a space reflection in the 4–7th directions $R_{4567}$, and $Z_2(\Omega R_{89})$ a $Z_2$-symmetry generated by a space reflection $R_{89}$ in the 8th and 9th directions along with an exchange of two Chan–Paton indices.

In the type IIB string theory with a D3–D7 system $U(1)$-dimensional U(1) is realized on thirty $U(1)$ the 8th and 9th directions along with an exchange of two Chan–Paton indices.

In the type IIB string theory with a D3–D7 system and O3- and O7-planes the maximal gauge group is $U(16) \times U(16)$ [13]. Each $U(16)$ is realized on thirty two D3-branes and thirty two D7-branes, respectively. Gauge group becomes smaller if the D-branes cluster in several places. The gauge group on D7-branes can be

$$\prod_{i=1}^{4} U(n_i) \times \prod_{j} U(n_j) \left( \sum n_i + \sum n_j = 16 \right).$$

The former $\prod_{i=1}^{4} U(n_i)$ come from D7-branes that are fixed under the $R_{89}$ and the latter $\prod_{j} U(n_j)$ come from the rest of the D7-branes, which are not fixed under the $R_{89}$ (i.e., that are located away from the O7-planes).

The gauge group on D3-branes can be

$$\prod_{i=1}^{64} U(m_i) \times \prod_{j} \text{Sp}(2m_j) \times \prod_{k} U(m_k) \times \prod_{l} U(m_l)$$

$$\times \left( \sum m_i + 2 \sum m_j + \sum m_k + 2 \sum m_l = 16 \right).$$

The $\prod_{i=1}^{64} U(m_i)$ come from D3-branes fixed both under the $R_{4567}$ and the $R_{89}$, the $\prod_{j} \text{Sp}(2m_j)$ from those fixed only under the $R_{89}$, the $\prod_{k} U(m_k)$ from those fixed only under the $R_{4567}$ and the $\prod_{l} U(m_l)$ from the rest of the D3-branes [13]. Gauge group can easily be $U(n) \times U(m) \times \cdots$. Therefore, it is quite natural to incorporate $SU(5)_{\text{GUT}} \times U(3)_H \times \cdots$ gauge group in this framework.

Now let us consider the orbifolding of the $T^6/(Z_2(R_{4567}) \times Z_2(\Omega R_{89}))$ to reduce the $N = 2$ SUSY down to the $N = 1$ SUSY. We assume $Z_N$-symmetry of the six-dimensional torus $T^6$ as a candidate of the orbifold group. In order to preserve the $N = 1$ SUSY, the $Z_N$ rotational symmetry, which belongs to the six-dimensional rotational group $SO(6) \simeq SU(4)$, must be included in $SU(3) \subset SU(4)$ that rotates three complex planes holomorphically [14,15]. Thirteen candidates that preserve $N = 1$ SUSY are listed in [14]. Among these, $Z_N$ that contains $Z_2(R_{4567})$ as a subgroup and that breaks the $N = 2$ down to the $N = 1$ SUSY is desirable for our purpose. In this Letter, we use $Z_{12}(\sigma')$ where $Z_2(R_{4567}) \subset Z_2(\sigma') \subset Z_{12}(\sigma)$; namely, we consider $T^6/Z_{12}$ orientifold model (i.e., $T^6/(Z_{12}(\sigma) \times Z_2(\Omega R_{89}))$). Other possibilities are discussed in a future publication [16].

There are three $(7 + 1)$-dimensional $Z_{12}$ fixed loci, among which one coincides with an O7-plane and the other two are mirror images of each other under the $\Omega R_{89}$ (see Fig. 1). We put a pair of six + six D7-branes on this mirror pair of fixed loci on which a $U(6)$ gauge group is obtained. The reason why we do not choose the one on the O7-plane but the latter fixed loci will be explained later. Before operating the $Z_{12}$ orbifold projection the gauge theory on the D7-branes consists of an $N = 4 \ U(6)$ vector multiplet—an

![Fig. 1. This figure shows a picture of the 3rd complex plane $z_3$ of the $T^6/(Z_{12}(\sigma) \times Z_2(\Omega R_{89}))$ geometry. Circles are O7-plane positions and dots are $Z_{12}(\sigma)$ fixed loci. We put six + six D7-branes for the $SU(5)_{\text{GUT}}$ on two dots without the circle. These two fixed loci are a $\Omega R_{89}$-mirror pair and so are these six + six D7-branes.](image)

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4 There are two $Z_{12}$ candidates. The $Z_{12}$ we use in this Letter is the one written as $Z_{12}$ in [15] and not the one written as $Z_{12}$ in [15]. We adopt the notation of the orbifold group in [15].
\( N = 1 \) vector multiplet \( \Sigma(0) \equiv W^{1(6)} \) and three \( N = 1 \) chiral multiplets \( \Sigma(b) \) \((b = 1, 2, 3)\)—which is, however, subject to the \( \mathbf{Z}_{12} \) orbifold projection, since the D7-branes are fixed under the \( \mathbf{Z}_{12} \). This \( \mathbf{U}(6) \) group is the origin of the \( \text{SU}(5)_{\text{GUT}} \), as derived in the following. Remaining twenty D7-branes are supposed to be away from these twelve D7-branes.

We impose a \( \mathbf{Z}_{12}({\sigma}) \) orbifold projection on the D7–D7 sector fields \((\Sigma(a)) \) \((a = 0, \ldots, 3)\) on the twelve D7-branes we are interested in. The generator \( \sigma \) is given by
\[
\sigma = e^{- 2\pi i \text{diag}(\nu)} \in \text{SU}(3) \subset \text{SU}(4),
\]
which rotates three complex planes as
\[
z_b \rightarrow e^{2\pi i \nu_k} z_b \quad (b = 1, 2, 3),
\]
where \( z_b \) denotes \( b \)th complex coordinate of the six-dimensional torus. The massless spectrum of the gauge theory in the orientifold geometry is given by partial components of the \( \mathbf{U}(6) \) \( N = 4 \) multiplet \( \Sigma(a) \) that satisfy the \( \mathbf{Z}_{12}({\sigma}) \) orbifold projection condition \([15]\):
\[
\Sigma^k(a) = e^{2\pi i\nu_k} \langle \tilde{\gamma}_{\sigma,7} \Sigma^k(a) \tilde{\gamma}_{\sigma,7}^{-1} \rangle_k,
\]
where we take the \((6 \times 6)\) matrix \( \tilde{\gamma}_{\sigma,7} \) as
\[
\tilde{\gamma}_{\sigma,7} = \text{diag}(e^{-\pi i/4}, e^{-\pi i/12}, e^{-\pi i/2}, e^{-\pi i/12}, e^{-\pi i/4}).
\]
Notice that the \((12 \times 12)\) matrix \( \gamma_{\sigma,7} \) acting on twelve D7 Chan–Paton indices that appears frequently in literatures is given by
\[
\gamma_{\sigma,7} = \text{diag}(\tilde{\gamma}_{\sigma,7}, \tilde{\gamma}_{\sigma,7}^{-1}).
\]
Eq. (8) is chosen so that the surviving gauge group is \( \text{U}(5) \times \text{U}(1)_6 \). The \( \text{SU}(5) \) subgroup of the \( \text{U}(5) \times \text{U}(1)_6 \) is identified with the \( \text{SU}(5)_{\text{GUT}} \). We see easily that only an \( N = 1 \) chiral multiplet \( \Sigma^{1_6} \) which transforms as \( \Sigma^{1_6} \) under the gauge group \( \text{U}(5) \times \text{U}(1)_6 \) survives the orbifold projection besides the \( N = 1 \) vector multiplets from the \( \Sigma(0) \). This multiplet comes from the fluctuation mode of the D7-branes (a massless mode of D7–D7 open string) in their transverse directions \( z_3 \). Anomaly cancellation conditions and the fate of the two remaining \( \text{U}(1) \) gauge symmetries are discussed later. We will identify the \( N = 1 \) chiral multiplet \( \Sigma^0_1(5^*) \) with one of the Higgs multiplets, \( \tilde{H}_i(5^*) \), in later arguments.

Now let us discuss how we obtain suitable massless spectrum from the D3–D3 sector and the D3–D7 sector. If the \( \text{U}(3)_\text{H} \) D3-branes are located in the bulk, unwanted NG modes destroy the gauge-coupling unification. If the D3-branes are located at an orbifold fixed point, however, then the \( N = 2 \) multiplet structure required in the \( \text{U}(3)_\text{H} \) sector might be lost. One way out of this difficulty is given by noticing that nature of all the fixed points is not necessarily the same.

Here, we introduce a notion of “\( N = 2 \) fixed point”. There is an isotropy subgroup \( G_x \subset \mathbf{Z}_{12} \) for each point \( x \) of the \( \mathbf{T}^6 \); \( G_x \) is given by elements of the \( \mathbf{Z}_{12} \) that fix the point \( x \). If \( G_x \) rotates only the first two complex planes \( z_1, z_2 \), or in other words, \( G_x \) is included in an \( \text{SU}(2) \subset \text{SU}(3) \) subgroup that rotates only the \( z_1, z_2 \), then we call such point \( x \) as an “\( N = 2 \) fixed point”.

Suppose that the D3-branes are located at an “\( N = 2 \) fixed point”.\(^5\) There exist mirror images under the \( \mathbf{Z}_{12}/G_x \). Orbifold projection due to the \( \mathbf{Z}_{12}/G_x \) identifies all mirror images, and the identified D3–D3 and the D3–D7 sectors are subject only to the remaining orbifold projection of the \( G_x \). Since the \( G_x \) at the D3-brane position belongs to the \( \text{SU}(2) \subset \text{SU}(3) \), the \( N = 2 \) multiplet structure in the D3–D7 system survive the orbifold projection. Moreover, we expect that the unwanted NG modes associated with the \( \text{U}(3)_\text{H} \) D3-brane positions are eliminated since the D3-branes can be “fixed” at that “\( N = 2 \) fixed point”.

We put the \( \text{U}(3)_\text{H} \) D3-branes at a point \( x \) on the \( \text{SU}(5)_{\text{GUT}} \) D7-branes where the isotropy subgroup \( G_x \) is \( \mathbf{Z}_3(\sigma^3) \subset \mathbf{Z}_{12}(\sigma) \). It is easy to see that they are “\( N = 2 \) fixed points”. There are only six such points, which form a coset space \((\mathbf{Z}_{12}(\sigma) \times \mathbf{Z}_2(\Omega_{80})) / \mathbf{Z}_4(\sigma^3)) \) (see Eq. (6)); namely, all these six points are mirror images of each other. Therefore, there is essentially only one candidate for the position of the \( \text{U}(3)_\text{H} \) D3-branes. We put three D3-branes at each mirror image and hence we need eighteen D3-branes.\(^6\)

The gauge groups \( \text{U}(3) \times \text{U}(3) \times \text{U}(3) \) arising from

\(^5\) There is no \( N = 2 \) multiplet structure on the D3-branes at other fixed points.

\(^6\) We put the rest of the D3-branes (there are twelve D3-branes left since we use two more D3-branes in our model construction later), for example, on the remaining D7-branes. Those D3- and D7-branes may be used for the dynamical SUSY breaking.
these eighteen D3-branes in the $T^6/Z_2 \times R_{4567} = \sigma^6$ orientifold\(^7\) are identified as a single U(3) after the orbifold projection of the $Z_{12}(\sigma)/Z_{4}(\sigma^3)$.

After this identification under the $Z_{12}(\sigma)/Z_{4}(\sigma^3)$, the D3–D3 and the D3–D7 sector fields consist of a U(3) $N = 2$ vector multiplet $(X_{(0)} \equiv \mathcal{W}_a^{U(3)}$, $X_{(3)})$ and a U(6) $\times$ U(3) D3 bifundamental $N = 2$ hypermultiplet $(\mathcal{O}^k, \mathcal{Q}^k)_{k=1, \ldots, 6 \alpha=1,2,3}$. The former comes from an open string massless mode that starts and ends on the three D3-branes and the latter is an open string that starts from three D3-branes and ends on the three D3-branes and the latter is an oriented geometry.

However, the projection conditions

$$X_{(0),(3)} = \tilde{\gamma}_{\sigma,3} X_{(0),(3)} \tilde{\gamma}_{\sigma,3}^{-1}, \quad (10)$$

$$Q^k = e^{i\beta(\gamma_{\sigma,3} \mathcal{O}^k)_{\sigma,3}^{-1}} e^{-i(\gamma_{\sigma,3} \mathcal{O}^k)_{\sigma,3}^{-1}} \quad (11)$$

remove none of the above $N = 2$ multiplets if the (3 $\times$ 3) matrix $\tilde{\gamma}_{\sigma,3}$ is taken as

$$\tilde{\gamma}_{\sigma,3} = \text{diag}(e^{-i\frac{\pi}{4}}, e^{-i\frac{\pi}{4}}, e^{-i\frac{\pi}{4}}). \quad (14)$$

Here, we note that

$$\tilde{\gamma}_{\sigma,7} = (\tilde{\gamma}_{\sigma,7})^3. \quad (15)$$

\(^7\) These D3-branes are fixed under the $R_{4567}$, while nonfixed under the $R_{89}$. This is the reason why the gauge group is U(3) $\times$ U(3) $\times$ (3 3).

\(^8\) The (18 $\times$ 18) matrix $\gamma_{\sigma,3}$ acting on eighteen D3 Chan–Paton indices that frequently appears in literatures is given by

$$\gamma_{\sigma,3} = \begin{pmatrix} 0 & s & 0 \\ 0 & 0 & 0 \\ 0 & 0 & s^{-1} \\ s & 0 & 0 \\ 0 & s^{-1} & 0 \\ 0 & 0 & 0 \\ \end{pmatrix} \quad (12)$$

with (3 $\times$ 3) matrices $s \cdot s' \cdot s'' = s' \cdot s'' \cdot s = s'' \cdot s' \cdot s = \tilde{\gamma}_{\sigma,3}$ so that

$$(\gamma_{\sigma,3})^3 = \gamma_{\sigma,3}$$

$$= \text{diag}(\tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}, \tilde{\gamma}_{\sigma,3}). \quad (13)$$

We identify the U(3)$_{D3}$ gauge group with the U(3)$_H$ and we see that the phenomenologically required $N = 2$ multiplets of the U(3)$_H$ sector are fully obtained at this $\mathcal{N} = 2$ fixed points. D3-branes are really fixed at that “$N = 2$ fixed point” and there is no unwanted massless field that destroys the gauge-coupling unification.

We show in [16] that similar arguments are possible in the case of $Z_6$ and $Z_6'$ orbifolding.

Unwanted matter multiplets would actually arise if $\Omega R_{89}$-mirror images of the D3-branes were not separated from themselves, and that is why we put the D7-branes for the SU(5)$_{\text{GUT}} \subset U(6)$ (and of course D3-branes, too) away from the O7-planes.

### 3.2. Triangle anomaly cancellation

Since we have started our model construction based on the matter content predicted by the type IIB string theory, there is no inconsistency before the $Z_{12}$ orbifold projection. Inconsistencies appear only through the orbifolding process, and they are expected to localize at orbifold fixed points [11,17]. Indeed, for example, the matter content derived so far has four-dimensional gauge anomalies, and a calculation of gauge triangle anomalies like that given in [18] shows that the anomalies localize only at four-dimensional $N = 1$ fixed points (see [16] for a detailed calculation).

The cancellation condition of these anomalies is stronger than that in the ordinary four-dimensional field theories. Not the total sum of the anomalies from all fixed points but also the anomalies at each fixed point must be canceled. The $Z_{12}$ model is special in that all triangle anomalies appear only at a single fixed point on the U(6) D7-branes, that is, the $Z_{12}(\sigma)$ fixed point [16].

\(^9\) This is also the reason why we do not choose $Z_4$, $Z_6$, $Z_6'$ or $Z_{12}$ as an orbifold group. They have no $(7 + 1)$-dimensional fixed locus which does not coincide with the O7-planes.

\(^10\) There is another model in which there is no triangle anomaly by the matter content derived from the D-branes. There, we put D3-branes at a point on the D7-branes where the isotropy subgroup is $Z_2(\sigma^6)$. It is possible in this model to choose $\gamma_{\sigma,7}$ and $\gamma_{\sigma,3}$ so that no $N = 1$ chiral multiplet from the D7–D7 sector survives the orbifold projection. Gauge group is SU(5)$_{\text{GUT}} \times U(2)_{H}$ instead of the SU(5)$_{\text{GUT}} \times U(3)_{H}$ and the weak-doublet Higgs multiplets are identified with the composites $Q^6 \mathcal{O}^7$ and $Q^5 \mathcal{O}^7 \mathcal{O}^6$. See [16] for details.
We put first one D3-brane at the $Z_{12}(\sigma)$ fixed point (along with its $O7$-mirror image) to cancel the SU(5)$_{\text{GUT}}$ triangle anomaly. The D3–D7 matters must satisfy the orbifold projection condition

$$\Psi_k = e^{\pi i s}(\tilde{\psi}_{\alpha,3}^i \psi^{\tilde{\alpha},7}_{\alpha,3}^{(-1)})_k,$$

$$\overline{\psi}^k = e^{\pi i s}(\tilde{\psi}_{\alpha,7}^i \psi^{\tilde{\alpha},3}_{\alpha,7}^{(-1)})^k,$$

where $k = 1, \ldots, 6$. We can easily see that only one $N = 1$ SUSY chiral multiplet $\overline{\psi}^i(5,0)$ ($i = 1, \ldots, 5$) of the D7–D7 U(5) $\times$ U(1)$_6$ gauge group satisfies this condition when\(^{11}\)

$$\tilde{\psi}_{\alpha,3}^i = e^{\frac{1}{2} \pi i}.$$

(17)

The SU(5)$_{\text{GUT}}$ triangle anomaly is completely canceled by this $\overline{\psi}^i(5)$. We can identify the multiplet $\overline{\psi}^i(5)$ with the Higgs multiplet $H^i(5)$. D3–D3 sector provides another (anomalous) $U(5)_L$ of the remaining two $U(5)_L$ symmetries, respectively, only a linear combination $U(1)_6 - U(1)_X$ has mixed anomalies $U(1)$ $\cdot$ [SU(5)]$^2$ and $U(1)$ $\cdot$ [grav]$^2$. These anomalies are canceled by an introduction of a field at the $Z_{12}$ fixed point that shifts under this $U(1)$ gauge symmetry\(^{13}\) [19]. Each of the remaining two $U(1)$ symmetries can be identified with the $U(1)_B-L$ symmetry. Triangle anomaly of this $U(1)$ symmetry vanishes. Mixed anomalies between the $U(1)_6 - X$ and the $U(1)_B-L$ can be canceled by the shifting field introduced above. Thus, all triangle anomalies are canceled out.

We introduce the quarks and leptons $3 \times (5^* + 10)$ also at this $Z_{12}(\sigma)$ fixed point. Although the $U(1)_{B-L}$ symmetry has a triangle anomaly, this anomaly is canceled by three families of right-handed neutrinos.

\(^{11}\) The $(20 \times 20)$ matrix $\gamma_{6,3}$ is given by $(18 \times 18$ part Eq. (13)) $\oplus\text{diag}(\gamma_{6,3}^i, \gamma_{6,3}^{(-1)})$.

\(^{12}\) This pair of two D3-branes is put at a pair of points, which is fixed by $R_{567}$ and not by $R_{89}$. Thus the gauge symmetry is $U(1)_6$.

\(^{13}\) Such an introduction of matter into a four-dimensional fixed point with only $N = 1$ SUSY would not lead to an inconsistency of higher-dimensional field theories.

4. Phenomenology

We identify the $N = 1$ chiral multiplet $\Sigma^6_{(3)_4}$ ($(5^*,+1)$) under the $U(5) \times U(1)_6$ from the D7–D7 sector with $\overline{H}$. Since the origin of the $\Sigma^6_{(3)_4}$ is the fluctuation of the six D7-branes in their transverse $(z_3)$ directions, interactions in the D3–D7 system give rise to a superpotential

$$W = \sqrt{2} g_{\text{GUT}} Q^a_{\alpha} \Sigma^6_{(3)_4} \overline{Q}^a_{\alpha},$$

along with the “$N = 2$ gauge interaction”

$$W = \sqrt{2} g_{\text{H}} \overline{Q}^b_{\alpha} X^a_{\alpha} \Sigma^6_{(3)_4} \overline{Q}^a_{\alpha}.$$  

(18)  

Eq. (18) automatically provides the first term in the second line of Eq. (1). We have to remember that in supersymmetric higher-dimensional theories $R$-symmetry has its geometrical interpretation. $R$-symmetries arise from the local Lorentz symmetry and the transformation property under the rotational symmetry determines the $R$ charge of each field. Symmetry of the $T^6/(\mathbb{Z}_{12} \times \mathbb{Z}_2(\Omega R_{89}))$ geometry contains $Z_4$ subgroup that rotates the third complex plane by angle $\pi$, since the $\Omega R_{89}$ symmetry connects $z_3$ with $-z_3$. Charge assignment under this rotation for the particles derived from the D3–D3, D3–D7 and D7–D7 sectors is given by $+2$ for $X^a_{\beta} = X^a_{(3)_4}$, 0 for $Q^a_{\alpha}$ and $\overline{Q}^a_{\alpha}$ ($k = 1, \ldots, 6$), and $+2$ for $\overline{H} = \Sigma^6_{(3)_4}$. Low energy $Z_4$ $R$-symmetry is considered to be a linear combination of such rotational symmetry and some $U(1)$ gauge symmetries. Now we have $U(1)_5$ and $U(1)_{6+X}$ symmetry. It is an easy task to determine the contribution of these two $U(1)$ symmetries so that the charges of the $Q_i, \overline{Q}^i$ ($i = 1, \ldots, 5$) and $Q_6, \overline{Q}^6$ are those given in Table 1. Then, we find that the $Z_4$ $R$ charge of the $\overline{H}$ becomes automatically 0, the charge given in Table 1. This nontrivial fact is not coincident, but rather inevitable, since any relevant symmetry cannot forbid the missing partner mass term Eq. (18).

Another $N = 1$ chiral multiplet $\overline{\psi}^i(5)$, which comes from the D3–D7 sector at the $Z_{12}$ fixed point, can be identified with the $H(5^*_i)$ because of its gauge charge. Although $H(5^*_i)$ and $Q^a_i, \overline{Q}^a_i$ localize at different fixed points, we expect that exchange of particles whose masses are of order of the cut-off scale $M_*$ provides the superpotential $\overline{Q}^a_i Q^a_i H(5^*_i)$. The exchange of such particles has a suppression factor
due to the Yukawa damping of the wave function. However, the suppression factor is of order 0.1, as we see below.

In order to explain the disparity of the two gauge coupling constants, as described in Section 2, the volume in which the D7–D7 sector fields propagate must be larger than the order $M_s^{-4}$. The four-dimensional volume in the 4–7th directions is $3L^4$ when the distance between the $Z_{12}(\sigma)$ fixed point and the $Z_4(\sigma^3)$ fixed point is given by $L$. Gauge-coupling unification condition

$$\left(1/a_{3H}, 1/a_{H} \sim 1/\alpha_s^{(3)} \right) \lesssim 10^{-2} \left(1/a_{\text{GUT}} \sim 3(ML_s)^4/\alpha_s^{(3)} \right)$$

(20)

determines the length of four-dimensional torus in the 4–7th directions. In particular, we find $e^{-M_sL} \simeq 0.1$ for $(M,L)^4 \simeq 30$, which is the Yukawa damping of wave functions due to the separation between the $Z_{12}(\sigma)$- and $Z_4(\sigma^3)$-fixed point for a particle of mass $M_s$.

In this model, $H^i(5^*)$ propagates in a bit large extra dimensions while $H^i(5)$ does not. This leads to a large wave function renormalization of the $H^i$ because of the large volume, which may result in suppressed type Yukawa couplings $\gamma_{\text{Yuk}}^{ij}$ compared with up type Yukawa couplings $\gamma_{\text{Yuk}}^{3i}$.

We discussed the triangle anomaly cancellation on the orbifold geometry. This is one of consistency conditions to be checked, but is not the only one. There are further consistency conditions, and they are discussed in a future publication [16].

Since the origins of the $Z_4$ R-symmetry are (local Lorentz and U(1)$_{\text{h}}$) gauge symmetries, the low energy $Z_4$ R-symmetry is also gauged. Thus, the discrete anomaly of the $Z_4$ R-symmetry must also be canceled out. Fortunately, it is known that the $Z_4$ R-symmetry of Table 1 has vanishing anomaly with a minimal extension of the present model as discussed in [20].

5. Conclusions and discussion

In Ref. [7] it has been shown that various features of the semi-simple unification model are simultaneously explained in a type IIB orientifold with a D3–D7 system. In this Letter, we show that the $T^6/Z_{12}$ orientifold model provides exactly the phenomenologically required matter content of the U(3)$_{\text{H}}$ sector (GUT-breaking sector) without unwanted light particles. $Z_4$ $R$ charge of each field determined from its property under the rotation of the extra-dimensional space can be the same as the desired one. Most of the superpotential relevant to the GUT breaking dynamics and the missing partner mechanism are obtained from the interactions of the D3–D7 system.

Extension to the type IIB string theory gives us a geometrical interpretation of what is happening in this model. First of all, the presence of the “$N = 2$ gauge interaction” term and the missing partner term becomes clear in the string theory. This is because both the $X_{\alpha \beta}^4$ and the $H^i = \Sigma_{(\alpha \beta)}^i$ are fluctuations of the D3-branes and D7-branes, respectively, in the D7-transverse directions, and expectation values of these fields (i.e., separation between D3- and D7-branes) must provide masses to the D3–D7 open strings $Q$ and $\tilde{Q}$ [8]. Secondly, the GUT breaking is regarded as a bound state formation of the three D3-branes (U(3)$_{\text{H}}$) with five D7-branes (SU(5)$_{\text{GUT}}$). Three of the five D7-branes form a bound state together with the three D3-branes, while two of them keep the original nature, and that is how the triple-double splitting takes place. Indeed, the massless field $X_{\alpha \beta}^4 = X_{(\alpha \beta)}^4$ acquires a mass with $\{X\} = 0$ after the GUT breaking, which means that the D3-branes that move freely in the D7-transverse directions are no longer able to leave the D7-brane position [21].

Thirdly, there is a suggestion on the background geometry from the presence of the FI term. Recall that the type IIB string theory predicts an existence of twisted sector fields on orbifold fixed points. Discussion in [22] shows that there is a bilinear coupling between the $\sigma^3$-twisted sector fields and the U(1)$_{\text{H}} N = 2$ vector multiplet fields since $\text{tr}(\gamma_{\sigma^3}^\dagger \gamma_{\sigma^3})_{18 \times 18} \neq 0$ (see Eqs. (13) and (14)). In particular, the origin of the FI $F$ term that induces the GUT breaking is interpreted as an expectation value of the $\sigma^3$-twisted NS–NS sector field. Under this interpretation, the FI $F$ term indicates that the back ground geometry is not exactly the orbifold,
but rather, some of the orbifold singularities are blown up and topologically nontrivial two cycles appear instead of the singularities. Finally, the U(3)_{11} D3-branes at an “N = 2 fixed point” are regarded as fractional branes [23]. They wrap the topologically nontrivial cycles discussed above and cannot move away from that place into D7-tangential directions, and that is how the unwanted NG modes are eliminated.

The above points are very encouraging facts in the string theory. However, even in the string theory it seems very difficult to obtain the three families of matter multiplets (5^* + 10) at the \(Z_{12}\) fixed point. This also strongly suggests that our manifold is not exactly the orbifold limit but rather it has more complicated structure around the fixed point. If it is the case, it may be very difficult to derive further consistency conditions originated from the string theory. Nevertheless, we believe the connection of the present semi-simple unification model in supergravity to the type IIB string theory to be pursued in future investigations.

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