Information Technology and Quantitative Management (ITQM 2015)

**Associated Credit Risk Contagion Between Enterprises Based on the Imperfect Immunization Scenario**

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**Abstract**

This paper giving the associated credit risk model with the imperfect immunization scenario in the enterprises cluster based on assets-related under the framework of the small world network, we apply mean field theory and principle of epidemiology, to reveal the relation among associated credit risk contagion probability with imperfect immunization and asset-related in enterprises community network. The study has found that the threshold value of the associated credit risk contagion is reduced under the condition of the immune failure and immune invalidity, thereby, increasing the likelihood of related credit risk contagion; and the assets correlation between enterprises will influence the associated credit risk contagion and immune effect.

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Peer-review under responsibility of the Organizing Committee of ITQM 2015

**Keywords:** assets-related; small world network; associated credit risk; imperfect immunization; contagion;

1. Introduction

Enterprises form a complicated relationship via equity, guarantees, mutual insurance, connected transaction, financial derivatives, supply chain and the multiple identities of management, etc. In the presence of some kind of connection (such as asset-related, connected transaction or interpersonal relationship, etc.) between enterprises, some corporate defaults will cause the other related corporate defaults or increase default probability, this kind of credit risk is said to be the associated credit risks for the companies. Enterprises having correlation relationship is called the associated enterprises for short, and the related enterprises form the associated enterprises group by correlation relationship, the network composed of the associated enterprises community is called the associated enterprises network.

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The associated credit risk between the enterprises often aggravate a complicated situation in the economic crisis, especially the enterprise with large and complex credit risk. Hardly this kind of enterprise violate the treaty when it will adversely affect the other associated enterprises and even the whole society, leading to a series of chain reaction. For example, the bankruptcy of Zhejiang Tianyu construction co., LTD. cause mutual insurance crisis among Zhejiang enterprises at the beginning of 2012, in July 2014, the "security incident" of the Sichuan Huitong credit financing guarantee co., LTD., brought about Sichuan non-governmental financial crisis, and so on. Therefore, the associated credit risk will affect the healthy and sustainable development of all the companies in the associated enterprise community.

In reality, some of the affiliated enterprises have a direct asset-related relationship, while most of the affiliated enterprises are indirectly associated. The indirect association relationship refers to the process of forming asset-related relationship by several correlation connection among enterprises. In the affiliated enterprises network, "relational degree" generally refers to the number of the related enterprises which have a direct asset connection of the enterprise. If most of the enterprises are not directly connected, but related by direct and indirect correlation in the network, and such characteristics of network structure is called a "small world network based on affiliated enterprises community". Using small world network[1] can not only portrays the performance characteristics of the association relationship in the affiliated enterprises community, but also simplify the complexity of the structure of the association relationship. Complex network studies is growing rapidly in academia, the development of the small world network theory also provides a studying tool for the research of the complex network of the enterprise management, such as equity relationship network between enterprises, index futures network etc. [2-3], all of which have the characteristics of small-world networks.

The survival environment of the economic enterprise is similar to the living environment of the organisms, both of which are full of complexity, uncertainty, infectious characters and immune effect. Facing the expectancy and uncertainty of the associated credit risk, the enterprise can judge accurately and take action accordingly to avoid being infectious, therefore the enterprise is to possess immune features of associated credit risk with immunity, and the ability of this self-treatment is called the enterprise immune governance capabilities. In recent years, some domestic scholars discussed the risk contagion and organizational immunity of the enterprises by using the biological infectious disease model. For example, Ma Yuanyuan et al.[4] studies the complex network structure characteristics of listed companies by using SIR model; Ma Yinghong et al. [5]discuss the function of the weighted weaken immunity to help to the control of the spread of information in the network;Lv Ping[6]proposed organization immunization behaviour, organization immunization performance and organization performance from the perspective of biological immunology; Xu hui et al.[7] apply organization immunization idea and enterprise risk management theory to study a technological small and mid-sized enterprise environmental risk. Up to now, not much literature is concerning the associated credit risk propagation between the affiliated enterprises, few scholars study the associated credit risk contagion effects, and less analyse small world network model and epidemic model.

In the process of the enterprise associated credit risk immune management, if the associated credit risk is highly contagious or the immune strategy of the enterprise is mistaken, may causing the failure of the enterprise immune governance, which is said to be the immune failure of the enterprise. If the enterprise has immunized successfully but lost immunity with a certain probability after a period of time, it’s called the immune invalidation of the enterprise. If some of the enterprise broke out credit risk, how does asset relationship affect credit risk contagion? What are the effects of the imperfect immunization scenario for the associated credit risk contagion in the affiliated enterprises community network? These problems is a major issue of concern in the current research of the associated credit risk.

This paper studies the affiliated enterprises community by asset-related relationship, and assumes that complex networks structure that is composed of the affiliated enterprises community network belongs to the small world network. Under the framework of the small world network and the principle of epidemiology, this
paper constructed the associated credit risk contagion model between enterprises based on the imperfect immunization scenario. The study has found out that the associated credit risk contagion threshold is reduced under the condition of the immune failure and immune invalidity, thus increasing the likelihood of related credit risk contagion; and the assets correlation between enterprises will influence the associated credit risk contagion and immune effect. This paper is helpful for the prevention and control of the spread of the associated credit risk in the affiliated enterprises community, it is significant in reality and more valuable in theory for promoting the healthy development of the enterprises cluster and maintaining the stability of the social economy.

2. The associated credit risk contagion model under the imperfect immunization scenario

Consider the affiliated enterprises group network is the small-world network for $N$ existing assets relationship of the enterprises, nodes represent the enterprises, edges denote the presence of relationship between company’s assets. $\langle k \rangle$ is the average correlation degree of the affiliated enterprises group, the ratio between the associated assets of the enterprise and it’s total assets is called asset-related ratio, Denoted $\eta$. For the sake of analysis, This article assumes that the enterprise can be any one of the following four states in the network: (1) “Healthy” status $S$: It means the enterprises don’t happen or can’t be contagious by the associated credit risk but susceptible; (2) “Unhealthy” status $I$: It is expressed as the enterprises have happened or has been infected by the associated credit risk and thus being infectious, but it can recover “healthy” status after the effective rescue; (3) “removed” status $R$: It represents the enterprises have happened or have been infected with the associated credit risk, but still cannot be converted to “healthy” status after a variety of rescues; (4) “quasi-immune” status $Im$: It is indicated that the associated credit risk occurred in the enterprises, it may avoid or mitigate credit risk further deterioration after the process of self-help.

This convention $\alpha(0 < \alpha < 1)$ is the density of "quasi-immune" companies in the affiliated enterprises community network whose size is $N$, immune success rate is $\beta(0 < \beta < 1)$, namely the proportion of the enterprise of immunizing successfully is $\alpha \beta$. Usually, due to various enterprises have different "immune" management ability, thus causing the immune success rate is also different, in order to discuss conveniently, this article assumes $\beta$ is all the "quasi-immune" enterprises success rate.

If at time $t$, the credit risk of the "unhealthy" enterprises increases and begins to spread in the affiliated enterprises group, at time $t+1$, "healthy" and "quasi-immune" enterprises were infected by "unhealthy" enterprise with probability $\gamma$, which $\gamma$ represented the associated credit risk contagion probability. Along with the associated credit risk contagion effect is further increased, the companies immune successfully lost immunity ability at a certain probability after a certain period of time, namely this kind of enterprise is immune failure. This paper ignores the difference of immunizing between the enterprises, and assumes all the "quasi-immune" enterprises failure probability are $\sigma$. For not having immunity ability and not infected enterprises, they are still thought to be in the "quasi-immune" status. The features of the associated credit risk contagion process can be described as follows: the "healthy" companies follow the transformation process $S \rightarrow I \rightarrow R$, the "quasi-immune" enterprises also follow the transformation process $Im \rightarrow I \rightarrow R$.

In time $t$, we suppose the proportion of the "healthy" enterprises in the enterprises group network is called the density of the "healthy" enterprise, and write for $S(t)$. Take the proportion of the number of the "unhealthy" companies in the network as the density of the "unhealthy" enterprise, and denoted by $\rho(t)$;Regard the proportion of the number of the default enterprises in the network as the density of the "removed" enterprise, and denoted by $R(t)$;The proportion of the number of the "quasi-immune” companies in the network is said to be the density of the "quasi-immune" enterprise, and denoted by $\tau(t)$, and satisfy the conditions $S(t) + \rho(t) + R(t) + \tau(t) = 1$.

According to the characteristics of the small world network, the "relational degree" of each enterprise is approximately equal to the average correlation of the network, so we can assume asset-related ratio of each
enterprise the same. If the association enterprises group network burst out the related credit risk contagion, the contagion intensity will be connected with the average correlation degree, the asset-related ratio, and so on of the affiliated enterprises group network. Based on these facts this paper apply the dynamic mean field theory to get the evolution equation of all sorts of enterprises with time \( t \) in the network:

\[
\frac{d\rho(t)}{dt} = -\rho(t) + \gamma < k > \eta \rho(t) S(t) + \sigma \gamma < k > \eta \rho(t) \tau(t) \tag{1}
\]

\[
\frac{dS(t)}{dt} = -\gamma \eta < k > \rho(t) S(t), \quad \frac{d\tau(t)}{dt} = -\sigma \gamma \eta < k > \rho(t) \tau(t), \quad \frac{dR(t)}{dt} = \rho(t). \tag{2}
\]

In the formula (1), the first item says the "unhealthy" enterprise with unit rate of default; The second item is the "healthy" enterprise can be contagious by the "unhealthy" enterprise default and thus we get the average density of the new "unhealthy" enterprise, it’s associated with \( \gamma , < k > , \eta \) and \( S(t) \), and so on; The last item showed under the influence of immune failure the "quasi-immune" enterprises generate the average density of the new "unhealthy" enterprise, it is in proportion to \( < k > , \eta , \gamma , \sigma \) and \( \tau(t) \) of the "quasi-immune" enterprise.

Assume that the "unhealthy" enterprise have a very small proportion in the affiliated enterprises group network at the initial moment of the associated credit risk contagion, and randomly distributed in the whole network. The "quasi-immune" enterprise density and immune success rate is \( D \) and \( E \) respectively, and get the initial conditions for the associated credit risk contagion \( \rho(0) = 0, R(0) = 0, \tau(0) = \alpha \beta, S(0) = 1 - \alpha \beta \).

By introducing an auxiliary function

\[
\phi(t) = \int_0^t \rho(u) du = R(t). \tag{3}
\]

Then \( \phi(t) \) is equal to the density of the "removed" enterprise \( R(t) \), in (3) the differential with respect to time \( t \), so

\[
\frac{d\phi(t)}{dt} = 1 - (1 - \alpha \beta) \exp(-\gamma \eta < k > \phi(t)) - \phi(t) - \beta \eta \exp(-\sigma \gamma < k > \phi(t)). \tag{4}
\]

As time tends to infinity, the affiliated enterprises group network is in a stable condition, at this point, there are not any "unhealthy" enterprise in the network, namely \( t \to \infty \), then \( \rho(\infty) = 0 \).

By (3)

\[
\lim_{t \to \infty} \frac{d\phi(t)}{dt} = 0. \tag{5}
\]

Combine (4) and (5), then

\[
H(\phi(\infty)) = 1 - (1 - \alpha \beta) \exp(-\gamma \eta < k > \phi(\infty)) - \alpha \beta \exp(-\sigma \gamma \eta < k > \phi(\infty)). \tag{6}
\]

Here \( H(\phi(\infty)) \) is an auxiliary function, (6) is obtained trivial solution \( \phi(\infty) = 0 \), and when \( \phi(\infty) = 0 \), by (3), so \( R(\infty) = 0 \), this shows when the associated credit risk contagion reaches a steady state, there is no infectious enterprise, then it does not exist the association of credit risk contagion effects in the affiliated enterprises group network.

Further \( R(\infty) \) represent the associated credit risk contagion effects when the enterprise network is in a stable state, then when \( \gamma < \gamma_c \), due to the network haven’t contagion of the associated credit risk, therefore \( R(\infty) = 0 \); When \( \gamma \geq \gamma_c \), by (3) and (6)

\[
R(\infty) = 1 - (1 - \alpha \beta(1 - \sigma)) \exp(-\gamma \eta < k > R(\infty)) - \alpha \beta \exp(-\sigma \gamma \eta < k > R(\infty)). \tag{7}
\]

In the case of ignoring the higher order term, using Taylor expansion formula further reduction (7)

\[
R(\infty) \approx [1 - \alpha \beta(1 - \sigma)] \gamma \eta < k > R(\infty) - \frac{1}{2} (1 - \alpha \beta + \beta \beta \sigma^2) [\gamma \eta < k > R(\infty)]^2. \tag{8}
\]
By (8)

$$R(\infty) = \begin{cases} 2(1-\alpha\beta(1-\sigma))\gamma\eta < k > 2 \\ [1-\alpha\beta(1-\sigma^2)]\gamma^2\eta^2 < k > 2 \end{cases}$$ (9)

From (9) can be found the correlation credit risk contagion effects after reaching a steady state, the density of the "removed" enterprise is the function of the density of $\alpha$, $\beta$, $\sigma$, $\gamma$ and $\eta$. Therefore, given the associated credit risk contagion probability, the density of the initial immune enterprise and the enterprise asset correlation ratio, the immune failure and invalidity coexist at the same time will impact the associated credit risk contagion process in the affiliated enterprises group network, thus weakening the immune effect.

3. Concluding discussion

The associated credit risk is an important part of modern enterprise risk management, the affiliated enterprises community network has small world characteristics through asset correlation channels between enterprises. Dynamic mean field theory and principle of infectious diseases is applied in this article, combined with the complex network theory, constructs the associated credit risk contagious model in the incomplete immune scenario for the enterprises cluster network. The Study has found that the incomplete immune is helpful to reduce the critical probability of the associated credit risk contagion, impact the transmission process of the associated credit risk, and weaken the enterprise immunity governance effects; Once the associated credit risk contagion effects strengthen in enterprise group network, the number of business failures will also increase. Therefore, the manager should not overestimate the effect of governance, and should take well charge of the related-assets between enterprises in the immunity governance processes in the face of the modern enterprise associated credit risk. Research conclusion of this paper will offer references to identify the related credit risk between companies and supervision for the relevant government departments and financial institutions, which also adds a new perspective to the management the associated credit risk.

Acknowledgements

This work was financially supported by National Natural Science Foundation of China (71271043) and Humanity and Social Science Research Program of Chongqing Municipal Education Commission (14SKC03).

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