Lorentz invariance without trans-Planckian physics?

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A R T I C L E   I N F O

A B S T R A C T

We explore the possibility that, in a quantum field theory with Planck scale cutoff $\Lambda \simeq m_\mathrm{p}$, observable quantities for low-energy processes respect the Lorentz symmetry. In particular, we compute the one-loop radiative correction Π to the self-energy of a scalar field with $\lambda\phi^4$ interaction, using a modified (non-invariant) propagator which vanishes in the trans-Planckian regime, as expected in the "classicalisation" scenario. We then show that, by imposing the result does not depend on Λ (in the limit $\Lambda \to m_\mathrm{p}$), an explicit (albeit not unique) expression for Π can be derived, which is similar to the one simply obtained with the standard Feynman propagator and a cutoff $\Lambda = m_\mathrm{p}$.

1. Introduction

It is usually believed that quantum gravitational effects should become relevant at energy scales of the order of the Planck mass, $m_\mathrm{p} \simeq 10^{16}$ TeV, or higher. This conclusion is easily reached by considering that the Einstein–Hilbert action is proportional to the Newton constant $G_\mathrm{N} = \ell_\mathrm{p}/m_\mathrm{p}$,[1] and gravitational perturbations on a given background therefore couple to matter sources with a strength proportional to $\ell_\mathrm{p}/m_\mathrm{p} \simeq m_\mathrm{p}^{-2}$. The strength of the matter–gravity coupling can also be seen directly in the semiclassical Einstein field equations,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi\, G_\mathrm{N} \langle \hat{T}_{\mu\nu} \rangle, \tag{1.1}$$

where the expectation value $\langle \hat{T}_{\mu\nu}\rangle$ of the energy-momentum (operator) tensor on a given quantum state of matter has replaced its classical counterpart $T_{\mu\nu}$.

A clear exception is given by purely classical vacuum solutions of Eq. (1.1), for which $\langle \hat{T}_{\mu\nu}\rangle \simeq T_{\mu\nu} = 0$, so that G_N apparently drops from the calculation. In fact, G_N can re-enter as part of an integration constant proportional to the mass m of a spin-less point-like source, and turns it into a length, namely the Schwarzschild radius

$$R_M = 2 G_\mathrm{N} m \equiv 2M. \tag{1.2}$$

On the other hand, for such a particle, quantum mechanics introduces an uncertainty in spatial localisation, typically of the order of the Compton (de Broglie) length,

$$\lambda_M \simeq \frac{\ell_\mathrm{p} m_\mathrm{p}}{m} = \frac{\ell_\mathrm{p}^2}{M}. \tag{1.3}$$

Given that quantum physics is a more refined description of reality than classical physics, the clash of the two lengths, R_m and λ_m, implies that the former only makes sense provided it is significantly larger than the latter,

$$R_M \gtrsim \lambda_M \quad \Rightarrow \quad m \gtrsim m_\mathrm{p}, \tag{1.4}$$

or $M \gtrsim \ell_\mathrm{p}$. Note that this argument employs the flat space Compton length (1.3), and it is likely that the particle's self-gravity will affect it. However, it is still reasonable to assume the condition (1.4) holds as a rough, order of magnitude, estimate. In fact, one can alternatively consider the "mean energy density" inside the Schwarzschild radius,

$$\mathcal{E}_H \simeq \frac{m}{R_H^3} = \frac{m_\mathrm{p}^3}{\ell_\mathrm{p}^3 m^2}, \tag{1.5}$$

and require that it does not exceed the Planck scale,

$$\mathcal{E}_H \lesssim \ell_\mathrm{p}^{-3} m_\mathrm{p}, \tag{1.6}$$

which again leads to Eq. (1.4).

Overall, the above-mentioned consideration that quantum gravity effects become relevant for m of order m_p or higher now appears questionable, since the condition (1.4) implies that such a system should be fairly well described in classical terms. This is indeed at the core of the recent ideas of UV self-completeness of gravity and "classicalisation" [1], as well as it had previously inspired Generalised Uncertainty Principles (GUPs) [2]. More or less

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[1] We shall always use units with $c = 1$ and $\hbar = \ell_\mathrm{p} m_\mathrm{p}$.

implicitly, such scenarios require the existence of a preferred (inertial) reference frame in which the components of four-momenta reach Planck size, thus breaking Lorentz covariance at face value.[2] Our main aim in this work will be precisely to investigate the possibility that Lorentz symmetry at low-energy and "classicalisation" – or, more precisely, a total suppression of trans-Planckian quantum modes, can be effectively reconciled.

2. Gravitational renormalisation

There are many reasons to indulge in the possibility that the gravitational interaction causes Quantum Field Theory (QFT) propagators to depart from their flat-space expressions at high energy. Whatever the reason, we then need an explicit implementation in order to compute physical predictions. Classicalisation induced by black hole formation and the GUPs are proposals we have already mentioned above. Alternatively, a set of "diagrammatic rules" was presented in Ref. [4] to effectively (and non-perturbatively) include (self-)gravity in the standard perturbative QFT of matter and other interactions. Since such rules will not be explicitly needed here, we will just recall the basic idea: in Feynman diagrams, each flat-space Feynman propagator of momentum p, should be replaced by the propagator in the curved space–time sourced by all the other (real or virtual) particles (say, with total momentum q) in the same diagram, to wit

$$G(p) \to G_q(p; m_\mathrm{p}), \qquad (2.1)$$

where we also allowed for an explicit dependence on the Planck scale m_p, as a reminder that (self-)gravity is to be included. These propagators could, in principle, be obtained perturbatively, by summing over infinitely many graviton exchange diagrams or, non-perturbatively – but perhaps, equivalently, by solving the semiclassical equations (1.1), although this task is likely unattainable without some other approximation. For example, in Ref. [4], a modified scalar propagator was derived, under the working assumption that the Schwarzschild metric can be approximated by a conformally flat metric for (short-lived) virtual processes. The one-loop correction to the four-point correlation function for the scalar field with $\lambda\phi^4$ interaction was then shown to contain no Ultra-Violet (UV) divergences.

In the following, we shall assume that classicalisation works, with no quantum degree of freedom propagating above the Planckian scale, and just focus on the requirements the propagator must consequently satisfy to build a theory consistent with low-energy Lorentz symmetry. To this purpose, we shall not (totally) specify the modified propagator in Eq. (2.1), but assume that when any component k^μ of the internal momenta reaches the Planck scale [so that condition (1.4) is roughly satisfied], a classical configuration forms, whose contribution as a radiative correction is negligible. This assumption can be effectively formalised by introducing a UV cutoff Λ in the integrals over components of the virtual momenta at the Planck scale, say $\Lambda \simeq m_\mathrm{p}$. This rule also seems to require a preferred reference frame. For example, one may consider the rest frame of the (virtual) forming black hole, in which the spatial components of its four-momentum vanish, that is $k^\mu = (E, 0, 0, 0)$, and apply a continuous change of frame while performing the integration over virtual momenta.[3] We shall here opt for a simpler picture, we are now going to illustrate with an example.
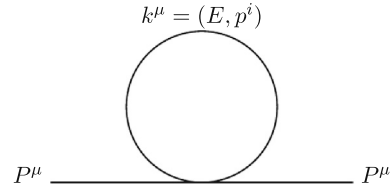

Fig. 1. One-loop correction to the mass from $\lambda\phi^4$.

3. Gravitationally renormalised self-energy

We wish to test the above rule on the self-energy of a scalar field with $\lambda\phi^4$ interaction. We shall first assume, for the sake of simplicity, that there exists a global inertial frame in which the cutoff is isotropic, and then estimate the possible corrections induced by local fluctuations of the cutoff itself.

3.1. Global isotropic cutoff

We shall here assume there exists a global inertial reference frame where the momentum cutoff is given by the isotropic four-vector

$$\Lambda^\alpha = (\Lambda, \Lambda, \Lambda, \Lambda). \qquad (3.1)$$

The one-loop contribution is therefore represented by the tadpole diagram (see Fig. 1), and reads

$$\Pi_\Lambda(P^2) = \lambda \left(\prod_{\mu=0}^{3} \int_{-\Lambda}^{+\Lambda} \mathrm{d}k^\mu \right) G_P(k; m_\mathrm{p}), \qquad (3.2)$$

where P^μ is the (on-shell) four-momentum of the scalar particle, with $P^2 \equiv P_\mu P^\mu = m_0^2$, and $G_P(k; m_\mathrm{p})$ is the modified propagator from Eq. (2.1). In a different inertial frame, the cutoff four-vector will be $\bar{\Lambda}^\alpha = L^\alpha{}_\beta \Lambda^\beta$ (where L is a Lorentz matrix), and we must likewise have

$$\Pi_{\bar{\Lambda}}(P^2) = \lambda \left(\prod_{\mu=0}^{3} \int_{-\bar{\Lambda}^\mu}^{+\bar{\Lambda}^\mu} \mathrm{d}k^\mu \right) G_P(k; m_\mathrm{p}), \qquad (3.3)$$

where $\bar{P}^\alpha = L^\alpha{}_\beta P^\beta$ is the boosted external momentum, again with $\bar{P}^2 = m_0^2$. If the result has to be invariant under (small) changes of the cutoff, $\Pi_\Lambda(P^2) = \Pi_{\bar{\Lambda}}(P^2)$ for $\Lambda \simeq \bar{\Lambda} \sim m_\mathrm{p}$, at least when the components $|P^\mu| \ll m_\mathrm{p}$, we must then have

$$\left. \frac{\partial \Pi_\Lambda(P^2)}{\partial \Lambda} \right|_{\Lambda = m_\mathrm{p}} = 0, \qquad (3.4)$$

which can be more explicitly written as

$$\sum_{\mu=0}^{3} \left[\left(\prod_{\nu \neq \mu} \int_{-m_\mathrm{p}}^{+m_\mathrm{p}} \mathrm{d}k^\nu \right) G_P(k; m_\mathrm{p}) \right]_{k^\mu = \pm m_\mathrm{p}} = 0. \qquad (3.5)$$

Clearly, Eq. (3.5) would hold if the modified propagator $G_P(k; m_\mathrm{p})$ vanished when the components $k^\mu = \pm m_\mathrm{p}$, and does not hold with the usual Feynman propagator.[4] Further, since we are interested in

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the low-energy regime for the external particles, all the components $|P^\mu| \ll m_\mathrm{p}$, and in Eq. (3.5) we can approximate

$$G_P(k; m_\mathrm{p}) \simeq G_{P=0}(k; m_\mathrm{p}) \equiv G(k^\mu; m_\mathrm{p}), \tag{3.6}$$

where Greek indices run from 0 to 3 (Latin indices $i = 1, 2, 3$ and $a = 1, 2$), and obtain

$$\sum_{\mu=0}^{3} \left[\left(\prod_{\nu \neq \mu} \int_{-m_\mathrm{p}}^{+m_\mathrm{p}} \mathrm{d}k^\nu \right) G(k^\alpha; m_\mathrm{p}) \right]_{k^\mu = \pm m_\mathrm{p}} = 0. \tag{3.7}$$

Since the "preferred" direction P^μ was dropped, we can now employ homogeneity of Minkowski space–time in order to write the above as

$$0 \simeq 2 \int_{-m_\mathrm{p}}^{+m_\mathrm{p}} \mathrm{d}k^1 \, \mathrm{d}k^2 \, \mathrm{d}k^3 \, G(k^\mu; m_\mathrm{p}) \bigg|_{k^0 = m_\mathrm{p}}$$

$$+ 6 \int_{-m_\mathrm{p}}^{+m_\mathrm{p}} \mathrm{d}k^0 \, \mathrm{d}k^1 \, \mathrm{d}k^2 \, G(k^\mu; m_\mathrm{p}) \bigg|_{k^3 = m_\mathrm{p}}$$

$$= 8\pi \int_0^{m_\mathrm{p}} p^2 \, \mathrm{d}p \, G(m_\mathrm{p}, p^i; m_\mathrm{p})$$

$$+ 12\pi \int_0^{m_\mathrm{p}} \mathrm{d}E \int_0^{m_\mathrm{p}} q \, \mathrm{d}q \, G(E, q^a, m_\mathrm{p}; m_\mathrm{p})$$

$$\equiv I_1 + I_2. \tag{3.8}$$

We further assume

$$G(k^\mu; m_\mathrm{p}) = \frac{g(E, p; m_\mathrm{p})}{E^2 - p^2 - m_0^2}, \tag{3.9}$$

where the function $g(E \lesssim m_\mathrm{p}, p \lesssim m_\mathrm{p}; m_\mathrm{p}) \simeq 1$, in order to recover the standard Feynman propagator at low momenta. The only non-vanishing contribution to the right hand side of Eq. (3.8) then comes from values of the integrands around $\Lambda \sim m_\mathrm{p}$, namely $E \simeq p \simeq m_\mathrm{p}$. In fact,

$$I_1(g=1) \simeq 8\pi \int_0^\Lambda \frac{p^2 \, \mathrm{d}p}{m_\mathrm{p}^2 - p^2 - m_0^2}$$

$$\simeq 8\pi \left[m_\mathrm{p} \operatorname{arctanh}\left(\frac{\Lambda}{m_\mathrm{p}} \right) - \Lambda \right]$$

$$\simeq 4\pi m_\mathrm{p} \ln\left(\frac{m_\mathrm{p}}{m_\mathrm{p} - \Lambda} \right) + \mathcal{O}(m_\mathrm{p} - \Lambda), \tag{3.10}$$

for $\Lambda \to m_\mathrm{p}^-$, and neglecting the bare mass $m_0 \ll m_\mathrm{p}$. Likewise,

$$I_2(g=1) \simeq 12\pi \int_0^\Lambda \mathrm{d}E \int_0^\Lambda \frac{q \, \mathrm{d}q}{E^2 - q^2 - m_\mathrm{p}^2 - m_0^2}$$

$$\simeq 12\pi m_\mathrm{p} \operatorname{arctanh}\left(\frac{\Lambda}{m_\mathrm{p}} \right)$$

$$- 12\pi \sqrt{m_\mathrm{p}^2 + \Lambda^2} \operatorname{arctanh}\left(\frac{\Lambda}{\sqrt{m_\mathrm{p}^2 + \Lambda^2}} \right)$$

$$+ 12\pi \Lambda \ln\left(\frac{m_\mathrm{p}^2 - \Lambda^2}{m_\mathrm{p}^2} \right)$$

$$\simeq 12\pi m_\mathrm{p} \left[\ln(2) - \sqrt{2} \operatorname{arccoth}(\sqrt{2}) \right]$$

$$+ \mathcal{O}(m_\mathrm{p} - \Lambda). \tag{3.11}$$

Since Eq. (3.10) diverges for $\Lambda \to m_\mathrm{p}$, the function g must be at least of order $(m_\mathrm{p} - \Lambda)$, for $p \sim q \sim E \sim \Lambda$, in order to cure the divergence and satisfy (3.8).

We shall therefore replace all the UV cutoffs at m_p with a general $m_\mathrm{p}^2 \gtrsim \Lambda^2 \gg P^2$, assume

$$g(E, p^i; m_\mathrm{p}) = \frac{1}{\alpha} \left[\frac{m_\mathrm{p}^2 - p^2}{m_\mathrm{p}^2} + (\alpha - 1) \frac{m_\mathrm{p}^2 - E^2}{m_\mathrm{p}^2} \right]$$

$$= 1 + \frac{k^2 - \alpha E^2}{\alpha m_\mathrm{p}^2} + \mathcal{O}\left(\frac{k^3}{m_\mathrm{p}^3} \right), \tag{3.12}$$

and take the limit $\Lambda \to m_\mathrm{p}^-$ at the end of the calculation. From this ansatz, we obtain

$$\frac{I_1}{4\pi} = \frac{m_0^2}{\alpha m_\mathrm{p}^2} \sqrt{m_\mathrm{p}^2 - m_0^2} \operatorname{arctanh}\left(\frac{\Lambda}{\sqrt{m_\mathrm{p}^2 + m_0^2}} \right)$$

$$+ \frac{\Lambda}{3\alpha m_\mathrm{p}^2} (\Lambda^2 - 3m_0)$$

$$= \frac{\Lambda^3}{3\alpha m_\mathrm{p}^2} + \mathcal{O}\left(\frac{m_0^2}{m_\mathrm{p}^2} \right), \tag{3.13}$$

and

$$\frac{I_2}{\pi} = \frac{\Lambda}{m_\mathrm{p}^2} \left[\frac{3 - 2\alpha}{\alpha} \Lambda^2 + (\Lambda^2 - 3m_\mathrm{p}^2) \ln\left(\frac{m_\mathrm{p}^2 - \Lambda^2}{m_\mathrm{p}^2} \right) \right]$$

$$+ \frac{2(\Lambda^2 - 2m_\mathrm{p}^2)}{m_\mathrm{p}^2} \sqrt{m_\mathrm{p}^2 + \Lambda^2} \operatorname{arctanh}\left(\frac{\Lambda}{\sqrt{m_\mathrm{p}^2 + \Lambda^2}} \right)$$

$$+ 4m_\mathrm{p} \operatorname{arctanh}\left(\frac{\Lambda}{m_\mathrm{p}} \right) + \mathcal{O}\left(\frac{m_0^2}{m_\mathrm{p}^2} \right). \tag{3.14}$$

Taking the limit $\Lambda \to m_\mathrm{p}^-$ and substituting I_1 and I_2 into Eq. (3.8) therefore yields an equation for the parameter α, which can be easily solved, that is

$$\alpha = \frac{13}{6[1 + \sqrt{2} \operatorname{arccoth}(\sqrt{2}) - 2 \ln 2]} \simeq 2.5, \tag{3.15}$$

or

$$g(E, p^i; m_\mathrm{p}) \simeq 0.4 \left(\frac{m_\mathrm{p}^2 - E^2}{m_\mathrm{p}^2} + 1.5 \frac{m_\mathrm{p}^2 - p^2}{m_\mathrm{p}^2} \right)$$

$$\simeq 1 + 0.4 \frac{k^2 - 2.5 E^2}{m_\mathrm{p}^2}. \tag{3.16}$$

Note that the function g does not only depend on the Lorentz scalar $k^2 = E^2 - p^2$, but also on the energy E. The propagator $G_P(k; m_\mathrm{p})$ is therefore not a Lorentz scalar. This seems a necessary price to pay in order to compensate for the Lorentz dependence of the cutoff Λ, and perhaps not such a costly one, since the propagator is not an observable *per se*.[5] More specifically, the constant $\alpha \neq 0$ signals the departure (of order E^2/m_p^2) of $G_P(k; m_\mathrm{p})$ from being a Lorentz scalar. On the other end, the function g is singular in the limit $\alpha \to 0$ for m_p finite, and the Lorentz violating correction does not depend on α in the low-energy limit. One may

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[5] Strictly speaking, the self-energy is hardly observable either, however we chose this quantity as a reasonably simple toy case to test our line of reasoning.

therefore argue that fixing the UV scale will bring down necessary modifications to the low energy regime (a form of IR–UV mixing).

With the condition (3.8) satisfied, we can finally estimate the mass correction, namely

$$
\begin{aligned}
\Pi_{m_{\mathrm{p}}}(m_0^2) &= \lambda \lim_{\Lambda \to m_{\mathrm{p}}} \int_0^\Lambda \mathrm{d}^4 k \, \frac{g(k^\mu; m_{\mathrm{p}})}{k^2 - m_0^2} \\
&= 2\frac{\pi}{3}\left(\frac{2}{\alpha} - 3\right)\lambda m_{\mathrm{p}}^2\left[1 + \mathcal{O}\left(\frac{m_0^2}{m_{\mathrm{p}}^2}\right)\right] \\
&\simeq -4.6\lambda m_{\mathrm{p}}^2\left[1 + \mathcal{O}\left(\frac{m_0^2}{m_{\mathrm{p}}^2}\right)\right],
\end{aligned}
\tag{3.17}
$$

where the Planck mass m_{p} must here be viewed as a universal constant. The result is therefore a (low-energy, $m_0 \ll m_{\mathrm{p}}$) Lorentz scalar, like we wanted. Of course, one might argue that the chosen form of the weight function g in Eq. (3.12) is hardly the unique solution for the constraint (3.8), and the final expression (3.17) remains consequently ambiguous. However, if we compare with the result derived by using the standard Feynman propagator ($g = 1$),

$$
\begin{aligned}
\Pi_\Lambda(m_0^2) &= \lambda \int_0^\Lambda \frac{\mathrm{d}^4 k}{k^2 - m_0^2} \\
&= -2\pi\lambda\Lambda^2\left[1 + \mathcal{O}\left(\frac{m_0^2}{\Lambda^2}\right)\right],
\end{aligned}
\tag{3.18}
$$

and set $\Lambda = m_{\mathrm{p}}$, we see that we obtained a correction of the same form. The fact that our result (3.17) closely resembles (3.18) is suggestive that, perhaps, any reasonably behaved modified propagator $G_P(k; m_{\mathrm{p}})$ which solves (3.8) would lead to the same kind of mass correction. Eq. (3.17) also implies that $|\Pi_{m_{\mathrm{p}}}| \sim m_{\mathrm{p}}^2 \gg m_0^2$, unless $\lambda \sim m_{\mathrm{p}}^{-2}$, and one must still apply the usual subtraction at the renormalisation point in order to set the mass $\mu^2 \simeq m_0^2 - \Pi$ to the "observed value".

3.2. Fluctuating cutoff

One might question the existence of a global reference frame in which the momentum cutoff takes the isotropic form in Eq. (3.1). For example, there are models in which the space–time appears as a foam (of virtual black holes) at the microscopic level,[6] and it is therefore reasonable to consider a stochastic dependence of the cutoff four-vector on position and time.

Previous results should then be corrected, for example, by taking an "ensemble average" over the stochastic distribution of cutoff around the Planck mass. This means that Eq. (3.4) should be replaced by

$$
\left\langle \frac{\partial \Pi_\Lambda(P^2)}{\partial \Lambda}\right\rangle \equiv \int \mathrm{d}m \, F_{m_{\mathrm{p}}}(m) \frac{\partial \Pi_\Lambda(P^2)}{\partial \Lambda}\bigg|_{\Lambda = m} = 0,
\tag{3.19}
$$

where $F_{m_{\mathrm{p}}}$ is a distribution peaked around the Planck scale $m \sim m_{\mathrm{p}}$ that could be specified given a microscopic model of the space–time, and after integration on the angular variables (to restore local isotropy). It is then easy to see that the final result (3.17) becomes, at least to leading order in m_0/m_{p},

$$
\Pi_{m_{\mathrm{p}}}(m_0^2) \simeq -4.6\lambda\left[1 + \mathcal{O}\left(\frac{m_0^2}{m_{\mathrm{p}}^2}\right)\right]\int \mathrm{d}m \, F_{m_{\mathrm{p}}}(m)m^2
$$

$$
\simeq -\beta\lambda m_{\mathrm{p}}^2\left[1 + \mathcal{O}\left(\frac{m_0^2}{m_{\mathrm{p}}^2}\right)\right],
\tag{3.20}
$$

where β is just a numerical coefficient (of order one) that depends on the details of the stochastic distribution $F_{m_{\mathrm{p}}}$.

To conclude, it is rather unlikely that the form of $F_{m_{\mathrm{p}}}$ is such that subtle cancellations occur in Eq. (3.4), so as to drastically change the final result, and we do not expect any significant modifications from the (more realistic) picture of a space–time dependent cutoff.

4. Final remarks

We have shown that, in the simple case of a (massive) scalar field, the self-energy correction Π can be computed in a QFT with a cutoff at the Planck scale m_{p}, and still obtain a Lorentz invariant result by modifying the propagator: the two non-invariances (of the cutoff and of the propagator) compensate each other and give rise to a (low-energy) frame-independent Π. Such a quantity naturally depends on m_{p}, which must be viewed as a universal (frame-independent) constant. Also, the correction differs just by numerical coefficients from the Π obtained from the usual Feynman propagator, which suggests that, if modifications to the propagator can be related to the scalar field self-gravitational interaction, the effect of the latter should be mild in this context. And that quantum gravitational effects might indeed have an almost irrelevant phenomenological impact on Standard Model predictions to all scales.

Of course, the above result is far from sufficient to prove that the question raised in the title of this Letter can be answered positively. For that purpose, one should generalise the above procedure and require Lorentz invariance of all quantities we can observe in particle physics (such as scattering cross-sections, etc.). In order to achieve this, it will be necessary that the deforming weight g in the propagator (3.9) contains enough degrees of freedom (or parameters, like α in the example above) to satisfy the equivalent of Eq. (3.8). This should not be difficult to accommodate in the spirit of the GUPs or of the rules of Ref. [4], since in diagrams with N internal lines, each corresponding propagators should depend (at least) on the $N - 1$ other virtual particles in the graph (and external real particles), and one expects to have at least $N - 1$ such parameters.

Finally, but not less important, there remains to see if gauge invariances and other symmetries of the Standard Model can be preserved as well, after imposing the low-energy Lorentz invariance of observable quantities. Addressing these crucial issues requires investigating more realistic gauge QFTs, rather than toy model scalar fields, and, unless one can find a systematic procedure, it will also involve a significant amount of work.

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[6] The literature on this subject is rather extensive (see, for instance, Ref. [6]).