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# Locating leaks in water distribution networks with simulated annealing and graph theory

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# Abstract

This paper addresses the problem of locating leaks in water distribution networks. The approach presented here is based on steady-state modelling, supported by monitoring tank flow and pressure at strategic nodes. The selection of the pressure monitoring nodes is done applying graph theory concepts adapted to water distribution networks. Pressure monitoring data is then used to build an optimization problem: the objective function is the minimization of the differences between estimated and measured pressures at the monitoring points, the constraints are the common energy and mass conservation laws and the decision variables are the leak locations and flows. This optimization problem is solved by a simulated annealing algorithm presented in a previous work. The application of this approach in a set of case studies produced some encouraging results and this paper describes the complete procedure and draws some conclusions. © 2015 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

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## 1. Introduction

Water losses in water distribution networks (WDN) affect water utilities and the society, and their negative impacts are: operational (lower service level), economic (lower income and higher operating and capital costs), environmental (higher amount of water and energy usage, and consequent higher water and carbon footprints), public health (potential focus of contamination) and social (service disruptions, traffic disturbances and damage to

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people and their belongings). For these reasons, water loss control is a major concern for water utilities. The increase of operational cost due to real losses, the loss of revenue due to apparent losses and water scarcity issues stimulate water utilities to implement water loss control programs.

It is considered good practice to approach the leak location problem with small sequential steps. Firstly, the WDN can be divided in small sectors called District Metering Areas (DMA). Then, water balance can be periodically applied to each DMA to assess water losses and conclude if it is necessary to implement a strategy to fight them. These water losses include apparent losses (unauthorized consumption and customer meter inaccuracies) and real losses (leakage on transmission and/or distribution mains, leakage and overflows at utility's storage tanks and leakage on service connections up to the point of customer metering point) [1]. Nevertheless, in a WDN (or DMA) a quicker way to detect leaks is to monitor the Minimum Night Flow (MNF). An increase of the MNF can be interpreted as the occurrence of a new leak in the WDN, and the leak flow corresponds to the change of the MNF.

Real losses due to pipe leakage can be reduced by implementing active leakage control, but this usually implies hard work and sophisticated and expensive equipment [2, 3]. Commonly, the leak location activities require on-field analysis with acoustic devices [4, 5, 6, 7], or the use of tracer gas, infrared imaging or ground penetrating radar [8].

This paper presents a methodology to locate leaks based on modelling and optimization techniques. This methodology is almost automatic and uses affordable equipment [9, 10] to monitor flow and pressure during the MNF period. However, the success of methodologies based on pressure measurements is quite dependent on the number and placement of the pressure transducers [11, 12, 13, 14]. Thus, an innovative tool based on graph theory concepts was developed to help in selecting the nodes for pressure monitoring.

The potential of these methodologies is illustrated with a set of case studies, considering different flows in a single leak or in two leaks located in different parts of the WDN. The results presented here include: (I) a sensitive selection of nodes for pressure monitoring; (II) a sensitivity analysis of the leak flow on the quality of the final solutions; (III) a critical analysis of the solutions including pipes incorrectly selected as leaky pipes. The results are discussed and the solutions obtained show the benefits and drawbacks of the methodologies.

# 2. Methodology for identification of probable leaky pipes

First, a computational methodology was developed to help in locating unreported leaks and estimating its flow. This methodology results from linking a hydraulic simulation model [15] to an optimization model. The hydraulic simulation model performs steady state analysis to estimate the WDN behaviour. The optimization model aims to minimize the difference between estimated and measured pressures (monitoring nodes), and the decision variables are the locations and flow of unreported leaks.

Second, a selection of the nodes to be monitored is made with a methodology that resulted from the adaptation of graph theory concepts.

Third, it is assumed that the unreported leakage flow equals the change between consecutive MNF periods and a sensitivity analysis is performed with different leakage flows (from one or two simultaneous leaks) and locations. The results of the methodology are discussed to confirm if the leaky pipes are correctly identified.

Fourth, the final results allow a better problem diagnosis by identifying the leaky pipes. Pipes not identified in any solution can be discarded for the field survey with leak location equipment. The consequence is a quicker response due to an inspection work better supported by previous planning.

#### 2.1. Link between the hydraulic simulator and the optimization model

Given a WDN with a certain total leakage flow, the methodology proposed here is intended to identify the most probable leaky pipes with a known leakage flow. It uses a calibrated model of the WDN to predict its hydraulic behaviour assuming that water consumption during the MNF period is known.

The WDN is being monitored (tanks are equipped with flow meters and a set of nodes is equipped with pressure transducers). The information gathered in the past has been used to calibrate the WDN model and the actual pressure monitoring data will now be used to locate the leaks.

#### 2.2. Optimization model

The leak location is achieved by simulating hypothetical leakage scenarios. The scenarios presenting pressure values closer to those measured are selected. This is done by using an optimization model, which is solved by linking a simulated annealing algorithm with a hydraulic solver (Fig. 1). The objective function is the minimization of the differences between measured (monitoring data) and correspondent estimated (simulation results) pressures, for all monitored nodes:

$$MinF(x) = Min\sum_{i=1}^{NPT} \left(\frac{p}{\gamma}\right)_{i}^{measured} - \left(\frac{p}{\gamma}\right)_{i}^{estimated}$$
(1)

The constraints of the optimization model are the common hydraulic constraints, which are implicitly tackled by the hydraulic simulation model.



Fig. 1. Structure of the leak location methodology.

#### 2.3. Hydraulic simulation model

The WDN hydraulic behaviour is studied assuming steady state conditions, by using the energy and continuity equations. At the MNF period water demand is usually very low and less uncertain. A reduced demand is associated with low head losses, near maximum pressure head and leakage reach its maximum value. Although WDN hydraulic behaviour is always changing, under these conditions the steady state assumption does not introduce major inaccuracies. For each leakage scenario, the hydraulic simulation model [15], compiled in FORTRAN, uses the node equations to estimate the pressure for the monitored nodes, under steady state conditions. These equations are solved by the Newton-Raphson method, supported by a line search algorithm to optimize the step length in order to improve convergence and avoid the drawbacks of the original method. This method has been successfully applied in several previous works [16, 17, 18, 19]. The node equations implicitly ensure the continuity and the energy laws, and the head losses were estimated with the Hazen-Williams formula (2):

$$\Delta H_{ij} = 10.674. \frac{L_{ij} Q_{ij}^{1.852}}{CHW_{ij}^{1.852} D_{ij}^{4.87}}$$
(2)

The Hazen-Williams formula is an empirical and widely used formula to estimate the friction losses from turbulent water flows in circular pipes at normal temperatures. Due to its simplicity and accuracy, this formula has been frequently used in the design and analysis of WDN. It is valid for pipes bigger than 50 mm and flow velocities under 3 m/s.

#### 2.4. Simulated Annealing algorithm

The optimization model presented in Section 2.2 is non-linear, non-convex and contains continuous variables (leakage flow at each pipe). It was decided to use a Simulated Annealing algorithm [20, 21] to solve it, this means, a randomized search method that is known to quickly find good solutions, even in extended search spaces. This method has been used to solve different hydraulic problems, as documented in the literature.

The Simulated Annealing method was inspired by the physical annealing process. The temperature is increased to a value considered high enough to melt a solid, following a slow cooling process in order to allow the molecular structure to reorganize and reach the minimum energy state—A crystal. Although theoretically this method converges to the global optimal solution [22], in real world applications, due to the need to limit the execution time and the number of solutions to evaluate, it is impossible to guarantee its global convergence. The core of a Simulated Annealing algorithm requires the definition of an initial solution, an initial temperature, a procedure to build neighbourhoods, a cooling schedule (temperature decay and number of evaluations to be performed at each temperature), and a stopping criterion. The algorithm starts up with an initial solution, which at this stage also plays the roles of best solution found so far and current solution. Then the neighbourhood procedure is used to identify a candidate solution in the neighbourhood of the current solution. The hydraulic behaviour of this candidate solution is estimated with the hydraulic simulation model, and its quality is assessed by the objective function (1). The acceptance or rejection of each candidate solution is governed by the Metropolis criterion (if the candidate solution is better than the current solution it is accepted, otherwise it may be accepted with a certain probability). The algorithm implemented comprises the steps and the information presented in Table 1.

Tε	al	b	e	1.	Simu	lated	Anne	aling a	algorit	hm	imp	lementati	on.
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Part of the algorithm	Implementation			
Initial solution (x0)	Total leakage flow is split and assigned to selected pipes			
Initial temperature (Tqt initial)	$Tqt\_initial = -0.1 \times F(x0)/Log(0.5)$			
	If Pa > 80% Tqti+1 = $0.60 \times Tqti$	IAi+1 = 40		
	If Pa > 50% Tqti+1 = $0.75 \times Tqti$	IAi+1 =60		
Cooling schedule	If Pa $> 20\%$ Tqti+1 = 0.90 $\times$ Tqti	IAi+1 =80		
	If Pa $\leq$ 20% Tqti+1 = 0.95 $\times$ Tqti	IAi+1 =100		
Number of evaluations at each temperature Tqti	$IAi \times Number of pipes in the network$			
Stopping criteria	Pa < 5% and 2 temperatures without solution improvement			

Step 1: Choose the initial solution (x0), the initial temperature (Tqt\_initial) and fix the number of solutions to be evaluated at each temperature (IA multiplied by the number of pipes in the WDN). The initial solution assumes the roles of current solution (*x current*) and best solution found so far (*x best*);

Step 2: Generate a candidate solution accessible from the current solution (x\_candidate) by applying one of the next two processes: (I) Transfer an elementary unit of the leakage flow from one pipe to one of its adjacent pipes (diversification mechanism); (II) Randomly select a leaky pipe and concentrate in it all the leaks from its adjacent pipes (concentration mechanism);

Step 3: Check the hydraulic behaviour of the candidate solution and assess the solution quality with the objective function (1);

Step 4: Apply the Metropolis criterion: if the candidate solution is better than the current solution or fulfils the condition established in equation (3) it is accepted and becomes the next current solution (if this candidate solution is better than the best solution found so far, it takes its place), otherwise the candidate solution is rejected and the current solution is maintained:

random number 
$$< \exp\left(-\frac{F_{(x\_current)} - F_{(x\_candidate)}}{Tqt_i}\right)$$
 (3)

Step 6: Apply the cooling schedule (Table 1) that defines the temperature decrease (Tqti+1) and the number of solutions to be evaluated at the new temperature as a function of the percentage of accepted solutions (Pa) at the previous temperature (Tqti);

Step 7: If the stopping criteria is not met, restart the counter of solutions evaluated at the new temperature and return to Step 2, otherwise stop and the final solution is  $x_{best}$ .

#### 2.5. Methodology for pressure transducers placement in water distribution networks

The selection of the nodes to be monitored is based on graph theory concepts and it is expected to produce a good coverage of the WDN pressure with the set of available pressure transducers (observability). The goal is to place a specified number of pressure transducers on the nodes of a WDN so as to minimize the maximum distance to any of the other nodes. In the present context the WDN is a directed graph and the length of each pipe corresponds to its head loss during the MNF period.

The procedure designed to solve this problem starts by applying the Floyd-Warshall algorithm [23, 24] to find the lengths of the shortest paths between all pairs of nodes in the WDN. Then, with this information, a Simulated Annealing algorithm (similar to the one presented in Section 2.4) is used to find the placement of the pressure transducers in order to minimize the maximum head loss between them and the other nodes.

It is expected that this strategy spreads the pressure transducers along the WDN taking into consideration the observed head losses, resulting in a high pressure sensibility monitoring network.

#### 3. Computer application

The methodology described below was implemented as a computer application with a user interface to facilitate the communication between the user and the different modules. It assumes that there is a calibrated model of the WDN, actual total leakage flow can be estimated by comparing the actual MNF with historical data and the minimum night consumption can be estimated. The hydraulic simulation model has a stop criterion of one millimetre in pressure and one millilitre per second in flow. Taking into account the hydraulic simulation model and the pressure transducers accuracies, the pressure measurements were truncated at the millimetre.

The procedure starts by dividing the total leakage flow in a user defined number of flow units and the result is the elementary unit of leakage flow ( $\Delta q$ ).

The construction of the initial solution (x0) starts by assigning the water consumption to the junction nodes. Then, sequentially, each  $\Delta ql$  is assigned to a pipe using the following procedure: assign one  $\Delta ql$  to one of the pipes in the network (a half of each elementary unit of leakage flow is assigned to its end nodes), use the hydraulic simulation model to simulate the hydraulic behaviour of the WDN (considering both water consumption and leakage— $\Delta ql$  already assigned), assess the objective function (1), repeat the simulation for each of the pipes in the network and finally assign this  $\Delta ql$  to the pipe that obtained the lowest value of the objective function (1). Repeat the process until all  $\Delta ql$  are assigned. At this stage the total leakage flow is completely assigned and this is the initial solution to start solving the optimization model. The objective function value of the initial solution (F(x0)) is then used to calculate the initial temperature (Tqt initial) to start the Simulated Annealing algorithm.

Due to the stochastic nature of the Simulated Annealing algorithm, the robustness of the search process was tested by solving the optimization problem with twenty different sets of random numbers (different seeds). The search procedure evolves by generating different combinations of leakage flows assigned to pipes (candidate solutions) looking for the one that best fits the pressure measurements. Each set of random numbers produces one solution (if the search procedure is robust these solutions should be equal or, at least, similar) and at the end the methodology builds the final solution by combining the twenty solutions, identifying for each pipe the number of times that it was part of a solution. Pipes figuring more times in the twenty solutions are potential leaky pipes—greater probability of having leaks, while pipes that were not identified have low probability of being leaky pipes.

# 4. Case Study

The performance of the proposed methodology is illustrated with several case studies created specifically for this purpose. All the case studies use the same WDN (Fig. 2 - one tank, 100 junction nodes, 129 pipes and 29 loops). The cases considered one single leak or two simultaneous leaks, 20 different possible locations randomly chosen for the leak occurrence (identified in Fig. 2), with four leakage flow scenarios 0.25, 0.50, 1.0 and 1.5 L/s (corresponding to 0.9, 1.8, 3.6 and 5.4 m<sup>3</sup>/h). The pressure data is monitored at the set of monitoring nodes presented in Table 2 and identified in Fig. 2. The WDN length is longer than one hundred and eight kilometres (108.188 km) and during the MNF period the water consumption equals 4.630 L/s.

The use of the same WDN is intended to assess the methodology performance under the influence of different leakage flows, located in different pipes.

## 4.1 Results of the pressure transducers placement methodology

The pressure transducers placement methodology was applied to the WDN of the case study, considering 10 monitoring nodes, and the results are presented in Table 2 and illustrated in Fig. 2.



Table 2. Results of the pressure transducers placement methodology.

Fig. 2. Network layout, leaky pipes and pressure monitoring nodes.

# 4.2 Results of the methodology to identify the leaky pipes

As previously mentioned, the methodology starts by calculating twenty solutions, obtained with different sets of random numbers, and builds the final solution afterwards in which each pipe is represented by the number of times it was identified as a probable leaky pipe in those solutions. To facilitate the analysis, results are presented in the format of correct identification of the leaky pipe, identification of a pipe adjacent to the correct leaky pipe (one end node is shared) and pipes adjacent to the previous ones (two pipes away from the leaky pipe).

The results from the proposed methodology are presented in Table 3. The crossing of the leaky pipe (horizontally) with the leak flow (vertically) leads to the number of solutions with the correct identification of the

pipe with the leak (number of correct solutions with twenty different random seeds). Solutions identifying a pipe adjacent to the real leaky pipe are marked with \* and two pipes away from the real leaky pipe are marked with \*\*.

Independently of the leak flow, in pipe 127 the final solution identified an adjacent pipe as the real leaky pipe. In pipe 99 with 0.5 L/s of leakage flow, the final solution identified an adjacent pipe (pipe 67) as the real leaky pipe, but with 0.25 L/s of leakage flow the final solution identified as leaky pipe a pipe adjacent to pipe 67 (marked with \*\* in Table 3). The 0.25 L/s leak in pipe 92 was also identified in a pipe adjacent to both adjacent pipe 92.

Leaky	Leak flow (L/s)				
pipe ID	0.25	0.5	1.0	1.5	
1	20	20	20	20	
5	20	20	20	20	
8	20	20	20	20	
12	20	20	20	20	
15	20	20	20	20	
17	20	20	20	20	
21	20	20	20	20	
23	20	20	20	20	
26	20	20	20	20	
55	20	20	20	20	
57	20	20	20	20	
63	20	20	20	20	
79	20	20	20	20	
90	20	20	20	20	
92	**	20	20	20	
99	**	*	20	20	
115	20	20	20	20	
117	20	20	20	20	
125	20	20	20	20	
127	*	*	*	*	

Table 3. Number of solutions that only identified the correct leaky pipe (single leak).

\* pipe adjacent to the real leaky pipe; \*\* one pipe away from the real leaky pipe

# 4.3 Results of the methodology to identify a pair of leaky pipes

To check the performance of the methodology, eleven pairs of the previous leaky pipes were investigated. The total leakage flow is 2.0 L/s and each pipe has a leak of 1.0 L/s. It was assumed an elementary unit of leakage flow equal to 0.50 L/s. For each situation twenty different sets of random numbers were evaluated.

The pairs 5–15, 8–92, 55–57 and 115–127 are pipes very far away from each other. On the contrary, the pairs 63–117, 79–90 and 117–125 are adjacent pipes. The remaining pairs are leaky pipes with 1 pipe between them.

Final results can be observed in Table 4, where the pairs of leaky pipes are always identified except in one situation (pair 79–90). When non-leaky pipes are identified in the final solutions, they are always adjacent to the real leaky pipes or are one pipe away from them.

1 <sup>st</sup> Leaky	1 <sup>st</sup> Leaky 2 <sup>nd</sup> Leaky		Number of solutions identified					
pipe ID	pipe ID	1 <sup>st</sup> Leaky pipe	2 <sup>nd</sup> Leaky pipe	Adjacent pipes*	One pipe away*			
5	15	20	15	6 (2 pipes)	4 (2 pipes)			
8	57	20	20	0	0			
8	92	20	20	0	0			
17	23	20	20	0	0			
17	115	18	18	14 (2 pipes)	0			
55	57	20	20	0	0			
63	117	20	20	0	0			
63	125	18	18	6 (3 pipes)	0			
79	90	0	20	40 (2 pipes)	0			
115	127	20	17	9 (2 pipes)	0			
117	125	20	20	0	0			

Table 4. Number of solutions identifying the leaky pipes, adjacent pipes and one pipe away (two simultaneous leaks).

\* parenthesis represent the number of non-leaky pipes mentioned in the final solutions

#### 4.4 Discussion of results

The methodology presented here to locate leaks demonstrated its ability to identify one leaky pipe or a pair of leaky pipes in a WDN. It showed its ability to deal with different leakage flows and spatial distributions.

Results confirm that the methodology is effective and robust. In fact, results have very little dispersion and the correct leaky pipes were identified in most of the case studies (with one leak or two simultaneous leaks).

Despite the generally good results, it becomes evident that pipes with greater leakage flows were better identified than the others. The search procedure tends to get focused on the most problematic pipes (bigger leakage flows) and in those cases the stronger signature of the leak makes easier the identification. This observation highlights the importance of the leak flow in the searching process.

The attempt to obtain a good coverage of the WDN led to locate the pressure transducers in the central part of the network. Even a reduced number of monitoring nodes was enough to identify the majority of the probable leaky pipes avoiding the pipes without leakage problems. Results also showed that leaky pipes further away from the monitoring nodes were more difficult to identify and this can be explained by the almost null impact of the leak on the monitoring nodes pressure.

It is remarkable that the methodology with the minimum leakage flow studied (0.25 L/s) was able to identify correctly 85% of real leaky pipes; 90% of real leaky pipes were identified with the leakage flow of 0.50 L/s and, with higher leakage flows, 95% of real leaky pipe were correctly identified.

It is also noticeable that most of the times the methodology was able to narrow the inspection area to the real leaky pipes and in only a few situations it spread to the adjacent pipes or to one pipe away from the leaky pipes, in the presence of a single or two simultaneous leaks.

The introduction of more monitoring nodes certainly would contribute to increase the quality of the solutions, but on the other hand it would also increase the cost of acquiring the equipment and gathering data. An alternative is to maintain the number of monitoring nodes and improve their effectiveness by artificially increase the head loss in the WDN [8], but this is a subject to be explored in a future work.

#### 5. Conclusions

Managing water losses is an important task for any water utility. This paper presented a methodology that uses a hydraulic model of the WDN to quickly and economically identify probable leaky pipes. This methodology is based on pressure measurements and explores the exchange of information between an optimization model and the hydraulic simulation of the WDN in steady state conditions. The pressure transducer placement is carried out by a novel tool based on the graph theory concepts, specifically developed to select the monitoring nodes, demonstrating the ability to gather the necessary information for the leak location methodology to work. The optimization model used to identify the leaky pipes is solved by a simulated annealing algorithm, a reliable, fast, and easy to implement method.

The results obtained with a single or with two simultaneous leaks at different locations are very encouraging. The implementation of the methodology does not affect the regular activity of the WDN and it is able to help active leakage control activities. Results showed that, in general, the methodology identifies a small number of pipes, confining the on-field works to a considerably reduced extension of the WDN.

Future works include the analysis of the WDN in quasi-steady state conditions and apply it to a real WDN to confirm its effectiveness.

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