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On recognizing Cartesian graph bundles

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Abstract

Graph bundles generalize the notion of covering graphs and graph products. In Imrich et al. (Discrete Math. 167/168 (1988) 393–403.) an algorithm that finds a presentation as a nontrivial Cartesian graph bundle for all graphs that are Cartesian graph bundles over triangle-free simple base was given. In this paper we extend this algorithm to recognize Cartesian graph bundles over a $K_4\backslash e$-free simple base, without induced $K_{3,3}$. Finally, we conjecture the existence of algorithm for recognition of Cartesian graph bundle over a $K_4\backslash e$-free simple base.

MSC: 05C60

Keywords: Graph bundles; Cartesian graph bundles

1. Introduction

Knowledge of the structure of a graph often leads to faster algorithms for solving combinatorial problems on these graphs. In general, an efficient algorithm for recognizing a special class of graphs may allow us to compute certain graph invariant faster. For example, the chromatic number of a Cartesian product is the maximum of the chromatic numbers of the factors. Computing the chromatic number is in general an NP-hard problem, but factoring can be done in polynomial time. Hence, if the graph is a Cartesian product, we can save computation time by first factorizing and then computing the chromatic number of the factors. Here we shall be concerned with the structure of Cartesian graph bundles over a $K_4\backslash e$-free simple base.

In topology, bundles are objects which generalize both covering spaces and Cartesian products [4]. Analogously, graph bundles generalize the notion of covering graphs

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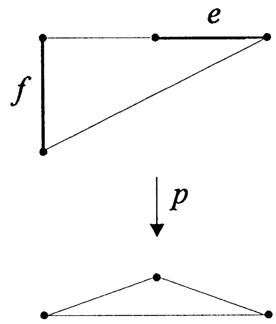

Fig. 1. Degenerate and nondegenerate edge in relation δ^*.

and graph products. Graph bundles can be defined with respect to arbitrary graph products [14]. (For a classification of all possible associative graph products, see [5].) Various problems on graph bundles were studied recently, including edge coloring [13], maximum genus [12], isomorphism classes [10], characteristic polynomials [11,16] and chromatic numbers [8,9].

It is well-known that finite connected graphs enjoy unique factorization under the Cartesian multiplication [15] and recently a number of polynomial algorithms for recognizing Cartesian product graphs have been published [3,17,2]. On the other hand, a graph may have more than one presentation as a graph bundle. Natural questions therefore are to find all possible presentations of a graph as a graph bundle or to decide whether a graph has at least one presentation as a nontrivial graph bundle. As recognizing covering graphs is NP-hard [1], we will restrict our attention to cases where fibres are connected (see also survey [18]).

In [6] an algorithm that finds a presentation as a nontrivial Cartesian graph bundle for all graphs that are Cartesian graph bundles over triangle-free simple base was given. The main result of [6] follows from properties of the 'local Cartesian product relation' δ^* defined among edges of a graph. Not surprisingly, this relation was, sometimes implicitly or under different names, used in work related to recognition and uniqueness of factorization of Cartesian product graphs [15,3,7]. An induced cycle of four vertices is called a *chordless square*. Relation δ^* is defined to be the reflexive and transitive closure of a relation which is defined as follows: $e\delta f$ if edges e and f are incident and span no chordless square and $e\delta f$ if e and f are opposite edges of a chordless square. Unfortunately, δ^* may fail to separate degenerate and nondegenerate edges in some cases. For example, the graph $K_{3,3}$ is a Cartesian graph bundle, in which all edges are in the same δ^* equivalence class. The reason for this is intuitively clear from Fig. 1 (see also [6]).

In this paper we modify the definition of δ^* slightly to obtain a relation δ_b^* which can be used for recognition of a larger class of Cartesian graph bundles. After recalling some definitions and facts in Section 2, we state some properties of Cartesian graph bundles with induced $K_{3,3}\backslash e$ or $K_{3,3}$ in Section 3. In Section 4 we prove the main result of this note:

Theorem 1.1. *There is a polynomial algorithm for recognition of Cartesian graph bundles over a* $K_4\backslash e$*-free simple base, without induced* $K_{3,3}$.

We believe that a similar approach would also work for general case, but the details seem to be much more involved, therefore we only discuss the general case in the concluding section.

2. Preliminaries

In this section we begin with definitions and well-known or easily proved facts. We will consider only connected simple graphs, i.e. graphs without loops and multiple edges.

We say that two edges are *adjacent* if they have a common vertex. Furthermore, $G \cong H$ denotes graph isomorphism, i.e. the existence of a bijection $b : V(G) \to V(H)$ such that vertices v_1, v_2 are connected in G exactly if $b(v_1), b(v_2)$ are connected in H. Vertex $x \in V(G)$ is an *universal vertex* if it is connected with every vertex in $V(G)\backslash\{x\}$.

The *Cartesian product* $G\square H$ of graphs G and H has as vertices the pairs (v, w) where $v \in V(G)$ and $w \in V(H)$. Vertices (v_1, w_1) and (v_2, w_2) are connected if $\{v_1, v_2\}$ is an edge of G and $w_1 = w_2$ or if $v_1 = v_2$ and $\{w_1, w_2\}$ is an edge of H.

Let B and F be graphs. A graph G is a (*Cartesian*) graph bundle with fibre F over the base graph B if there is a mapping $p : G \to B$ which satisfies the following conditions:

(1) It maps adjacent vertices of G to adjacent or identical vertices in B.
(2) The edges are mapped to edges or collapsed to a vertex.
(3) For each vertex $v \in V(B)$, $p^{-1}(v) \cong F$, and for each edge $e \in E(B)$, $p^{-1}(e) \cong K_2\square F$.

In this paper we will consider only Cartesian graph bundles over simple bases. A mapping satisfying just the first two conditions above is called a *graph map*. For a given graph G there may be several mappings $p_l : G \to B_l$ with the above properties. We say an edge is *degenerate* if $p(e)$ is a vertex. Otherwise we call it *nondegenerate*. A factorization of a graph G is a collection of spanning subgraphs H_i of G such that edge set of G is partitioned into the edge sets of the graphs H_i. In other words, the set $E(G)$ can be written as a disjoint union of the sets $E(H_i)$. The projection p induces a factorization of G into the graph consisting of isomorphic copies of the fibre F and the graph $\tilde{G}$ consisting of all nondegenerate edges. This factorization is called the *fundamental factorization*. It can be shown that the restriction of p to $\tilde{G}$ is a covering projection of graphs; see, for instance, [13] for details.

Common examples of Cartesian products are squares, hypercubes, prisms (Cartesian products of n-gons by an edge) or the square lattice as the product of two infinite paths. Intuitively, graph bundles can be seen as 'twisted products'. A small example

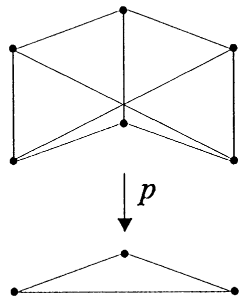

Fig. 2. A small example of a Cartesian graph bundle, $K_{3,3}$.

of a Cartesian graph bundle is the graph $K_{3,3}$ in Fig. 2. It is a discrete analog of the well-known Möbious band, which is a topological bundle (base is a circle, fibres are lines).

In [6] the equivalence relation δ^* defined on the edge-set of a graph is used for recognizing Cartesian graph bundles over a triangle-free simple base. In this paper we introduce its modification which differs on induced subgraphs isomorphic $K_{3,3}\backslash e$ and use it for recognizing Cartesian graph bundles over $K_4\backslash e$-free simple base. The new equivalence relation δ_b^* helps us to avoid joining degenerate and nondegenerate edges in the same equivalence class.

In next section we will prove some properties of induced $K_{3,3}$ or $K_{3,3}\backslash e$ which we will use in definition of relation δ_b^*.

3. Induced $K_{3,3}$ or $K_{3,3}\backslash e$

Let G denote Cartesian graph bundle with fibre F over a simple base graph B and K any induced subgraph in G isomorphic to $K_{3,3}$ or $K_{3,3}\backslash e$. Let (K_1, K_2) be the partition of $V(K)$.

Lemma 3.1. *Any two vertices from the same partition subset of $V(K)$ are endpoints of at least two edge-disjoint P_3 induced in K.*

Proof. Clear. □

Lemma 3.2. *If any two vertices u and v from the same partition subset of $V(K)$ lie in the same copy of fibre F, K intersects only one copy of fibre F.*

Proof. Let $u, v \in K_1$ lie in F_1 (a copy of fibre F). Since u and v are in the same partition subset, it is easy to see that they cannot be both endpoints of the missing edge e.

If neither u nor v is an endpoint of the missing edge e, they are both connected with each vertex in K_2. Since every $x \in K_2$ have two neighbors in F_1, the vertex set K_2 lies entire in F_1, because of 2-convexity of fibres. By the same reasoning we see that the third vertex in K_1 lies in F_1.

In the other case, when one of the two vertices, say u, is an endpoint of e, two common neighbors $x, y \in K_2$ of vertices u and v lie in F_1. Vertices x and y are both connected with third vertex $w \in K_1$ which therefore lies in F_1. Since v and w are both neighbors of each vertex in K_2, every vertex from K_2 also lies in F_1. □

Corollary 3.3. *An induced K in G either intersects each copy of fibre F in at most two vertices or lies entire in one copy of fibre F.*

Proof. If at least three vertices intersect one copy of fibre F, at least two vertices lie in the same partition subset of $V(K)$. Therefore by Lemma 3.2 K lies entire in one copy of fibre F. □

Corollary 3.4. *Induced K in G cannot intersect exactly two copies of fibre.*

Proposition 3.5. *If K intersects four or five copies of fibre F, then at least one square with chord lies in B.*

Proof. If K intersects exactly five copies of fibre F, from Lemma 3.2 it follows that two vertices u, v from different partition subsets of $V(K)$ lie in the same copy of fibre F. Since every vertex in $V(K)$ is connected with u or v, $p(u) = p(v)$ is an universal vertex in projection of K on base graph B. Projection of $K\backslash\{u, v\}$ is $K_{2,2}$ or $K_{2,2}\backslash e$ (see Fig. 3) on which universal vertex causes at least one square with chord.

If K intersects exactly four copies of fibre F, from Lemma 3.2 and Corollary 3.3 it follows that two pairs $\{u_1, v_1\}, \{u_2, v_2\}$ of vertices from different partition subsets of $V(K)$ lie in the same copy of fibre F. For the same reason as before $p(u_1) = p(v_1)$ and $p(u_2) = p(v_2)$ are universal vertices of $p(K)$ in B. Two universal vertices in graph on four vertices ($p(K)$) induce a square with chord (see Fig. 3). □

Corollary 3.6. *If G is Cartesian graph bundle over a $K_4\backslash e$-free simple base graph, induced K with at least one degenerate and one nondegenerate edge intersects three copies of fibre F in exactly two vertices.*

From Corollaries 3.4 and 3.6 we see that induced K in G over a $K_4\backslash e$-free simple base graph can lie entire in one copy of fibre F or intersect six copies each in one vertex or it can intersect three copies of fibre F, each in two vertices. We call those types of induced $K_{3,3}$ or $K_{3,3}\backslash e$ *fibre-K*, *base-K* and *bundle-K*, respectively.

Let in the rest of the paper G denote a graph bundle over a $K_4\backslash e$-free simple base graph.

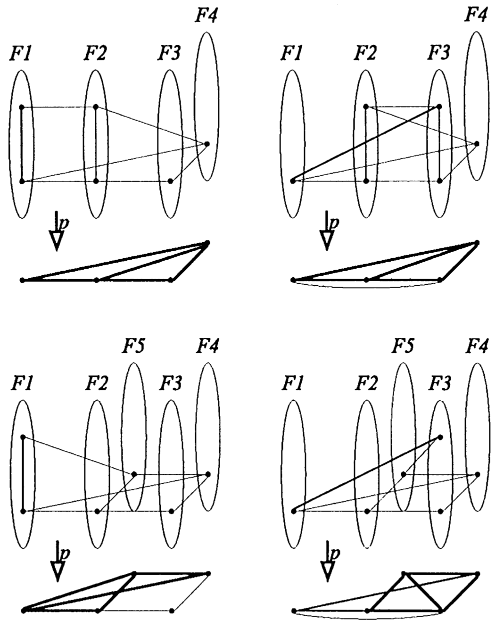

Fig. 3. The projections of K intersecting four or five copies of F.

4. Case $K_{3,3}\backslash e$

Lemma 4.1. *Let K and K' be induced subgraphs of G isomorphic to $K_{3,3}\backslash e$. If K and K' intersect in P_3, they are both of the same type.*

Proof. A fibre-K can intersect only another fibre-K, since from Corollary 3.3 any other K' has at most two vertices in the same fibre. Hence if one of K and K' is a fibre-K, then both must be fibre-K's.

It remains to prove that a base-K intersecting a bundle-K' implies existence of a square with chord in B. Assume K is a base-K, K' is a bundle-K and their intersection is a P_3. Since vertices on intersection K with K' lie in different copies of fibre F, their projections on base graph B induce K_3, therefore two vertices from different partition subset of vertex set of graph $p(K) \cong K_{3,3}\backslash e$ are connected in B. By Lemma 3.1 the new edge meets two triangles in B, hence there is an induced square with chord in B. Since we assumed G has $K_4\backslash e$-free base, this completes the proof. □

Lemma 4.2. *Let K and K' be induced bundle-K isomorphic to $K_{3,3}\backslash e$. If K and K' intersect in graph $P \cong P_3$, then they intersect in all three copies of fibre F in only nondegenerate edges.*

Proof. Since K and K' intersect in three vertices, they intersect in at least two copies of fibre F. Since $p(K)$ and $p(K')$ represent two triangles in B with common edge they only can intersect the same copies of fibre F because we assume B is $K_4\backslash e$-free.

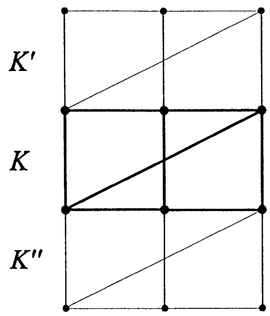

Fig. 4. Bundle-K isomorphic to $K_{3,3}\backslash e$.

Now we will prove, that K and K' intersect only in nondegenerate edges. Let us assume that one edge in P is degenerate. Therefore P intersects only two copies of fibre F. Every vertex on P has a neighbor in the third copy of the fibre. Furthermore, any two vertices of K which are in the same fibre are always in different partition sets of K. These two vertices are connected with at least three of four other vertices of K. Consequently, at least two of three edges connecting P with the third copy of fibre must be included in any bundle-K isomorphic to $K_{3,3}\backslash e$ which intersects P. Therefore at least one edge is part of both graphs K and K'. This contradicts the fact that K and K' intersect in P. $\square$

Theorem 4.3. *For any induced bundle-K, isomorphic to* $K_{3,3}\backslash e$, *there exist two induced bundle-K,* K' *and* K'', *isomorphic to* $K_{3,3}\backslash e$, *which intersect K in two disjoint graphs isomorphic to* P_3 (*see Fig. 4*).

Proof. It is easy to see that vertices of K from the same copy of fibre F are connected and that each vertex of K is connected with exactly one vertex from any copy of F intersecting K. Let us denote copies of fibre F intersecting K with F_1, F_2, F_3 and their vertices with $\{a_i, b_i\} \in V(F_i) \cap V(K)$. Let $a_i \in F_i$ and $b_j \in F_j$, $i \neq j$, $i, j \in \{1,2,3\}$ be the endpoints of missing edge e in K. They have no neighbors from F_j and F_i, respectively, in K. Therefore, there exists a vertex $c_j \in F_j$, connected with a_i and a_j and a vertex $d_i \in F_i$, connected with b_i and b_j.

The fact that $p^{-1}(c) \cong K_2 \Box F$ for any $e \in E(B)$ completes the proof. $\square$

From Lemma 4.2 and Theorem 4.3 we see that degenerated edges of K are the only three disjoint edges (perfect matching) between two paths P_3 as defined in statement of Theorem 4.3. With these terms we can now define an auxiliary binary relation δ_b. For any $e, f \in E(G)$ we set $e \delta_b f$ if at least one of the following conditions is satisfied:

1. e and f are incident and there is no chordless square spanned on e and f.
2. e and f are the opposite edges of a chordless square which is not subgraph of any induced $K_{3,3}\backslash e'$.

3. e and f are the edges of an induced subgraph K isomorphic to $K_{3,3}\backslash e'$ and
   (a) there exist two induced subgraphs isomorphic to $K_{3,3}\backslash e'$ which intersect K in two disjoint paths P' and P'' isomorphic to P_3 and
      (i) e and f are both edges of the perfect matching between P' and P'' or
      (ii) neither e nor f is edge of the perfect matching between P' and P''.
   (b) there are no subgraphs isomorphic to $K_{3,3}\backslash e'$ which intersect K in two disjoint paths of length 2.

By δ_b^* we denote the reflexive and transitive closure of δ_b. Since δ_b is symmetric, δ_b^* is an equivalence relation.

It may be interesting to note that any pair of adjacent edges which belong to distinct δ_b^*-equivalence classes span one or more chordless squares, therefore δ_b^* does not have the *square property* as defined in [7,6]. From definition also follows $\delta_b^* \subseteq \delta^*$.

Remark. Note that using Theorem 4.3 gives us not only the partition of edges of an induced $K_{3,3}\backslash e$ into two classes, but we also know which of the two classes will be degenerate and which will be nondegenerate in the fundamental factorization, provided this K is a bundle-K. (In the definition this information is not used.)

The relation δ^* joins degenerate and nondegenerate edge in the same equivalence class only on induced subgraphs isomorphic to $K_{3,3}\backslash e$ (note that G is a Cartesian graph bundle over a $K_4\backslash e$-free simple base, without induced $K_{3,3}$). Therefore the relation δ_b^* separates degenerate and nondegenerate edges of G.

5. The algorithm

In this section we briefly show, how analogous algorithm as in [6] can be used for recognition of Cartesian graph bundles over $K_4\backslash e$-free simple base.

Let R be any equivalence relation on edge set of G. The 2-convex R-closure $\mathscr{C}_2(\varphi, R)$ of a set of edges φ relative to equivalence relation R is the subset ρ of the edge set $E(G)$, such that ρ is the minimal union of equivalence classes of R, that satisfies the following two conditions: (1) $\varphi \subseteq \rho$ and (2) ρ is 2-convex in G. It is known [6] that $\mathscr{C}_2(\varphi, R)$ can be computed in polynomial time.

Lemma 5.1. *Let G be a Cartesian graph bundles over a $K_4\backslash e$-free simple base, without induced $K_{3,3}$ and let φ be any equivalence class of δ_b^* containing only degenerate edges. If $\rho := \mathscr{C}(\varphi, \delta_b^*) \neq E(G)$, then G is a Cartesian graph bundle with fibres being the connected components of G_ρ.*

Proof. Since base graph B is $K_4\backslash e$-free, any two triangles cannot share an edge. Let D denote a minimal set of edges in B, such that $B\backslash D$ is triangle-free. Graph $G_\triangle := G\backslash p^{-1}(D) \subseteq G$ is a Cartesian graph bundle over triangle-free simple base. From the definition it follows that $\delta_b^* = \delta^*$ on $G_\triangle$. Because B is $K_4\backslash e$-free, any two incident nondegenerate edges, which are projected into a triangle in B, cannot span a chordless square in $G_\triangle$. Therefore they are δ_b^*-equivalent in $G_\triangle$. Recall that by Lemma 6 of [6], we have: if $\rho := \mathscr{C}(\varphi, \delta^*) \neq E(G_\triangle)$ then $G_\triangle$ is a Cartesian graph bundle with fibres being the connected components of $G_{\triangle_\rho}$.

Finally, note that the edges of $p^{-1}(D)$ are nondegenerate and observe, using the definition of δ_b^*, that they are δ_b^*-equivalent to nondegenerate edges of $G_\triangle$. □

Lemma 5.2. *Let G be a Cartesian graph bundle with fibre F. Let γ be any equivalence class of δ_b^*. If a connected component of the graph determined by γ is contained in a fibre, then also the connected component of the* 2*-convex closure $\mathscr{C}_2(\gamma, \delta_b^*)$ is contained in a fibre. In particular, the graph determined by the* 2*-convex closure of γ has at least two connected components.*

Lemma 5.2 can be proved along the same lines as the Lemma 7 of [6] using Lemma 5.1.

If there is a graph B with no $K_4 \backslash e$, such that (G, p, B) is a Cartesian graph bundle for some p, we can now give a polynomial algorithm which finds at least one representation of G as a bundle. In fact, by computing the closure of all δ_b^* equivalence classes, we can find all minimal representations of G as a Cartesian graph bundle.

Algorithm *CGB*:
Input: G: graph;
Output: C set of degenerate edges of some bundle presentation.

1. compute δ_b^*
2. **for all** equivalence classes φ of δ_b^* **do**
    2.1 **if** $C := \mathscr{C}_2(\varphi, \delta_b^*) \neq E(G)$ **then return**(C)
3. **return**('G is not a Cartesian graph bundle over $K_4 \backslash e$-free base'.)

For any representation with $K_4 \backslash e$-free base, the equivalence classes of the relation δ_b^* contain either only degenerate or only nondegenerate edges. Let φ be an equivalence class of δ_b^* with degenerate edges. Each connected component must be contained in one fibre and by Lemmas 5.1 and 5.2 the closure $\mathscr{C}_2(\varphi, \delta_b^*)$ is the set of degenerate edges for a representation of G a Cartesian graph bundle. This proves the correctness of the algorithm CGB.

Note that the algorithm CGB here and algorithm B of [6] differ only in step 1, where relation δ^* is replaced by δ_b^*.

Now we will prove that δ_b^* can be also computed in polynomial time. In arbitrary graph with n vertices there exist at most as many induced subgraphs isomorphic to $K_{3,3} \backslash e$ and $K_{3,3}$ as in complete bipartite graph $K_{\lceil n/2 \rceil, \lfloor n/2 \rfloor}$. It is easy to see that this number is polynomialy bounded in n, more precisely, it is less than $N := n^6/2^8 3^2$, i.e. $\mathcal{O}(n^6)$. In definition of δ_b^* we consider each induced subgraph isomorphic to $K_{3,3} \backslash e$ at most $N-1$ times (all possible intersections of induced subgraph isomorphic to $K_{3,3} \backslash e$). Therefore the complexity of computation the relation δ_b^* is at most $\mathcal{O}(n^{12})$.

Using the fact that the algorithm B in [6] is polynomial, it follows that complexity of modified algorithm CGB is also bounded by polynomial in n, the number of vertices of G. Hence Theorem 1.1.

Theorem 1.1. *There is a polynomial algorithm for recognition of Cartesian graph bundles over a* $K_4\backslash e$*-free simple base, without induced* $K_{3,3}$.

6. Conclusion

We conclude with a discussion of a more general case, recognition of Cartesian graph bundles over $K_4\backslash e$-free base. Unfortunately, Theorem 4.3 cannot be generalized to bundle-K isomorphic to $K_{3,3}$. However, we believe that enough information can be obtained by using some properties of the neighborhoods of induced $K_{3,3}$. Therefore we state

Conjecture 6.1. There is a polynomial algorithm for recognition of Cartesian graph bundles over a $K_4\backslash e$-free simple base.

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