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Procedia Computer Science 54 (2015) 849 – 853

Procedia Computer Science

Eleventh International Multi-Conference on Information Processing-2015 (IMCIP-2015)

Noise Reduction using Wavelet Transform and Singular Vector Decomposition

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Abstract

Signals do not exist without noise. It must be removed from the data for further data analysis. The search for efficient noise reduction methods is still going on. Wavelet Transform (WT) is a powerful tool for removal of noise from various signals. Combining WT with other noise reducing techniques may result in further reduction of noise. Similar to WT, Singular Vector Decomposition (SVD) is also an effective noise reduction tool. In this experiment, wavelet transform is used along with Singular Vector Decomposition (SVD) for noise reduction. The results of noise reduction using Wavelet transform and combination of SVD with WT are compared. Error between coefficients of WT of original signal and coefficients of noise added signal SVD along with WT is found to be less as compare to WT operated alone on noisy signal.

Peer-review under responsibility of organizing committee of the Eleventh International Multi-Conference on Information Processing-2015 (IMCIP-2015)

Keywords: Wavelet Transform (WT); Fourier Transform (FT); Noise reduction; Singular Vector Decomposition (SVD).

1. Introduction

Signals never exist without noise. Removal of noisy is necessary from the data in order to proceed with further data analysis. In literature, many methods are mentioned for removal of noise. It is generally classified into two categories: denoising in the original signal domain (e.g., time or space) and denoising in the transform domain (e.g., Fourier or WT). For stationary signals (no change in frequency over a period of time), traditional Fourier Transform (FT) is suitable. But, many times the information can be seen in the frequency domain which is not easily seen in the time-domain.[1] For non-stationary signals, to get the time-frequency characteristics of the signals is very important. For such signals, WT is particularly suitable. For signal and image processing the Wavelet Transform (WT) is a powerful tool. In various research fields like signal processing, image compression, pattern recognition etc., WT is quite useful. Continuous Wavelet Transform (CWT) gives reliable and detailed time-scale information as compare to classical short time Fourier transform (STFT). So, researchers are using (CWT) instead o STFT.[2] Singular Vector decomposition is also used for noise reduction.[3] Combining WT with other noise reduction techniques or system can further reduce noise in the signal. Motivation for this experiment to have better signal to noise ratio which may find applications many fields like bio-medical signals, image processing etc.

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Peer-review under responsibility of organizing committee of the Eleventh International Multi-Conference on Information Processing-2015 (IMCIP-2015)
doi:10.1016/j.procs.2015.06.099

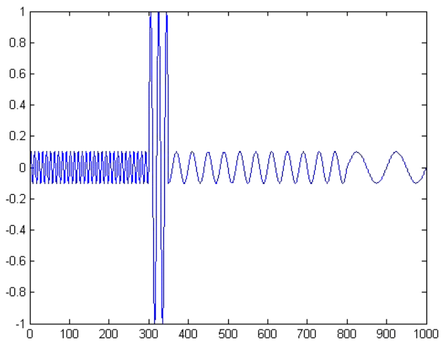

Fig. 1. Signal of 50 Hz along with other frequencies 100 Hz, 25 Hz and 10 Hz.

2. Why Wavelet Transform?

2.1 Fourier transform (FT)

FT and WT are reversible transforms. So, one can get back required information from processed (transformed) signals. But, one cannot get frequency information from the time-domain signal, and while no time information is available in the Fourier transformed signal. FT gives information about each frequency that exists in the signal, but it does not tell us timing of various frequency components. For stationary signal, this information is not required.

Mathematically, FT equation is[1,6]:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \quad (1)$$

and

$$x(t) = \int_{\infty}^{\infty} X(f) e^{j2\pi f t} df \quad (2)$$

t - Time,
f - Frequency,
x - Signal in time domain.
X - Signal in frequency domain.

The signal $x(t)$, is multiplied with an exponential term, at some certain frequency "f" , and then integrated from $-\infty$ to $+\infty$. If the signal has a same component of frequency "f", then that component term will coincide, and the product of them will give a (relatively) large value. But, if frequencies do not match then product will give zero value. So, Fourier transform is not suitable for time varying frequency, i.e. for the signal is non-stationary.

As shown in Fig. 1 original signal contain 10, 25, 50 and 100 Hz. FT of signal is given in Fig. 2 which gives information about frequency components in signal. But, it is not giving any information about timing of different frequency components. So, FT fails to detect timing of different frequency components i.e. it is not suitable for non-stationary signals.

2.2 Short time fourier transform (STFT)

The limitation of FT to detect time location of desired frequency signal is prevailing over by Short Time Fourier Transform (STFT). There is a small difference between STFT and FT. In STFT, the signal is divided into small segments. Here, these segments (portions) of the signal are understood to be stationary. For this purpose, a fixed window function is chosen. The width of this window must be equal to the segment of the signal. Next step is to shift

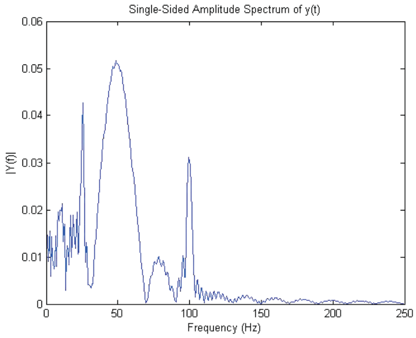

Fig. 2. FT Spectrum of signal Fig. 1.

this window (say '$t1$' seconds) to a new location, multiplying with the signal, and taking the FT of the product. This procedure is repeated again and again; till the end of the signal is reached by shifting the window with "$t1$" seconds intervals.

$$STFT_x^w(t', f) = \int_0^t x(t)w^*(t - t1)e^{-j2\pi ft}dt \tag{3}$$

$x(t)$ - signal
$w(t)$ - window function, and
$*$ - complex conjugate

For every "$t1$" and a new STFT coefficient is computed. But, due to fix window size, narrow windows give good time resolution, but poor frequency resolution. While wide windows result in good frequency resolution, but poor time resolution.[1]

2.3 Wavelet transform

For time localization of the spectral components, a transform which can provide the time-frequency representation of the signal is needed. The Wavelet transform fulfills this criterion. It is capable of providing the time and frequency information simultaneously. There are two main differences between the STFT and the CWT[1]:

a. The FT of the windowed signals are not considered, and so single peak will appear corresponding to a sinusoid, i.e., negative frequencies are not computed.
b. The width of the window is changed as the transform is calculated for every single spectral component. This is the most important feature of the wavelet transform.

The continuous wavelet transform is defined as follows[1,5]:

$$CWT_x^\Psi(\tau, s) = \Psi_s^\Psi(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t)\Psi^*\left(\frac{t - \tau}{s}\right) dt \tag{4}$$

Here, the transformed signal is a function of two variables, τ and s, the translation and scale parameters, respectively.

$\Psi(t)$ is the transforming function, and it is called the mother wavelet. Here, if the signal has a spectral component that corresponds to the current value of s the product of the wavelet with the signal at the location where this spectral component exists gives a relatively large value. If the spectral component that corresponds to the current value of s is not present in the signal, the product value will be relatively small, or zero.

As shown in Figs. 1 and 2 original signal contain 10, 25, 50 and 100 Hz. FT of signal is given in Fig. 2 which gives information about frequency components in signal. But, it is not giving any information about timing of different frequency components. While Fig. 3 WT of signal is showing not only frequency components, but also timing of different frequency components.

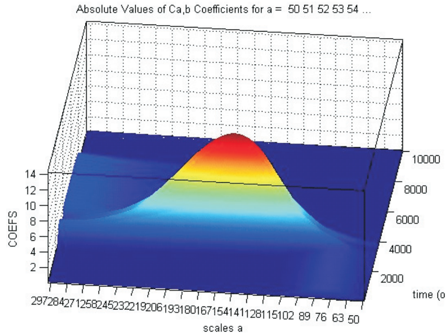

Fig. 3. WT spectrum of signal Fig. 1.

3. Singular Value Decomposition

In Singular value decomposition (SVD), one can identify and order the dimensions along which data points exhibit the most variation. So, it is possible to find the best approximation of the original data points using fewer dimensions. Hence, SVD is seen as a method for data reduction or noise reduction. It is based on a theorem from linear algebra. Here, rectangular matrix A can be broken down into the product of three matrices – an orthogonal matrix U, a diagonal matrix S, and the transpose of an orthogonal matrix V. Mathematically, it is shown as:

$$Amn = UmmSmnV^Tnn \tag{5}$$

where $U^TU = I$, $V^TV = I$; the columns of U are orthonormal eigenvectors of AA^T, the columns of V are orthonormal eigenvectors of A^TA, and S is a diagonal matrix containing the square roots of Eigen values from U or V in descending order.

Here, by deleting elements representing dimensions which do not give meaningful variation, one can effectively remove noise in the representation. Now the vectors are shorter, and contain only the elements that account for the most significant correlations in the original dataset. The deleted elements had the effect of diluting these main correlations by introducing potential similarity along dimensions of questionable significance.[6]

4. Experimental Setup

Programs were written in MATLAB 2011a. A standard signal Fig. 1 was generated using equation given below:

$$x(t) = 0.1 * \sin(2 * pi * 100 * (t1/1000)) + \sin(2 * pi * 50 * (t2/1000)) + 0.1 * \sin(2 * pi * 25 * (t3/1000)) + 0.1 * \sin(2 * pi * 10 * (t4/1000) \tag{6}$$

$t1$, $t2$, $t3$ and $t4$ – Time slots for which respective signal appear.

Then, Gaussian noise having signal to noise ratio of 1 db, 5 db and 10 db was introduced in $x(t)$. Morlet wavelet is used for Wavelet transform. Coefficients of original signal, noisy signal using WT were calculated. Coefficients of noisy signal were further processed using SVD as per equation (5). This process is repeated 100 times and each time difference between coefficients of original signal and processed signal was calculated.

5. Results and Discussion

Difference in WT coefficients of original signal and processed signal were calculated. Coefficients of processed signal mean WT coefficients of noisy signal and SVD of WT coefficients of noisy signal.

$$\text{Difference1} = \text{WT Coefficient of original signal} - \text{WT coefficient of noisy signal} \tag{7}$$

$$\text{Difference2} = \text{WT Coefficient of original signal} - \text{Optimum SVD of WT coefficient of noisy signal} \tag{8}$$

Table 1. Difference between coefficients of original signal and noisy signal after processing. WT vs. WT+SVD on noisy signal for 100 cases.

		Error below 1%		Error below 2%	
Sr. no.	SNR in db	WT	WT+SVD (max.)	WT	WT+SVD (max.)
1	1	1	58	25	87
2	5	0	21	21	93
3	10	13	61	100	100

Results in Table 1 show the comparison between combinations of Wavelet transform (WT) and Singular vector decomposition (SVD) vs. WT. It clearly indicates that combination of WT and SVD increases signal to noise ratio. The results of WT and WT+SVD combination are tabulated for different values of SNR. For a particular percent SVD along with WT, noise in signal becomes minimum i.e. combination of WT and SVD results in higher signal to noise ratio.

6. Conclusion

Above results clearly indicate that the combination of SVD and WT gives 4–5 times better signal to noise ratio than WT alone. Inherent property of WT which reduces noise and property of SVD to separate redundant data together give better result than WT alone. So, combination of SVD and WT will be helpful for reduction of noise from data signal.

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