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The entangled accelerating universe

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ABSTRACT

Using the known result that the nucleation of baby universes in correlated pairs is equivalent to spacetime squeezing, we show in this Letter that there exists a T-duality symmetry between two-dimensional warp drives, which are physically expressible as localized de Sitter little universes, and two-dimensional Tolman–Hawking and Gidding–Strominger baby universes respectively correlated in pairs, so that the creation of warp drives is also equivalent to spacetime squeezing. Perhaps more importantly, it has been also seen that the nucleation of warp drives entails a violation of the Bell's inequalities, and hence the phenomena of quantum entanglement, complementarity and wave function collapse. These results are generalized to the case of any dynamically accelerating universe filled with dark or phantom energy whose creation is also physically equivalent to spacetime squeezing and to the violation of the Bell's inequalities, so that the universe we are living in should be governed by essential sharp quantum theory laws and must be a quantum entangled system.

There exist many kinds of spacetime entities which are denoted by the term universe. The so-called Friedmann–Robertson–Walker universes, the parallel universes, the de Sitter universe or the baby universes Wick rotated from Euclidean wormholes, to quote just a few. In this report we shall show however that all of such universes correspond actually to a unique scale-invariant quantum cosmic scenario that has no classical counter-part. It will be also shown that such a cosmic scenario is equivalent to violating the Bell's inequalities [1] and, therefore, talking about any of the conventional models for quantum cosmology is rather redundant and indeed meaningless because quantum theory and cosmology are actually equivalent descriptions of the same deep physical reality.

Several years ago the idea was advanced that the nucleation of correlated baby universes, taken to be the Lorentzian sector of Euclidean wormholes, is equivalent to squeezing the spacetime [2]. These baby universes can first be represented as Tolman–Hawking closed spaces which are described by the metric

$$ds^2 = a(\eta)^2\left(-d\eta^2 + d\Omega_3^2\right), \tag{1}$$

where $\eta = \int dt/a(t)$ is the conformal time, $d\Omega_3^2$ is the unit metric on the three-sphere, and $a(\eta)$ is the scale factor

$$a_b(\eta) = R_0 \cos\eta, \tag{2}$$

with R_0 the maximum radius of the baby universe and $a_b(t) = \sqrt{R_0^2 - t^2}$.

Now, it has been more recently shown [3] that the two-dimensional metric of a warp drive can be expressed in terms of the conformal time in the manifestly cosmological form

$$a_w(\eta) = \frac{R_0'}{\cos\eta}, \tag{3}$$

where R_0' is the maximum radius of a spatially closed de Sitter like local space. We may be therefore uncovering an $a \to 1/a$ duality symmetry between two-dimensional warp drives and two-dimensional baby universes which, if confirmed to hold, would entitle us to accomplish the conclusion that the creation of warp drives is also equivalent to spacetime squeezing, such as it is currently believed [4]. That this is actually the case can be checked by showing that the purely gravitational part of the Hilbert–Einstein two-dimensional action corresponding to baby universes and warp drives is the same. Generally, for the relevant geometric sector of the two-dimensional Hilbert–Einstein action one may write

$$S = M_p^2 \int dx^2\, a^2 R - 2M_p^2 \int dx\, a\, \mathrm{Tr}\, K, \tag{4}$$

in which M_p is the Planck mass, $R \equiv R(a)$ is the Ricci curvature scalar and K is the second fundamental form on the one-dimensional boundary. Computing this action sector for the scale factors (2) and (3) the result is immediately derived that such an action is in fact the same for both metrics and given by

$$S_b = S_w = 6M_p^2 S_2 \tan\eta, \tag{5}$$

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with S_2 the proper surface of the given baby (or warp) universe. It then follows that, relative to the geometric part of the action, the $a \to 1/a$ duality symmetry holds for two-dimensional baby and warp universes having any relative sizes, and therefore, since most of the physics of these two spacetime constructs is concentrated on two dimensions, all of their observable physical properties are indistinguishable from one another and hence they are both equivalent to squeezing the spacetime. Now, since squeezing is a quantum phenomenon devoid of any classical counterparts [5], it follows that the baby universe spacetime and the warp universe spacetime also are both quantum in nature, such as it must happen with the scale-factor duality symmetry between them.

The case for Giddings–Strominger axionic baby universes [6] is a little more difficult to show but from the very onset we know that it is at the end of the day transformable in the one for the Tolman–Hawking case [7] and hence one would expect it to have the same properties. The metric of a Giddings–Strominger baby universe is given as in Eq. (1), with

$$a_{GS} = R_0^{GS} \cos^{1/2}(2\eta). \tag{6}$$

There would then exist a solution such that

$$a_d(\eta) = \frac{R_0^d}{\cos^{1/2}(2\eta)} \tag{7}$$

whose two-dimensional version must be dual to the two-dimensional version of metric (6). The corresponding geometric sectors of the Hilbert–Einstein actions are in fact the same and again given by expression (5). In terms of the Robertson–Walker time the scale factor (7) can be expressed as an elliptic function

$$a(t) = R_0^d \,\mathrm{nc}\left(\frac{\sqrt{2}t}{R_0^d}\right), \tag{8}$$

with $0 < t < R_0^d K(1/\sqrt{2})/\sqrt{2}$ and $K(x)$ the complete elliptic integral of the first kind. However, the metric given by Eq. (8) in terms of time t does no longer describe a Friedmann–Robertson–Walker metric as it can be checked by embedding this two-dimensional spacetime as the three-hyperboloid

$$-T^2 + S^2 + X^2 = R_0^{d2}, \tag{9}$$

with the Lorentzian metric

$$ds^2 = -dT^2 + dS^2 + dX^2. \tag{10}$$

This embedding can be achieved by exhibiting the new coordinates in terms of the elliptic functions sc and nc in the form

$$T = R_0^d \,\mathrm{sc}\left(\frac{\sqrt{2}t}{R_0^d}\right), \qquad S = R_0^d \,\mathrm{nc}\left(\frac{\sqrt{2}t}{R_0^d}\right)\sin\rho,$$
$$X = R_0^d \,\mathrm{nc}\left(\frac{\sqrt{2}t}{R_0^d}\right)\cos\rho, \tag{11}$$

with which we in fact get a manifestly non Friedmann–Robertson–Walker metric

$$ds^2 = -2\mathrm{dc}^2\left(\frac{\sqrt{2}t}{R_0^d}\right)dt + R_0^{d2}\,\mathrm{nc}^2\left(\frac{\sqrt{2}t}{R_0^d}\right)d\rho^2, \tag{12}$$

dc being still another elliptic function. A Friedmann–Robertson–Walker metric can then be obtained by re-defining the time so that

$$\theta = \sqrt{2}\int \mathrm{dc}\left(\frac{\sqrt{2}t}{R_0^d}\right)dt = R_0^d\left[\mathrm{nc}\left(\frac{\sqrt{2}t}{R_0^d}\right) + \mathrm{sc}\left(\frac{\sqrt{2}t}{R_0^d}\right)\right], \tag{13}$$

with which the metric becomes finally

$$ds^2 = -d\theta^2 + R_0^{d2}\cosh^2\left(\frac{\theta}{R_0^d}\right), \tag{14}$$

where $0 < \theta < \infty$. Metric (14) can then be again interpreted as that for a two-dimensional warp drive and hence we again derive the same result as for a Tolman–Hawking baby universe.

Let us now consider the possible connection between baby universes and warp drives with the essential property behind quantum entanglement, complementarity and wave function collapse, that is the Bell's inequalities [1]. Starting with a detailed comparison of the original intentions of Bohr and Einstein in their development of quantum mechanics and general relativity, Sachs showed [8] that the goals of general relativity are more insightful and subsume those of quantum mechanics, with "general relativity playing the role of a forest and quantum mechanics that of its trees". In what follows we shall use the properties discovered above in order to investigate whether localized warp drives cropped up in the universe are also connected to a violation of the Bell's inequalities [1]. If such a task led to that connection then the very much dismissed Einstein's dream that it is general relativity where the deepest roots of quantum theory reside [8] would be re-opened and mark one more example of the tremendous Einstein insight, leading this time to a new avenue to unity quantum mechanics and gravitation.

Violation of Bell's inequalities is attained when the following inequality holds [5]

$$C = \frac{\langle a^\dagger a a^\dagger a\rangle}{\langle a^\dagger a a^\dagger a\rangle + \langle (a^\dagger)^2 (a)^2\rangle} \geqslant 0.707, \tag{15}$$

where the a's are Fock annihilation and creation quantum operators.

Now, from inequality (15) and the definition

$$g_n^{(2)} = \frac{\langle n^2\rangle - \langle n\rangle}{\langle n^2\rangle} \geqslant 1 - \frac{1}{\langle n\rangle} \tag{16}$$

where use has been made of the condition $\langle n^2\rangle/\langle n\rangle \geqslant 1$, we get

$$C = \frac{1}{1 + \frac{\langle n\rangle^2}{\langle n^2\rangle} g_n^{(2)}} \geqslant \frac{1}{1 + g_n^{(2)}}. \tag{17}$$

We compute then the master equation for the second-order correlation function from the matrix elements in the baby universe Fock space of the matter field number states [2], in the diagonal representation

$$\dot{\bar{P}}_n(k,t) = -8\left(n + \frac{1}{2}\right)^2\left(N + \frac{1}{2}\right)\sinh(2k_0)\bar{P}_n(k,t), \tag{18}$$

in which $N = 0, 2, 4, \ldots$ denotes the initial number of baby universes, $\sqrt{2k}$ is the proper distance on the wormhole inner 3-manifold between the two correlated points at which two baby universes are created or annihilated, and $k_0 = \sqrt{2k/(R_0^2 - k)}$, with R_0 is the smallest value of the scale factor in the connected manifold, to obtain [2]

$$\dot{g}_n^{(2)} = P(N,k_0)\left[\frac{1}{4}\langle n\rangle g_n^{(2)2} + \frac{1}{4}\left(8\langle n\rangle^2 g_n^{(3)} - 7\right)g_n^{(2)} - \langle n\rangle\left(\langle n\rangle g_n^{(4)} + 6 g_n^{(3)}\right)\right], \tag{19}$$

where $g_n^{(3)}$ and $g_n^{(4)}$ are the third- and fourth-order coherence functions, respectively, and $P(N,k_0) = 8(N + \frac{1}{2})\sinh(2k_0)$. For the vacuum case, Eq. (19) admits the exact solution $g_0^{(2)}(0,k_0) = \exp(-\frac{7}{2}P(0,k_0)t)$, so that

$$C \geqslant \frac{1}{1+e^{-\frac{7}{2}P(0,k_0)t}}. \tag{20}$$

Thus, for the vacuum case there will always be a large enough time for which Bell's inequalities are violated. Such a time is smaller than or as most equal to $t_v = 2\ln 2.37/(7P(0,k_0))$. This conclusion is still valid for small, nonzero values of $\langle n\rangle$ and even in the limit of large $\langle n\rangle$ where

$$g_n^{(2)} \simeq \frac{1}{2}\big(1+\exp\big[2P(N,k_0)t\big]\big).$$

It follows that, though in his limit the right-hand side of expression (20) is in this case defined to be smaller than 0.707 even for $t=0$, the inequality relating it with C can leave still a residual room for the violation of Bell's inequalities in the situation where we usually expect the classical limit to hold, so allowing multiverse descriptions to call for a joint quantum treatment. The conclusion can then be drawn that warp drives entail the phenomena of quantum entanglement, complementarity and wave function collapse. Whether other special kinds of spacetime involving exotic matter with negative energy would also give rise to violations of the Bell's inequalities is a matter which deserves further consideration.

The point now is, provided a de Sitter space by itself implies the very essential phenomena of quantum entanglement, complementarity and wave function collapse, would any dynamical generalizations of a cosmological constant described by a quintessential or k-essential dark or phantom energy field also entail the deepest essentials of quantum theory by themselves?

Let us first assume from the onset that the nucleation of baby universes in pairs can be equivalently described by means of a duality symmetry transformation in terms of closed universes filled with an homogeneous and isotropic fluid, with equation of state $p = w\rho$, p and ρ being the pressure and the energy density, respectively, and w a parameter which for the sake of simplicity we take here to be constant. From the equation of the cosmic energy conservation,

$$d\rho = -3(p+\rho)\frac{da}{a}, \tag{21}$$

the solutions to the equation of motion can be computed for such closed universes. In conformal time η, they are given by [9]

$$\eta - \eta_0 = \pm\int \frac{da}{a\sqrt{\lambda_0^2 a^{2-3\beta}-1}} = \pm\frac{1}{\alpha}\arccos\frac{1}{\lambda_0 a^{\alpha}}, \tag{22}$$

with λ_0 a constant, $\beta = 1+w$ and $\alpha = 1-\frac{3\beta}{2} \neq 0$. Then, the considered baby universes resulting from the cosmic solutions when the duality symmetry holds can be taken as those described by the metric (1),

$$ds^2 = a^2(\eta)\big(-d\eta^2 + d\Omega_3^2\big),$$

with the scale factor given by

$$a(\eta) = R_0^{\alpha}\cos^{-\frac{1}{\alpha}}(\alpha\eta), \tag{23}$$

where $R_0^{\alpha} \equiv \lambda_0^{\frac{1}{\alpha}}$.

Let us then be concerned with the two-dimensional version of this type of baby universes, for which we take the slice that results at constant angular variables. As it has been pointed out previously, it will be now assumed that most of the relevant physics involved is concentrated on that slice. In such a case, the duality symmetry $a \to \frac{1}{a}$ corresponds in Eq. (23) just to a change of sign in the value of the parameter α. The action given by Eq. (4) becomes thus invariant under the transformation, $\alpha \to -\alpha$.

We next notice that there are two special values of the parameter w, which are equivalent to the baby universes which were considered before. First, the value $w = -1$ (which corresponds to the case of a positive cosmological constant), i.e., $\alpha = 1$, is associated by the duality symmetry to the closed Tolman–Hawking baby universe considered in Eq. (2). The second case, for a value $w = -\frac{5}{3}$ (which falls well inside the phantom energy regime [10]), i.e., $\alpha = 2$, corresponds to the Giddings–Strominger baby universe, given by Eq. (6). There is still another special value which the parameter w may take on, $w = -\frac{2}{3}$ (which describes an accelerating universe dominated by dark energy [11]), i.e $\alpha = 1/2$, amounting to a third kind of Euclidean wormholes characterized by a scale factor which in its Lorentzian sector is given by

$$a(\eta) = M\cos^2(\eta/2). \tag{24}$$

An Euclidean wormhole solution of the form $a \propto \cosh(\eta_E/2)$ can be derived from the Euclidean Friedmann–Robertson–Walker Einstein equations by simply adding an extra quantum term arising from the insertion of a minimum resolution distance in the background theory, in the case that no cosmological constant be included [12]. Now, similarly to how we have shown that the two two-dimensional cosmological solutions respectively associated with Tolman–Hawking and Giddings–Strominger two-dimensional baby universes are both convertible into the two-dimensional warp drive spacetime, one would expect the two-dimensional version of the cosmic scale factor associated with a two-dimensional baby universe given by Eq. (24) to be convertible into that for the two-dimensional warp drive, too. This expectation arises from the result [13] that the three kinds of baby universe considered above correspond to the only three existing Euclidean wormhole solutions and it was shown that they are physically equivalent to each other. In fact, the two-dimensional cosmic solution for the scale factor $a(\eta) = \frac{M}{\cos^2(\eta/2)}$ is once again physically equivalent to that of a two-dimensional warp drive as it can be readily shown that such a metric is conformal to the one associated with a closed de Sitter space when the former metric is expressed in terms of a Friedmann–Robertson–Walker time. Thus, defining first the time $t = \int \frac{M\,d\eta}{\cos^2(\eta/2)}$ and hence, $a(t) = M + t^2/(4M)$, and then a new time θ through $t = 2M\sinh(\theta/4M^2)$, we have the well-defined two-dimensional conformally Friedmann–Robertson–Walker line element

$$ds^2 = \cosh^2\big(\theta/4M^2\big)\big[-d\theta^2 + M^2\cosh^2\big(\theta/4M^2\big)\big], \tag{25}$$

with $0 \leqslant \theta \leqslant \infty$.

When combined with the above discussion on the violation of Bell's inequalities, if we take into account the restriction that $\alpha > 0$ which is required in order to obtain baby universe solutions, then the previous results leads inexorably to the conclusion that always we have a universe which expands in an accelerated fashion, no matter whether it is phantom, de Sitter or dark energy dominated, it entails the deepest essentials of quantum theory, meaning that such a universe, by itself, is a quantum, entangled system which has no classical analog whatsoever. The rather bizarre implication that the universe where we live in is of necessity a quantum universe appears to be made less surprising, at least when phantom energy is considered. In fact, the very essential features of a universe filled with such a kind of vacuum energy mark rather quantum footprints, which manifests in the fact that the parameter of its equation of state must be quantized and that the phantom energy density increases with time to tend to a classical singularity (which likely be smoothed out by quantum effects) in a finite time in the future. The feature that all accelerating ways to expand are at the end of the day physically equivalent makes then the above conclusion quite less bizarre.

Indeed, if the ultimate cause for the current speeding-up of the universe is a universal quantum entanglement, then one would expect that the very existence of the universe implied the violation of the Bell's inequalities and hence the collapse of the superposed cosmic quantum state into the universe we are able to observe, or its associated complementarity between cosmological and microscopic laws, and any of all other aspects that characterize a quantum system as well. The current dominance of the resulting quantum repulsion over attractive gravity started at a given coincidence time would then mark the onset of a new *quantum* region along the cosmic evolution, other than that prevailed at the big bang and early primeval universe, this time referring to the quite macroscopic, large universe which we live in. Thus, quite the contrary to what is usually believed, quantum physics not just govern the microscopic aspect of nature but also the most macroscopic description of it in such a way that we can say that current live is forming part of a true quantum system.

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