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Perturbative neutrino pair creation by an external source

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Abstract

We consider the rate of fermion–antifermion pair creation by an external field. We derive a rate formula that is valid for a coupling with arbitrary vector and axial vector components to first order in perturbation theory. This is then applied to study the creation of neutrinos by nuclear matter, a problem with astrophysical relevance. We present an estimate for the creation rate per unit volume, compare this to previous results and comment on the role of the neutrino mass.

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1. Introduction

Starting with Schwinger's classical account [1] of electron–positron pair creation by an external electric field, fermion pair creation has been the subject of continued interest. A variety of pair creation rates for specific external fields in quantum electrodynamics can be found in the literature, such as Refs. [2–10] and further references therein. The process exemplifies a true quantum field theory phenomenon: the creation of particles from the vacuum.

Because neutrinos carry weak charge, one expects that an external Z-boson field can produce neutrino–antineutrino pairs in a similar manner. The concept of an external Z-boson field can be seen as arising from a distribution of nuclear matter (in the sense of Ref. [11]). Neutron stars are a prime example of such a matter distribution and their neutrino emission by this mechanism was studied using non-perturbative methods [11–13]. Pair creation of neutrinos is also studied in relation to the stability of neutron stars, see Ref. [14] and references therein. Although Refs. [11–13] find typical neutrino fluxes that are too small to be observable, we believe it is worthwhile

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to study such a relatively unexplored neutrino source from a different point of view. In particular, we want to develop a method that is not limited to a specific source but allows us to draw conclusions with a broad applicability. This can then be applied to study, e.g., neutrino pair creation by non-standard model weakly interacting particles or domain walls.

In the present Letter, we study the creation of neutrino pairs in a perturbative way. We present a first order computation of the pair creation rate per volume, with a dynamical nuclear configuration acting as a source. The reasons for using perturbation theory are twofold. First, the axial coupling to the Z-boson prevents an easy generalization of non-perturbative QED methods. Second, non-perturbative methods generally consider a very specific source, or class of sources, from the start. The perturbative method is more flexible in the sense that a specific source is folded in at the end. This allows us to keep separate the physics of the pair creation process and that of a specific source.

In part, our computation was triggered by the results presented in Ref. [11], in which the creation of neutrinos by a time-dependent nuclear distribution is studied. One of the results in Ref. [11] is that the overall rate is proportional to the square of the neutrino mass. This implies that there can be no pair creation of massless neutrinos. The question arises whether this is a manifestation of a general chiral suppression mechanism or a consequence of the specific source considered. We shall see that the perturbative viewpoint contributes to a more complete understanding of this effect.

The Letter is organized as follows. In Section 2 we discuss the theoretical background of pair creation processes for fermions and introduce the relevant quantities. In Section 3, we discuss the perturbative computation. The result is then applied to neutrinos in Section 4 and we present our conclusions in Section 5.

2. Pair creation physics

We study fermions that are coupled to an external source j. The interaction Lagrangian reads

$$\mathcal{L}_{\text{int}} = j_\mu(x)\bar{\psi}(x)\Gamma^\mu\psi(x). \tag{1}$$

The source is fully prescribed and has no further dynamics. We choose the coupling of the general form

$$\Gamma^\mu = \gamma^\mu\big(c_V - c_A\gamma^5\big), \tag{2}$$

where c_V (c_A) is the vector (axial vector) coefficient; the coupling constant is absorbed in j.

Following Ref. [15], we introduce the overlap of asymptotic ‘in’ and ‘out’ vacua to describe the pair creation process:

$$S_0(j) = \langle 0,\infty|0,-\infty\rangle_j = \langle 0,\infty|S|0,\infty\rangle_j, \tag{3}$$

where S is the scattering operator and the subscript is a reminder that a source is switched on and off adiabatically somewhere between $t=-\infty$ and $t=\infty$. The probability that a system that started in the vacuum state will remain in the vacuum state is then expressed [15] as

$$\big|\langle 0,\infty|0,-\infty\rangle_j\big|^2 = \exp(-W) = \exp\left(-\int d^4x\, w(x)\right). \tag{4}$$

For a positive W, this probability is between zero and one which signals a non-zero probability for the creation of a fermion pair. Now suppose that $w(x)=\bar{w}$ is constant. We can embed the system in a box of size $V\times T$, write $W=\bar{w}VT$ and choose the box small enough such that $W<1$:

$$\big|\langle 0,\infty|0,-\infty\rangle_j\big|^2 \simeq 1-\bar{w}VT, \tag{5}$$

which supports the interpretation of the function $w(x)$ as the probability per unit time and volume to create a pair at space–time location x. Such a rate density is the physical quantity of interest. For QED, the Schwinger formula

[1] states that for a photon field of the form $A^\mu(x) = j^\mu(x) = (0, 0, 0, -eEt)$,

$$\bar{w} = \frac{\alpha E^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m^2}{|eE|}\right), \tag{6}$$

where m is the electron mass. We mention that Refs. [12,13] conclude that this result extends to the case of neutrino pair creation by a source of the same form.

To compute the rate density, we use perturbative quantum field theory:

$$\langle 0, \infty | 0, -\infty \rangle_j = Z[j] = \exp\bigl(iW[j]\bigr), \tag{7}$$

where $W[j]$ is the generating functional of connected n-point functions.[1] In this context, $W[j]$ is also the effective action for the external field j [10,15].

The interaction Lagrangian (1) only contains a vertex that couples to the external field. Therefore, $W[j]$ represents an infinite sum of fermion loop diagrams, labeled by the number of vertices which are all connected to the external field. In terms of W that was introduced in Eq. (4),

$$W = 2 \operatorname{Im} W[j]. \tag{8}$$

The fermion loop diagram with one external field vertex is zero by momentum conservation, so the first non-zero contribution is from the loop with two external field vertices, i.e. the two-point function. This is the object that we will compute in Section 3. Its contribution to the pair creation rate W_2 is found by folding in the sources according to the formula[2]

$$W_2 = -\int \frac{d^4p}{(2\pi)^4} j_\mu(p) j_\nu(-p) \operatorname{Im} \Sigma^{\mu\nu}(p), \tag{9}$$

where $\Sigma^{\mu\nu}$ represents the two-point function, with prefactors as chosen in Eq. (10). For time-like currents, $j_\mu(p) j_\nu(-p) \operatorname{Im} \Sigma^{\mu\nu}(p) < 0$ since a probability cannot exceed one. For a given W, the pair creation density follows by extracting the function $w(x)$.

There has to be enough energy in the source to put two virtual particles on-shell. For the perturbative mechanism that we describe, this implies a threshold energy for the source insertions. This is in contrast to the non-perturbative effect, which can be thought of as an infinite sum of loop diagrams with an increasing number of source insertions. This infinite amount of sources conspire to create a pair and the amount of energy per source insertion can be arbitrarily small.

For QED it is known that the real part of the sum of loop diagrams has a divergent structure, which can be used to extract non-perturbative results by performing a Borel transformation [16]. We do not know whether or not a similar procedure can be applied in this more general situation.

3. The two-point function

The two-point function without external sources is transcribed from Fig. 1. We find that, in dimensional regularization with $n = 4 - \epsilon$,

$$\Sigma^{\mu\nu}(p) = -i\mu^{(4-n)} \int \frac{d^nk}{(2\pi)^n} \frac{\operatorname{tr}[(\not{k} + m)\Gamma^\mu(\not{k} + \not{p} + m)\Gamma^\nu]}{(k^2 - m^2 + i\epsilon)((k+p)^2 - m^2 + i\epsilon)}, \tag{10}$$

[1] The use of W and $W[j]$ may be confusing, but both symbols are standard in the literature. The generating functional will always be denoted with its argument j.

[2] We use a metric tensor $g^{\mu\nu} = \operatorname{diag}(1, -1, -1, -1)$ throughout this Letter.

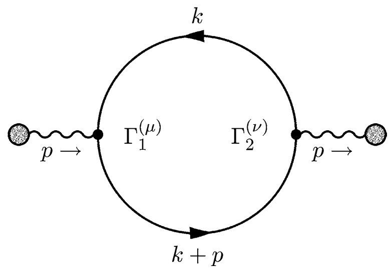

Fig. 1. Fermion loop diagram with two external sources attached. The external field couples directly (i.e., without propagators) to the loop.

where m is the fermion mass. From Eq. (9), we are interested in the imaginary part of this expression, which is finite. Note that we integrate over the fermion momentum; in the source's rest frame (where the particles are created back to back), the fermion and the antifermion each carry half of the energy.

Expression (10) is reduced to a linear combination of scalar integrals in the fashion of Passarino–Veltman [17]. A series expansion in ϵ reveals the divergent structure, and after some algebra the problem depends only on the one- and two-point scalar integrals. The one-point scalar integral is real, the two-point integral develops an imaginary part if $p^2 > 4m^2$ which means there should be enough energy in the source to create two fermions. If this is not satisfied, $\Sigma^{\mu\nu}$ is purely real and there is no pair creation. The final result is the following expression:

$$\operatorname{Im}\Sigma^{\mu\nu}(p) = \frac{1}{16\pi^2}\left[\left(c_V^2 - c_A^2\right)\Sigma_{\mathrm{I}}^{\mu\nu}(p) + \left(c_V^2 + c_A^2\right)\Sigma_{\mathrm{II}}^{\mu\nu}(p)\right]\theta\left(p^2 - 4m^2\right), \tag{11a}$$

$$\Sigma_{\mathrm{I}}^{\mu\nu}(p) = 4\pi m^2\sqrt{1 - \frac{4m^2}{p^2}}\, g^{\mu\nu}, \tag{11b}$$

$$\Sigma_{\mathrm{II}}^{\mu\nu}(p) = \frac{4}{3}\pi\left(p^2 g^{\mu\nu} - p^\mu p^\nu - \frac{2m^2}{p^2}p^\mu p^\nu - m^2 g^{\mu\nu}\right)\sqrt{1 - \frac{4m^2}{p^2}}. \tag{11c}$$

For some typical values of the parameters c_V and c_A, this result can be compared to the literature [15,18].

From expressions (11) we observe that for massless fermions $\Sigma_{\mathrm{I}}^{\mu\nu} = 0$, so that only the second term contributes. This means that the physics is qualitatively insensitive to different choices of c_V and c_A; only the square sum is quantitatively important. We conclude that the difference between the two-point functions with two different normalized sets of couplings (e.g., purely vector, purely axial vector) is proportional to m^2.

The contribution due to the three-point diagram should be interpreted with care. In QED it vanishes by Furry's theorem, but for axial couplings it contributes to the axial anomaly. This means one should verify that the final result does not depend on the regularization procedure. For the present calculation, this is not an issue.

4. Neutrino pair creation to first order

4.1. The general case

We specialize to neutrino pair creation by putting $c_V = c_A = 1/2$ in the expression for the two-point function (11). Combining Eqs. (9) and (11), we find

$$W_2 = -\frac{1}{24\pi}\int\frac{d^4p}{(2\pi)^4}\theta\left(p^2 - 4m^2\right)\sqrt{1 - \frac{4m^2}{p^2}}\left[\mathcal{F}_0(p, j) + m^2\mathcal{F}_1(p, j)\right], \tag{12a}$$

$$\mathcal{F}_0(p, j) = p^2\left[j(p)\cdot j(-p)\right] - \left[\left(p\cdot j(p)\right)\left(p\cdot j(-p)\right)\right], \tag{12b}$$

$$m^2\mathcal{F}_1(p,j) = -m^2\big[j(p)\cdot j(-p)\big] - \frac{2m^2}{p^2}\big[\big(p\cdot j(p)\big)\big(p\cdot j(-p)\big)\big]. \tag{12c}$$

Without loss of generality, we consider a source with a density component and a spatial current in the $\hat{z}$ direction:

$$j_\mu(p) = \big(j_0(p), 0, 0, j_3(p)\big), \qquad p_\mu = (E, \vec{p}_T, p_3). \tag{13}$$

Here p_μ labels the energy and momentum of the source. Though the current is directed in the $\hat{z}$ direction, we allow for a dependence on the transverse direction by leaving $\vec{p}_T$ unspecified. The two terms in (12) can be written as

$$\mathcal{F}_0(p,j) = -\vec{p}_T^{\,2}\big(|j_0|^2 - |j_3|^2\big) - |Ej_3 - p_3 j_0|^2, \tag{14a}$$

$$m^2\mathcal{F}_1(p,j) = -\frac{2m^2}{E^2 - \vec{p}_T^{\,2} - p_3^2}\big(E^2|j_0|^2 + p_3^2|j_3|^2 - Ep_3(j_0 j_3^* + j_0^* j_3)\big) - m^2\big(|j_0|^2 - |j_3|^2\big). \tag{14b}$$

We do not simplify these equations further, as we do not want to constrain the properties of the source.

It is instructive to analyze the massless limit in more detail. In this case only $\mathcal{F}_0$ in (12) contributes, so that

$$W_2(m=0) = -\frac{1}{24\pi}\int \frac{d^4p}{(2\pi)^4}\big[p^2\big(j(p)\cdot j(-p)\big) - \big(p\cdot j(p)\big)\big(p\cdot j(-p)\big)\big]. \tag{15}$$

In analogy with QED, we introduce a field strength $F_{\mu\nu}(p) = ip_\nu j_\mu(p) - ip_\mu j_\nu(p)$ and its 'electric' and 'magnetic' components E_i and B_i and find:

$$W_2(m=0) = -\frac{1}{48\pi}\int \frac{d^4p}{(2\pi)^4}\big[F_{\mu\nu}(p)F^{\mu\nu}(-p)\big] \tag{16a}$$

$$= \frac{1}{24\pi}\int \frac{d^4p}{(2\pi)^4}\big[E_i(p)E_i(-p) - B_i(p)B_i(-p)\big]. \tag{16b}$$

This is exactly half of the QED result [15] if we insert a factor e^2 from the coupling constants, which reflects the discussion in the previous section. In electrodynamics, $\vec{E}$ and $\vec{B}$ are the physical electric and magnetic fields and one can go to a frame in which $\vec{B} = 0$. Then Eq. (16) yields a positive result from which we conclude that the creation of massless particles by the two-point mechanism is in general possible. Eq. (16) is consistent with the massless limit of the first-order effective action in an axial background that was computed in Ref. [19].

It is interesting to compare this result to the creation of neutrinos by an external electromagnetic field as computed in Ref. [20]. In that case, the pair creation rate is proportional to m^2 and depends on the electromagnetic invariant $\vec{E}\cdot\vec{B}$.

4.2. The time-dependent density

We consider a time-dependent distribution of nuclear matter, described by the following source term:[3]

$$j_\mu(t) = \frac{G_F}{\sqrt{2}}\langle \bar{n}\gamma_\mu\big(1-\gamma^5\big)n\rangle = \big(j_0(t), 0, 0, 0\big), \tag{17a}$$

$$j_0(t) = \frac{G_F}{\sqrt{2}} n_N(t), \tag{17b}$$

where n_N is the number density of the nuclear matter distribution and G_F is Fermi's constant. This is the specific background that we refer to as a time-dependent density. Our main motivation for this source is to compare the perturbative results with the non-perturbative results of Ref. [11].

[3] This source originates from an effective four-fermion description, see Ref. [11]. Note that j^μ contains the axial current; since the neutrons are massive, axial symmetry is broken and the current need not be divergence-free.

For simplicity (and because any source can be decomposed into a trigonometric sum) we assume a monochromatic source: $j_0(t) = E_0 \cos \omega t$. In Fourier space, this is

$$j_0(p) = \frac{E_0}{2}(2\pi)^4 \delta(\vec{p})\left[\delta(E-\omega) + \delta(E+\omega)\right], \tag{18a}$$

$$E_0 = \frac{G_F}{\sqrt{2}} n_N(0). \tag{18b}$$

Inserting the source (18) into Eq. (9) results in products of delta functions. We employ a box normalization procedure to reduce these to a single delta function and a factor $V \times T$ and find

$$W_2 = -VT \frac{G_F{}^2 (n_N)^2}{8}\left[\operatorname{Im} \Sigma^{00}(\omega; \vec{p}=0) + \operatorname{Im} \Sigma^{00}(-\omega; \vec{p}=0)\right]. \tag{19}$$

Using Eq. (11), with $c_V = c_A = 1/2$, we see

$$\operatorname{Im} \Sigma^{00}(\pm\omega; \vec{p}=0) = -\frac{m^2}{8\pi}\sqrt{1 - \frac{4m^2}{\omega^2}}, \tag{20}$$

leading to the following pair creation probability per unit time and volume:

$$\bar{w}_2 = \frac{W_2}{VT} = \frac{m^2}{32\pi}\sqrt{1 - \frac{4m^2}{\omega^2}}\, G_F^2 n_N^2. \tag{21}$$

The rate density scales with the square of the nuclear density, as expected for the two-point mechanism.

We see that the rate is proportional to m^2, which could have been anticipated from Eq. (14) because the time-dependent density (17) is characterized by $j_3 = \vec{p}_T = p_3 = 0$ so that $\mathcal{F}_0(p, j) = 0$. Ref. [11] also finds the m^2 proportionality for sources with a time-dependent current in the $\hat{z}$ direction. Eq. (14) suggests that such sources can contribute to first order for a zero neutrino mass.

To derive an order-of-magnitude estimate for the number of created neutrinos per unit volume per unit time, we take the square root factor in Eq. (21) of order unity, use a neutrino mass of 0.1 eV and assume a 'reduced density' $G_F n_N/\sqrt{2} \sim 1$ eV, such as in a neutron star [11]:

$$\bar{w}_2 = \frac{(0.1\ \mathrm{eV})^2}{32\pi}\left(2\ \mathrm{eV}^2\right) \sim 10^{-4}\ \mathrm{eV}^4 \sim 10^{26}\ \mathrm{s}^{-1}\,\mathrm{cm}^{-3}. \tag{22}$$

At the pair creation threshold, this corresponds to an energy output of order 10^{13} erg cm^{-3} s^{-1}. Ref. [11] estimates the energy output of neutrinos that are created non-perturbatively by an oscillating neutron star to be of order 10^3 erg cm^{-3} s^{-1}. However, these numbers should not be compared because the (realistic) driving frequency that is considered in [11] is so low that the perturbative mechanism is not operational.

As follows from Eq. (11), there can only be pair creation by the two-point mechanism if $\omega^2 > 4m^2$. With a neutrino mass of 0.1 eV, the creation of a neutrino–antineutrino pair requires a driving frequency of at least 3×10^{14} Hz. The coherence length of such a system is roughly 10^{-4} cm, so it is not very feasible to look for an oscillating astrophysical object that would produce an appreciable amount of neutrinos with this mechanism. However, the value of our computation lies in its general applicability. We are not limited to this particular type of sources, and we believe it may be interesting to study sources of a more transient nature such as a forming neutron star. Alternatively, one could consider weakly interacting particles beyond the standard model or domain walls as a source.

5. Conclusion

We have described pair creation of fermions by an external field to first order in perturbation theory and found the contribution by the two-point mechanism for a general coupling. Our main result is Eq. (11), which should be interpreted in the context of Eq. (9). We observe that at this order in perturbation theory, the difference in pair creation rates between two sets of normalized coupling coefficients $\{c_V, c_A\}$ is proportional to the square of the fermion mass.

For the case of neutrino pair creation by a distribution of nuclear matter, we have derived expressions (12) and (14). From this result we observe that, to first order, neutrino pair creation is possible with a suitable source if neutrinos would have been massless particles. We then considered pair creation of neutrinos by the time-dependent density of Eq. (17). For this specific source, we conclude that the production rate due to the two-point contribution (21) is proportional to the square of the neutrino mass. This conclusion is in qualitative agreement with the non-perturbative result derived in [11].

The method that we presented in this Letter is suitable to study different types of sources and we intend to do so in the future.

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