Phases of augmented hadronic light-front wave functions

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ABSTRACT

It is an important question whether the final/initial state gluonic interactions which lead to naive-time-reversal-odd single-spin asymmetries and diffraction at leading twist can be associated in a definite way with the light-front wave function hadronic eigensolutions of QCD. We use light-front time-ordered perturbation theory at the lowest nontrivial order to obtain augmented light-front wave functions which contain an imaginary phase which depends on the choice of advanced or retarded boundary condition for the gauge potential in light-cone gauge. We apply this formalism to the wave functions of the valence Fock states of nucleons and pions, and show how this illuminates the factorization properties of naive-time-reversal-odd transverse momentum dependent observables which arise from rescattering. In particular, one calculates the identical leading-twist Sivers function from the overlap of augmented light-front wave functions that one obtains from explicit calculations of the single-spin asymmetry in semi-inclusive deep inelastic lepton–polarized nucleon scattering where the required phases come from the final-state rescattering of the struck quark with the nucleon spectators.

1. Introduction

Wave functions are key objects of the quantum world, specifying the structure of composite states in terms of their fundamental constituents. The conceptual extension of the non-relativistic wave functions of Schrödinger theory to relativistic hadron physics are the frame-independent light-front wave functions (LFWFs) of hadrons $\Psi_n^H(x_i, \vec{k}_{i\perp}, \lambda_i)$ where $x_i = \frac{k_i^+}{P^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$ are the light-front momentum fractions of the n constituents, $k_{i\perp}$ the transverse momentum components, and λ_i the parton helicities. The LFWFs are defined as constituent wave functions at fixed light-front time $\tau = x^+ = t + z/c$ and in the light-cone gauge $A^+ = A^0 + A^z = 0$ where A^μ represents the gauge field [1–4]. The LFWFs are obtained explicitly by computing the hadronic eigensolutions $|\Psi_H\rangle$ of the QCD light-front Hamiltonian H_{LF} projected on the free Fock basis $\Psi_n^H = \langle n|\Psi_H\rangle$.

Light-front wave functions in QCD describe the quark and gluon composition of hadron at a fundamental level, leading to a description of a wide range of hadronic and nuclear physics phenomena [2]. For example, the parton distribution functions measured in deep inelastic lepton–hadron scattering, including DGLAP evolution and their transverse momentum extensions, are defined from the sum over squares of the light-front wave functions. Form factors are given by the sum of overlap matrix elements of the initial and final LFWFs with the electroweak currents. The gauge-invariant distribution amplitudes $\phi(x_i, Q)$ which control hard exclusive reactions are the valence LFWFs integrated over transverse momenta $k_\perp^2 < Q^2$.

Recent theoretical developments have shown that final and/or initial-state interactions can generate a phase in scattering amplitudes which lead to novel single transverse spin asymmetries in high energy hadronic reactions at leading twist. A prime example of this rescattering physics in QCD is the Sivers single-spin asymmetry measured in semi-inclusive deep inelastic scattering and spin-dependent Drell–Yan lepton pair production [5–7]. Double initial-state interactions lead to an anomalous $\cos 2\phi$ azimuthal dependence of the production plane in unpolarized lepton pair hadroproduction, corresponding to the breakdown of the Lam–Tung relation in PQCD [8,9]. Similarly, diffractive deep inelastic lepton scattering $\ell p \to \ell' p' X$ arises from the exchange of gluons in the final state which occurs after the hard lepton–quark interaction [10]. Since nuclear shadowing involves diffractive deep inelastic processes, nuclear distributions are also dependent on rescattering processes.

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The wave functions of stable hadrons that are obtained by solving the Heisenberg problem $H_{\rm LF}^{\rm QCD}|\Psi\rangle = \mathcal{M}^2|\Psi\rangle$ have a real phase. As discussed in Ref. [11], one can distinguish "static" structure functions, the probabilistic distributions computed from the square of the light-front wave functions of the target hadron from the "dynamic" structure functions measured in deep inelastic lepton-hadron scattering which include the effects of rescattering associated with the Wilson line. Thus it is an important question whether the final/initial state gluonic interactions responsible for the dynamics of rescattering can be associated in a definite way with the light-front wave function eigensolutions of QCD. The resulting augmented LFWFs provide an important tool for understanding the factorization properties of dynamical hadronic phenomena including single-spin asymmetries and diffraction.

It has been shown that the light-cone gauge condition $A^+ = 0$ does not fix the gauge of Abelian or non-Abelian gauge fields completely [7]: one has to choose a boundary condition for the transverse component of the gauge potential at spatial infinity: $A_\perp(x^- = \pm\infty)$ [7]. The propagators of the gauge field which define the QCD light-front Hamiltonian in the Heisenberg problem are regulated using the principal value prescription. However, a different choice of boundary condition will lead to different properties of the light-front wave function amplitudes. In particular, if we choose a retarded $(A_\perp(x^- = -\infty) = 0)$ or advanced $(A_\perp(x^- = \infty) = 0)$ boundary condition, the resulting augmented light-front wave function will contain the necessary phase to generate the nonzero single spin asymmetry in hadronic reactions. We will demonstrate these properties, giving an explicit calculation in light-front time-ordered perturbation theory [2]. The result of our analysis provides the general structure of augmented LFWFs which is easy to apply to phenomenological applications. As an example, we will present results for the three-quark Fock state component of nucleon and the quark–antiquark component of pion at lowest nontrivial order. We can further simplify the result for the pion in terms of the distribution amplitudes. Given these light-front wave function amplitudes results, it is straightforward to calculate the pseudo-time-reversal-odd quark distributions of the nucleon and pion, by applying the overlap formalism derived in [12]. The light-cone gauge with retarded/advanced boundary condition has also been used to investigate the small-x physics [13–16], in particular, to study the evolution and factorization for nucleus–nucleus collisions [16].

The rest of this Letter is organized as follows. In Section 2, we present a general derivation of augmented LFWFs using light-front time-order perturbation theory within a lowest order formalism. In Section 3, we apply our method to the construction of augmented light-front wave function amplitudes for the three-quark Fock component of nucleon and quark–antiquark component of the pion. We summarize our Letter in Section 4.

2. General derivations

We start our derivation by constructing the general form for a Fock state expansion of any given hadron,

$$|P, S\rangle = \sum_n \int \prod_{i=1}^{n} d[i]\, \psi_n(x_i, k_{i\perp}, \lambda_i) a_1^\dagger a_2^\dagger \cdots a_n^\dagger |0\rangle, \qquad (1)$$

where P and S are the momentum and spin of the hadron, $d[i] = dx_i\, d^2 k_{i\perp}/(\sqrt{2x_i}(2\pi)^3)$ with the overall constraint on x_i and $k_{i\perp}$ implicit. For convenience, in following calculations we set the transverse momentum of hadron equal zero: $P_\perp = 0$. Because the wave functions are boost invariant, all our results can be extended to more general case with $P_\perp \neq 0$. In this Fock state, each parton is represented by the associated creation operator $a_i^\dagger(k_i)$ with

$k_i^\mu = (k_i^+, \vec{k}_{\perp i}) = (x_i P^+, \vec{k}_{\perp i})$, which contains certain longitudinal momentum $k_i^+ = x_i P^+$ and transverse momentum $k_{i\perp}$, whereas the minus component is determined by the on-shell condition $k_i^- = (k_\perp^2 + m_i^2)/k_i^+$. Implicitly, the above light-front wave function amplitude ψ_n depends on the orbital angular momentum projection from the constituents with the form of $(k_i^x \pm i k_i^y)$ [17]. Since the following derivation does not depend on this structure, we will not include it explicitly.

As discussed in the introduction, the definition of a light-front wave function amplitude ψ_n can be extended to include rescattering effects so its phase is not necessarily real. We can obtain the imaginary part (or the phase), by iterating the light-front wave function eigensolutions employing a particular boundary condition for the gauge field. In fact as we shall show, the phase of the augmented wave functions can be computed perturbatively by applying light-front time-ordered perturbation theory [2] analogous to the Lippmann–Schwinger method.

The first order correction to the LFWF can be obtained by iterating the light-front equation of motion:

$$\left(P^- - \sum k^-\right)\psi_n(x_i, k_{i\perp}) = \int d[i]'\, K[k; \ell] \otimes \psi_n'(y_i, \ell_{i\perp}), \qquad (2)$$

where $\sum k^-$ represents the sum of all partons energy k_i^-, $d[i]'$ represents the integral of $(y_i, \ell_{i\perp})$. The interaction kernel K can be calculated from the light-front time-order perturbation theory [2]. The wave functions ψ_n and ψ_n' may differ. From the above expression, we find that the phase of ψ_n may come from the wave function in the right-hand side ψ_n' or the interaction kernel K. In the following, we assume that the wave function ψ_n' is real, for example, from model calculation such as constituent quark model [18]. We will focus on the contribution from the interaction kernel. We will calculate, in particular, the one-gluon exchange contribution to the interaction kernel.

At the lowest order of the light-front time-order perturbation theory, we have one gluon exchange contribution to the interaction kernel. This can be expressed as a sum of all diagrams with gluon connection between all possible pair of constituents in the light-front wave function. For example, the contribution from the gluon exchange between the ith and jth quark can be written as

$K[k; \ell]_{ij}$

$$= \frac{\bar{u}_{\lambda_i}(x_i, k_{i\perp})}{\sqrt{x_i}} \gamma^\mu \frac{u_{\lambda_i'}(y_i; \ell_{i\perp})}{\sqrt{y_i}} d_{\mu\nu} \frac{\bar{u}_{\lambda_j}(x_j, k_{j\perp})}{\sqrt{x_j}} \gamma^\nu \frac{u_{\lambda_j'}(y_j; \ell_{j\perp})}{\sqrt{y_i}}$$

$$\times \left\{ \frac{1}{P^- - q^- - k_i^- - \ell_j^- - \sum_{\alpha \neq \{i,j\}} k_\alpha^- + i\epsilon} \frac{\theta(q^+)}{q^+} \right.$$

$$\left. + \frac{1}{P^- - q'^- - k_j^- - \ell_i^- - \sum_{\alpha \neq \{i,j\}} k_\alpha^- + i\epsilon} \frac{\theta(q'^+)}{q'^+} \right\}, \qquad (3)$$

where λ represents the helicity for the associated quarks, $q^+ = k_j^+ - \ell_j^+$ and $q'^+ = k_i^+ - \ell_i^+$, and the color factors are implicit in the above equation. Similar expression shall hold for the gluon constituent in the wave function, and so the final results. We illustrate the contribution in the above calculations for $i = 1$ and $j = 2$. The first term in the above bracket comes from Fig. 1(a), whereas the second term comes from Fig. 1(b). Moreover, at this particular order, quark number is conserved, such that $n = n'$. The gluon polarization tensor is defined as

$$d_{\mu\nu} = -g_{\mu\nu} + \frac{v_\mu \tilde{q}_\nu + v_\nu \tilde{q}_\mu}{[v \cdot q]}, \qquad (4)$$

where v is a light-like vector $v \cdot P = 1$, $\tilde{q}$ differs from q in the minus component to take into account the instant propagator contribution with $\tilde{q}^+ = q^+$, $\tilde{q}_\perp = q_\perp$, and $\tilde{q}^- = \sum_{\rm inc} k^- -$
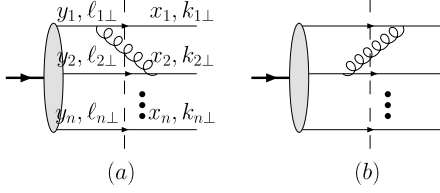
Fig. 1. Light-front time-order perturbation Feynman diagrams for the phase contribution from one-gluon exchange between two constituent quarks.

$\sum'_{\text{interm}} k^-$ [2]. Here, $\sum_{\text{inc}}$ denotes summation over initial particles whereas $\sum'_{\text{interm}}$ denotes summation over all particles in the intermediate state other than the gluon q. When one solves the Heisenberg eigenvalue problem for light-front QCD as in discretized light-front quantization [3], the light-cone gauge singularity is regulated by the principal value prescription, which corresponds to the antisymmetric boundary condition for the gauge potential $A_\perp(x^- = +\infty) + A_\perp(x^- = -\infty) = 0$. However, this prescription will not result into an imaginary part for the light-front wave function from the above interaction kernel. In the following calculation, we will choose the advanced boundary condition: $A_\perp(x^- = +\infty) = 0$ whereas $A_\perp(x^- = -\infty) \neq 0$ in order to construct the augmented light-front wave function. With this boundary condition, the light-cone singularity will be regulated by [7],

$$\frac{v_\mu \tilde{q}_\nu + v_\mu \tilde{q}_\nu}{[v \cdot q]}\bigg|_{Adv} = \frac{v_\mu \tilde{q}_\nu}{v \cdot q - i\epsilon} + \frac{v_\nu \tilde{q}_\mu}{v \cdot q + i\epsilon}, \tag{5}$$

where the momentum flow of q is toward to the vertex ν. Clearly, this term contains a phase. The imaginary part is simple, and proportional to a Delta function: $i\pi\delta(v \cdot q)$. Since we are only interested in the imaginary part of the light-front wave function amplitudes, we simply apply this Delta function to the interaction kernel in Eq. (3). In particular, we find that the dominant contribution comes from the $d_{+\perp}$ components of the $d_{\mu\nu}$ tensor [7,15]. All other contributions cancel out between the above two terms or by themselves. Another important consequence is that the helicities are conserved in the interaction kernel: $\delta_{\lambda_i \lambda'_i} \delta_{\lambda_j \lambda'_j}$. After a little algebra, we obtain a rather simple result for the imaginary part of the light-front wave function amplitude generated from lowest order perturbation theory,

$$\mathcal{I}\big[\psi_n(x_\alpha, k_{\alpha\perp})\big] = -\frac{\alpha_s}{2\pi}[\text{C.F.}] \int \frac{d^2 q_\perp}{\vec{q}_\perp^2} \left(1 - \frac{P^- - \sum \ell^-}{P^- - \sum k^-}\right)$$
$$\times \sum_{i \neq j} \psi_n^{(ij)}(x_\beta; \ell_{\beta\perp}), \tag{6}$$

where [C.F.] represents the color-factor for the Feynman diagram in Fig. 1 and $\psi_n^{(ij)} = \psi_n(x_\alpha; \ell_{i\perp} = k_{i\perp} - q_\perp, \ell_{j\perp} = k_{j\perp} + q_\perp, \ell_{\beta\perp}|_{\beta \neq i, j} = k_{\beta\perp})$. We emphasize that the wave function at the right-hand side only contains the real part.

The above equation is the main result of this Letter. It explicitly demonstrates that the light-front wave function amplitudes contain an imaginary part if we choose advanced boundary condition for the transverse component of the gauge potential. If we choose the retarded boundary condition, we obtain an opposite sign in the above equation.

3. Applications to pion and nucleon

The three-quark Fock state components have been classified in Ref. [12]. For these light-front wave function amplitudes, we can apply the derivation in the last section, and obtain the imaginary part as

$$\mathcal{I}\big[\Psi_{qqq}(x_i, k_{i\perp})\big]$$
$$= \frac{\alpha_s}{2\pi} C_B \int d^2\ell_{1\perp} d^2\ell_{2\perp} d^2\ell_{3\perp} \delta^{(2)}(\ell_{1\perp} + \ell_{2\perp} + \ell_{3\perp})$$
$$\times \Psi_{qqq}(x_i, \ell_{i\perp}) \left(1 - \frac{P^- - \sum \ell^-}{P^- - \sum k^-}\right)$$
$$\times \left[\frac{\delta^{(2)}(\ell_{3\perp} - k_{3\perp})}{(\vec{k}_{1\perp} - \vec{\ell}_{1\perp})^2} + (2 \leftrightarrow 3) + (1 \leftrightarrow 3)\right], \tag{7}$$

where $C_B = (N_c + 1)/2N_c$ and Ψ represents the general wave function amplitude constructed in Ref. [12]. By applying these results, we are able to formulate the naive time-reversal-odd quark distributions (such as the quark Sivers function) in terms of the light-front wave function amplitudes, by taking into account the above imaginary phase using the above derivation [12].

For the quark–antiquark Fock component of pion, the result can be further simplified as

$$\mathcal{I}\big[\psi(x, k_\perp)\big] = \frac{\alpha_s}{2\pi} C_F \int \frac{d^2 q_\perp}{\vec{q}_\perp^2} \psi(x, k_\perp - q_\perp)$$
$$\times \left(1 - \frac{x(1-x)M^2 - (k_\perp - q_\perp)^2 - m_q^2}{x(1-x)M^2 - k_\perp^2 - m_q^2}\right), \tag{8}$$

where we have chosen $P_\perp = 0$ and assumed that the quark and antiquark have the same mass m_q and $C_F = (N_C^2 - 1)/2N_C$. In particular, if we are interested in the large transverse momentum behavior of the light-front wave function amplitudes, we can expand the interaction kernel in terms of $\ell_\perp / k_\perp$, and keep the leading order contribution. By doing that, we will obtain

$$\mathcal{I}\big[\psi(x, k_\perp)\big] = \frac{\alpha_s}{2\pi} \frac{1}{k_\perp^2} C_F \phi(x), \tag{9}$$

and $\phi(x)$ is the leading-twist distribution amplitude for the pion, normalized by the leading Fock component light-front wave function $\phi(x) = \int d^2\ell_\perp \psi(x, \ell_\perp)$. Similar expressions can be found for the quark–diquark model [5].

Although light-front wave functions depend on the boundary condition of the gauge potential in the light-cone gauge, physical observables cannot depend on this choice because of gauge invariance [7,10]. In particular, the single-spin asymmetry in semi-inclusive deep inelastic polarized proton deep inelastic scattering $\ell p^{\updownarrow} \to \ell' q X$ and the associated quark Sivers function can be formulated simply as the overlap of augmented LFWFs using the advance boundary condition [12]. In particular, it is the phase difference between the LFWFs for the S and P-wave Fock components that contributes to the quark Sivers function in the quark–diquark model studied in Ref. [5]. The imaginary phases are calculated by using the general formalism Eq. (6) with similar expression as Eq. (8).

For the quark Sivers function and the single spin asymmetry in the Drell–Yan process $pp^{\updownarrow} \to \gamma^* X \to \ell^+ \ell^- X$, we have to choose the retarded boundary condition to have the LFWF overlap interpretation. Under this condition, the imaginary phase for the augmented wave function in Eq. (6) will change the sign. The resulting Sivers function and the spin asymmetry will change the sign as well. This is consistent with the opposite sign prediction between the quark Sivers function for the SIDIS and Drell–Yan processes by using the time-reversal transformation [6,7].

The result for the Sivers single-spin asymmetry using augmented LFWFs is identical to that found in Ref. [5] using conventional LFWFs (with the principal value boundary condition), together with an explicit calculation of the final state phases which arise from the rescattering of the struck quark with the spectator diquark after the lepton–quark interaction. This identity is possible

since the final-state phase due to rescattering is independent of the momentum transferred in the lepton–quark interaction in DIS process. On the other hand, if we choose the retarded boundary condition, the augmented wave function will have opposite imaginary part. However, under this boundary condition, we have to take into account the final state interaction effects (the transverse gauge link contributions at spatial infinity in the quark distribution definition in DIS process [7]), and we do not have the simple light-front wave function overlap interpretation for the quark Sivers function [12].

4. Summary and discussions

We have used light-front time-ordered perturbation theory at the lowest nontrivial order to obtain augmented light-front wave functions that contain an imaginary phase which depends on the choice of advanced or retarded boundary condition for the gauge potential in light-cone gauge. We have applied these results to construct augmented wave functions for the three-quark or quark–diquark Fock state components of nucleon and the quark–antiquark component of the pion. We obtain the leading-twist quark Sivers function from these augmented light-front wave functions by applying the overlap formalism [12]. The result is identical to the explicit calculation [5] of the single spin asymmetry in semi-inclusive deep inelastic lepton-polarized nucleon scattering where the required phases come from the final-state rescattering of the struck quark with the nucleon spectators.

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