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Bulk viscosity of gauge theory plasma at strong coupling

Alex Buchel [a,b,*]

[a] Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2J 2W9, Canada
[b] Department of Applied Mathematics, University of Western Ontario, London, Ontario N6A 5B7, Canada

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ABSTRACT

We propose an inequality on bulk viscosity of strongly coupled gauge theory plasmas that allow for a dual supergravity description. Using explicit example of the $\mathcal{N}=2^*$ gauge theory plasma we show that the bulk viscosity remains finite at a critical point with a divergent specific heat. We present an estimate for the bulk viscosity of QGP plasma at RHIC.

Recently, a holographic link between finite temperature gauge theories and string theory black holes emerged as a viable theoretical tool to model properties of strongly coupled quark gluon plasma (QGP) produced at RHIC [1–4]. While the precise holographic dual to QCD is still missing, a progress in study of string theory black holes made it possible to compare the thermodynamics of strongly coupled QCD-like gauge theories [5,6] with lattice results [7]. The dual holographic approach has been successful to address dynamical properties of QGP such as the shear viscosity [8] and the parton jet quenching [9,10], where few alternative techniques are available (see however of [11]). Intriguingly, dual string theory studies reveal certain universal features of gauge theory plasma dynamics. A notable example is the ratio of the shear viscosity η to the entropy density s. It was shown in [12–15] that

$$\frac{\eta}{s}=\frac{1}{4\pi}\quad\longrightarrow\quad\frac{\hbar}{4\pi k_B}\approx 6.08\times 10^{-13}\ \mathrm{K\,s},\tag{1}$$

in any gauge theory plasma at infinite 't Hooft coupling and infinite number of colors (or in the supergravity approximation), irrespectively of the dimensionality of the space, the microscopic scales of the theory, and chemical potentials for the conserved quantities. The universality of the shear viscosity ratio (1) in strongly coupled gauge theories at finite temperature led Kovtun, Son and Starinets (KSS) to conjecture a shear viscosity bound [16]

$$\frac{\eta}{s}\geqslant\frac{1}{4\pi},\tag{2}$$

for all physical systems in Nature. Empirically, the KSS bound indeed appears to be satisfied by all common substances [13]; moreover, it is correct at large (but finite) 't Hooft coupling in $\mathcal{N}=4$ Yang–Mills theory plasma [17,18].

We believe that it is such universal features of dual holographic models of gauge theories that might have some relevance to QCD. Thus, it is imperative to ask what are other generic properties of strongly coupled gauge theories. The question is complicated as neither the bulk viscosity [19] nor the quenching of parton jets [20] is universal for different gauge theory plasmas.

It this Letter we propose an inequality on bulk viscosity ζ of strongly coupled gauge theories that allow for a dual supergravity description. Based on holographically dual computations, we conjecture that a bulk viscosity in a strongly coupled gauge theory plasma in p space dimensions satisfies

$$\frac{\zeta}{\eta}\geqslant 2\left(\frac{1}{p}-c_s^2\right),\tag{3}$$

where c_s is the speed of sound. Notice that unlike the shear viscosity bound (2), our inequality (3) is dynamical: as the temperature varies, generically both the speed of sound and the ratio of bulk to shear viscosities will change. Our conjecture is that the inequality (3) is correct over all range of temperatures, but only in the regime of the validity of the supergravity approximation in the dual holographic description.

In the following we present evidence in support of the bulk viscosity inequality (3). First, we observe that the inequality is saturated by the $p+1$ space–time dimensional gauge theory plasma holographically dual to a stack of near-extremal flat Dp-branes [21], as well as in the hydrodynamics of Little String Theory [21,22]. Second, we point out that the inequality (3) remains saturated once the above p-space dimensional gauge theory is compactified on a $k<p$ space dimensional torus [21,23]. Third, we observe that the inequality is satisfied (but in general not saturated) in certain $3+1$ strongly coupled non-conformal plasma at

* Correspondence address: Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2J 2W9, Canada.
E-mail address: abuchel@perimeterinstitute.ca.

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high temperature [19,24]. Finally, we present results [25] for the bulk viscosity of the $\mathcal{N}=2^*$ gauge theory plasma [5,26–30] over a wide range of temperatures, and for various mass deformation parameters. We find that the bulk viscosity of the $\mathcal{N}=2^*$ plasma satisfies the inequality (3). As observed in [5], the $\mathcal{N}=2^*$ plasma with zero fermion masses undergoes an interesting phase transition with vanishing speed of sound. A detailed analysis of the critical point [25] reveals that at the transition point the specific heat diverges as $c_V \sim |1 - T_c/T|^{-1/2}$. We find that despite the divergent specific heat the bulk viscosity at criticality remains finite. We use results for the $\mathcal{N}=2^*$ gauge theory plasma to estimate the bulk viscosity of QGP at RHIC.

Bulk viscosity of D*p-brane gauge theory plasma.* $\mathcal{N}=4$ Yang–Mills plasma at strong coupling is holographically dual to near-extremal stack of D3-branes. In this case conformal invariance of the theory implies that

$$c_s^2 = \frac{1}{3}, \qquad \zeta = 0. \tag{4}$$

Eq. (4) was verified in supergravity approximation in [31] and beyond the supergravity approximation in [18]. Notice that $\mathcal{N}=4$ plasma trivially satisfies the inequality (3).

In [21] the authors generalized computation of [31] to $p+1$ space–time dimensional gauge theory plasma holographically dual to near-extremal stack of Dp-branes. They found the following dispersion relation for the sound waves

$$\mathfrak{w} = \sqrt{\frac{5-p}{9-p}}\,\mathfrak{q} - i\frac{2}{9-p}\mathfrak{q}^2 + \cdots, \tag{5}$$

where

$$\mathfrak{w} \equiv \frac{\omega}{2\pi T}, \qquad \mathfrak{q} \equiv \frac{q}{2\pi T}. \tag{6}$$

Hydrodynamics of a fluid with shear and bulk viscosities $\{\eta, \xi\}$ in p-space dimensions predicts the following sound wave dispersion

$$\omega = c_s q - i\frac{\eta}{sT}\left(\frac{p-1}{p} + \frac{\zeta}{2\eta}\right)q^2 + \cdots. \tag{7}$$

Using the universality of the shear viscosity (1), one can verify that the inequality (3) is saturated [21] in the hydrodynamics of the flat Dp-branes. It is saturated as well in the hydrodynamics of Little String Theory [21,22].

We point out now that the inequality (3) is saturated as well for above strongly coupled gauge theory plasmas compactified on a k-dimensional torus ($k < p$).[1] Indeed, upon such a compactification the dispersion relation (5) will not change—much like an equation of state it is sensitive only to the local properties of the background geometry:

$$\mathfrak{w}_{k<p} = \sqrt{\frac{5-p}{9-p}}\,\mathfrak{q} - i\frac{2}{9-p}\mathfrak{q}^2 + \cdots. \tag{8}$$

On the other hand, the hydrodynamics relation (7) is sensitive to the number of macroscopic (infinitely extended) directions:

$$\omega_{k<p} = c_s q - i\frac{\eta_{k<p}}{s_{k<p}T}\left(\frac{(p-k)-1}{(p-k)} + \frac{\zeta_{k<p}}{2\eta_{k<p}}\right)q^2 + \cdots. \tag{9}$$

Again, using the universality of the shear viscosity (1) we find (see also Eq. (5.2) of Ref. [21])

$$\frac{\zeta_{k<p}}{\eta_{k<p}} = 2\left(\frac{1}{p-k} - c_s^2\right). \tag{10}$$

[1] A saturation of the inequality (3) upon Kaluza–Klein compactification on k-dimensional torus was also observed in [21].

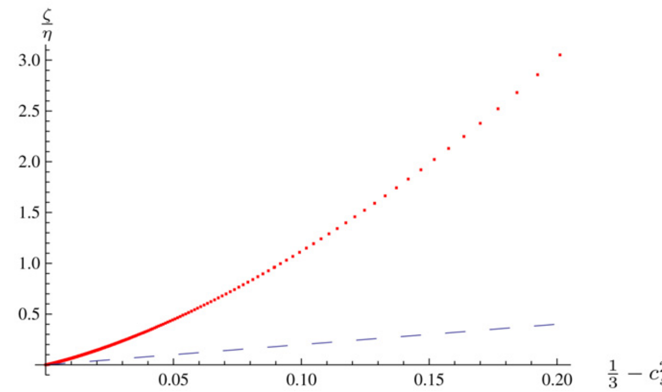

Fig. 1. Ratio of viscosities $\frac{\zeta}{\eta}$ versus the speed of sound in $\mathcal{N}=2^*$ gauge theory plasma with zero fermionic mass deformation parameter $m_f = 0$. The dashed line represents the bulk viscosity inequality (3).

It is precisely for the stated reason the inequality (3) is saturated in Sakai–Sugimoto model in the quenched approximation [23], even though

$$\frac{\zeta}{\eta}\bigg|_{\text{Sakai–Sugimoto}} = \frac{4}{15} \neq \frac{1}{10} = \frac{\zeta}{\eta}\bigg|_{\text{D4}}. \tag{11}$$

Bulk viscosity of non-conformal plasma at high temperatures. A much more nontrivial example is the bulk viscosity of non-conformal gauge theory plasma in four dimensions. The computation in the cascading gauge theory [32,33] produced [24]

$$\frac{\zeta}{\eta}\bigg|_{\text{cascading}} = 2\left(\frac{1}{3} - c_s^2\right) + \mathcal{O}\left(\left[\frac{1}{3} - c_s^2\right]^2 \sim \ln^{-2}\frac{T}{\Lambda}\right), \tag{12}$$

where Λ is the strong coupling scale of the cascading gauge theory.

Likewise, for $\mathcal{N}=2^*$ gauge theory plasma with bosonic and fermionic mass deformation parameters $m_b \ll T$ and $m_f \ll T$,

$$\frac{\zeta}{\eta}\bigg|_{m_f=0} = \frac{\pi^2\beta_b^\Gamma}{16}\left(\frac{1}{3} - c_s^2\right) + \mathcal{O}\left(\left[\frac{1}{3} - c_s^2\right]^2\right), \tag{13}$$

where $\beta_b^\Gamma \approx 8.001$ [19];

$$\frac{\zeta}{\eta}\bigg|_{m_b=0} = \frac{3\pi\beta_f^\Gamma}{2}\left(\frac{1}{3} - c_s^2\right) + \mathcal{O}\left(\left[\frac{1}{3} - c_s^2\right]^2\right), \tag{14}$$

where $\beta_f^\Gamma \approx 0.66666$.[2]

In all cases above we find that the viscosity inequality (3) remains true—in general, it is no longer saturated.

Bulk viscosity of $\mathcal{N}=2^*$ *plasma.* The strongest support for the bulk viscosity inequality (3) comes from study of the $\mathcal{N}=2^*$ bulk viscosity over the wide range of temperatures. Such analysis is a direct extension of the framework presented in [19]. The computations are extremely technical and will be detailed elsewhere [25]. Here, we report only the results of the analysis.[3]

Fig. 1 represents the ratio $\frac{\zeta}{\eta}$ versus the speed of sound in $\mathcal{N}=2^*$ gauge theory plasma with $m_f = 0$. This model reaches a critical point with vanishing speed of sound at $\frac{m_b}{T_c} \approx 2.32591$ [5]. Although near the critical point the specific heat diverges as $c_V \sim |1 - T_c/T|^{-1/2}$ [25] (also Fig. 8 of [5]), we find that the bulk viscosity remains finite, Figs. 2 and 3.

[2] There is a mistake in Eq. (4.37) in [19]: correspondingly to the connection coefficient of dZ_ψ^0/dx in Eq. (4.35), the connection coefficient of dZ_ψ^1/dx in Eq. (4.37) must be $12x^2(x^2-1)^2$. Fixing this mistake leads to the value of β_f^Γ presented [25].

[3] Numerical data is available from the author upon request.

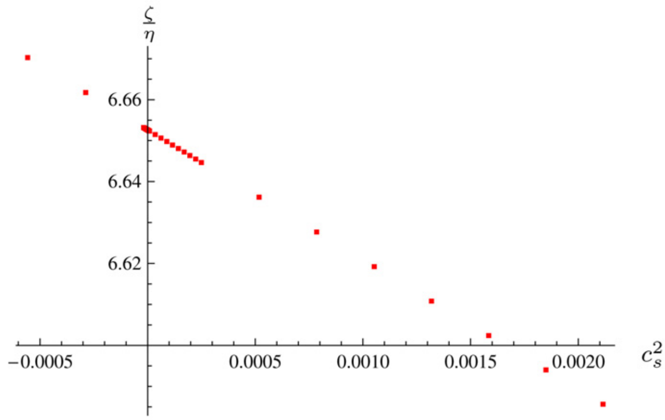

Fig. 2. Ratio of viscosities $\frac{\zeta}{\eta}$ in $\mathcal{N}=2^*$ gauge theory plasma near the critical point. Note that the critical point corresponds to $c_s^2=0$.

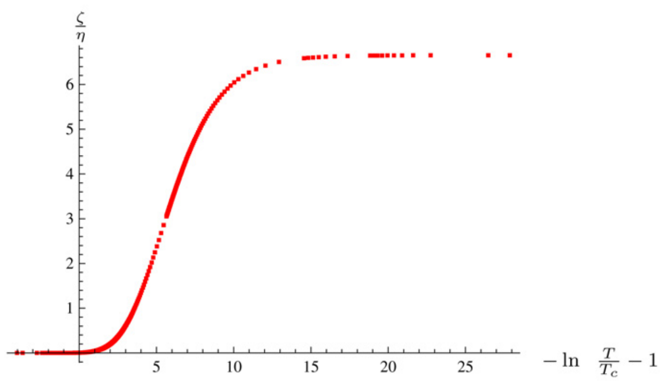

Fig. 3. Ratio of viscosities $\frac{\zeta}{\eta}$ in $\mathcal{N}=2^*$ gauge theory plasma with zero fermionic mass deformation parameter $m_f=0$.

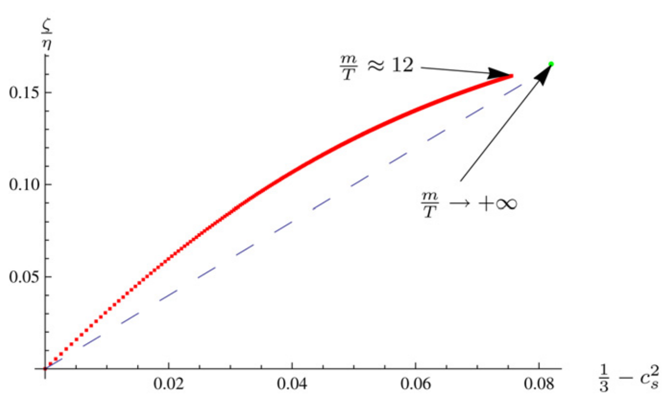

Fig. 4. Ratio of viscosities $\frac{\zeta}{\eta}$ versus the speed of sound in $\mathcal{N}=2^*$ gauge theory plasma with "supersymmetric" mass deformation parameters $m_b=m_f=m$. The dashed line represents the bulk viscosity inequality (3). We computed the bulk viscosity up to $m/T\approx 12$. A single point represents extrapolation of the speed of sound and the viscosity ratio to $T\to +0$.

Fig. 4 represents the ratio $\frac{\zeta}{\eta}$ versus the speed of sound in $\mathcal{N}=2^*$ gauge theory plasma with "supersymmetric" mass deformation parameters $m_b=m_f=m$. We did not find any phase transition in this system up to temperatures $T\approx\frac{m}{12}$.

The dashed line in Figs. 1 and 4 represents the bulk viscosity inequality (3). In both cases the inequality is satisfied.

Estimates for the viscosity of QGP at RHIC. It is tempting to use the $\mathcal{N}=2^*$ strongly coupled gauge theory plasma results to estimate the bulk viscosity of QGP produced at RHIC. For c_s^2 in the range 0.27–0.31, as in QCD at $T=1.5T_{\text{deconfinement}}$ [34,35] we find

$$\frac{\zeta}{\eta}\bigg|_{m_f=0}\approx 0.17\text{–}0.61,\qquad \frac{\zeta}{\eta}\bigg|_{m_b=m_f=m}\approx 0.07\text{–}0.15. \tag{15}$$

Since RHIC produces QGP close to its criticality, we believe that $m_f=0$ $\mathcal{N}=2^*$ gauge theory model would reflect physics more accurately. If so, it is important to reanalyze the hydrodynamics models of QGP with nonzero bulk viscosity in the range given by (15).

In this Letter we presented some evidence in support of the bulk viscosity inequality in strongly coupled gauge theory plasmas. It would be interesting to examine other holographic models and test the inequality. As in [13], it would be interesting to study applicability of the inequality in common substances realized in Nature. It appears that common liquids, like water, satisfy the inequality [36]. While the inequality is generically satisfied in polyatomic gases [37], it is violated in monoatomic gases [38]. The inequality also appears to be violated in high-temperature QCD at weak coupling [39]. In fact, experimental study of the bulk viscosity in argon at different densities [40] demonstrates that its ratio of bulk-to-shear viscosities violates/satisfies the inequality at small/large densities. All this indicates the relevance of the bulk viscosity inequality (3) to strongly coupled systems only.

We demonstrated that the bulk viscosity in the $\mathcal{N}=2^*$ plasma with vanishing fermionic masses has a finite value at the critical point with divergent specific heat. The corresponding critical exponent $\alpha=0.5$ ($c_V\sim|1-T_c/T|^{-\alpha}$) coincides with the mean-field universal value at the tricritical point [41]. Such a tricritical point is realized experimentally in solids [42]. It would be interesting to find a fluid with such a universal tricritical point and compare its bulk viscosity with that of the $\mathcal{N}=2^*$ plasma at criticality. For most physical substances the bulk viscosity is less than the shear viscosity, but for pure fluids at the critical point the bulk viscosity can reach a finite but sharp peak (see [43] for references), much like observed for the $\mathcal{N}=2^*$ plasma here. On the other hand, in mixtures at the critical point the bulk-to-shear ratio can be very large [44].

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