A New Foundation Model for Dynamic Analysis of Beams on Nonlinear Foundation Subjected to a Moving Mass

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Abstract

This paper proposes a new foundation model for dynamic analysis of beam on nonlinear foundation subjected to a moving mass. This model includes the linear and nonlinear Winkler parameters, the linear Pasternak foundation parameter, viscous damping and special consideration of the influence of a foundation mass parameter. The governing equation of motion of the system is formulated by finite element method and dynamic balance of principle and solved by the Newmark method in the time domain. The influences of foundation parameters on dynamic responses of the beam are investigated.

Keywords: Dynamic analysis; moving mass; nonlinear foundation; nonlinear dynamic foundation; foundation mass

1. Introduction

The dynamic responses of structures resting on foundation are commonly used in engineering application. Specifically, beams on such a foundation occupy a prominent place in structural mechanics such as those rated to rails, roadways and runway analysis, and design. Therefore, the dynamic response analysis problems of beams on the foundation under dynamic load have attracted attention of many researchers.

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Recently, the linear and nonlinear elastic foundations with or without viscous damping based on Winkler foundation model (1867) [1] are used in many behavior analysis problems of the beam resting on subgrade such as Kargarnovin et al. (2005) [2], Al-Azzawi et al. (2010) [3], Park and Jang (2014) [4], Castro Jorge et al. (2015) [5]. Although, the Winkler elastic foundation is one of the simplest foundations to describe behavior of soil, but one of the most important deficiencies of the Winkler model is that a displacement discontinuity appears between the loaded and the unloaded part of the foundation surface while the soil surface does not show any discontinuity in reality. To overcome the deficiency of Winkler model, several researchers proposed some different foundation models to describe more real response of soil by introducing some kind of interaction between the independent springs by visualising various types of interconnections without mass density such as: Hetenyi (1946) [6], Pasternak (1954) [7], Reissner (1958) [8], Vlasov and Leontiev (1966) [9]. Hence, the above foundation models also had many researches published in during many last decades such as Matsunaga (1999) [10], Avramidis and Morfidis (2006) [11], Challamela et al. (2010) [12], Ozgan (2012) [13], Calim (2009) [14], Zheng et al. (2014) [15].

In all the foundation models, the concept of a massless foundation has been followed and the effects of the mass density of foundation on the behavior of structures have been neglected. In reality, the foundation has mass density, so that the effect of the density of foundation on dynamic response of structures always exists in during vibration of structures. Hence, the dynamic responses of structures on foundations should be considered with attending of the mass density of foundation. This paper strongly proposes the new foundation model, called “Nonlinear dynamic foundation model” including Winkler nonlinear elastic spring, shear layer of Pasternak, viscous damping and mass density of foundation, and the effects of mass foundation parameters on dynamic responses of beams subjected to a moving mass are investigated clearly.

2. Problem formulation

2.1. A new nonlinear dynamic foundation model

The new nonlinear dynamic foundation model which fully describes dynamic characteristic parameters includes the Winkler linear $k_L$ and nonlinear $k_{NL}$ elastic parameters, the Pasternak shear layer parameter $k_S$, viscous damping $c_f$ and mass density of foundation $\rho_f$. In this model, the mass density of foundation is respectively replaced by lumped mass $m_f$ at the top of the elastic spring, shown in Figure 1. The mass $m_f$ is determined based on the balance principle of kinetic energy of the elastic spring element as considered as a straight rod with unit mass density $\rho_f$, is given by $m_f = \beta \rho_f$ with the dimensionless parameter $\beta$ depending on the effect of depth of foundation $H_f$.

![Fig. 1. The new foundation model (a) the basic model; (b) stress in the shear layer; (c) forces acting on the shear layer.](image)

The pressure-deflection relationship at the time $t$ due to a pressure $q(x, y, t)$ is determined based on dynamic balance principle, can be expressed mathematically as follows

$$\frac{\partial N_{x,t}}{\partial x} + \frac{\partial N_{y,t}}{\partial y} + q(x, y, t) - r_0(x, y, t) - m_0(x, y, t) - c_0(x, y, t) = 0$$  (1)
where

\[
N_{x,i} = \int_{0}^{L} r_{x,i}(z) \, dz = k_{x} \frac{\partial w(x, y, t)}{\partial x} \quad \text{and} \quad N_{y,j} = \int_{0}^{L} r_{y,j}(z) \, dz = k_{y} \frac{\partial w(x, y, t)}{\partial y} \quad \text{with} \quad r_{0}(x, y, t) = k_{L}w(x, y, t) + k_{NL}w^{3}(x, y, t)
\]

\[
m_{0}(x, y, t) = m_{f} \frac{\partial^{2} w(x, y, t)}{\partial t^{2}} \quad \text{and} \quad c_{0}(x, y, t) = c_{f} \frac{\partial w(x, y, t)}{\partial t}
\]

are the reaction of the Winkler elastic spring, inertia force of the density of foundation and viscous damping resistance at each time \(t\), respectively. Substituting equation (2) into equation (1), it can be expressed as follows

\[
q(x, y, t) = k_{eff} w(x, y, t) + c_{f} \frac{\partial w(x, y, t)}{\partial t} + m_{f} \frac{\partial^{2} w(x, y, t)}{\partial t^{2}} - k_{S} \nabla^{2} w(x, y, t)
\]

where \(k_{eff}\) is equivalent stiffness of the nonlinear dynamic foundation, it is determined based on derivative of the elastic foundation reaction \(r_{0}(x, y, t)\) with respect to the deflection of its. It can be seen that the equivalent stiffness \(k_{eff}\) depends mostly on the time history of deflection at each point of the nonlinear dynamic foundation and can be expressed as follows \(k_{eff} = k_{L} + k_{NL}w^{3}(x, y, t)\).

It can be concluded that this model fully describes dynamic characteristic parameters of foundation such as stiffness, damping and mass. This foundation model is similar to Filonenko-Borodich and Pasternak foundation model in case overlooks influence of the viscous damping and density of foundation. Moreover, this foundation model considers to the effect of depth of the foundation while the above different foundation models overlook influence of its excepted the Vlasov foundation model, and thus the two foundation model are homologous together. Therefore, it can be said that the dynamic foundation model accurately represents the characteristics of the soil in using to analyze dynamic responses of structures resting on the foundation.

2.2. Finite element formulation

Consider an Euler-Bernoulli beam on the new nonlinear dynamic foundation model subjected to a moving mass with constant velocity and a two-node uniform beam element, shown in Figure 2. In the Figure, the symbols \(l\), \(\rho\), \(A\), \(E\) and \(I\) are the element length, density, cross-sectional area, modulus of elasticity and inertia moment of area of the beam, respectively.

![Fig. 2. A beam and a two-node beam element to a moving mass.](image)

By means of finite element method, the vertical displacement and acceleration field of the beam element are represented by nodal displacement and acceleration vector as follows, respectively

\[
w_{e} = [N_{w}] \{u\}_{e} \quad \text{and} \quad \ddot{w}_{e} = [N_{w}] \{\ddot{u}\}_{e}
\]
Based on the strain energy, kinetic energy and dissipated energy of the beam element resting on the nonlinear dynamic foundation; stiffness, mass and damping matrices are determined as, respectively

\[
[K]_e = [K]_e^b + [K]_e^F + [K]_e^S, \quad [M]_e = [M]_e^b + [M]_e^F, \quad [C]_e = \int [N_u]^T c_f [N_u] \, dx
\] (5)

where \([K]_e^b\) is the normal bending stiffness matrix; \([K]_e^F\) and \([K]_e^S\) are the nonlinear elastic and shear layer stiffness matrix, respectively, \([M]_e^b\) and \([M]_e^F\) are the normal mass matrix of the beam and foundation element, are given by

\[
[K]_e^F = \int [N_u]^T k_{eff} [N_u] \, dx \quad [K]_e^S = \int [N_u]^T k_s [N_u] \, dx \quad [M]_e^F = \int [N_u]^T m_j [N_u] \, dx
\] (6)

where \([N_u]\) and \([N_s]\) are the matrix of interpolation functions for displacements and rotation, respectively, studied in many research related to finite element method.

### 2.3. Governing equation

The governing differential equation of the beam element \(i^{th}\) without material damping resting on the new foundation subjected to a moving mass \(M\) with constant velocity can be written as

\[
[M]_e \{\ddot{u}_i\} + [K]_e \{u_i\} = -[N_{u-z}]^T f_c
\] (7)

where \([N_{u-z}]\) is the values of the matrix of interpolation function in local coordinate \(\xi\) and \(f_c\) is contact force between the beam resting on the foundation and the moving mass depended on the coordinate \(\xi\) of the position of the moving mass on the beam element at the time \(t\), is given by

\[
f_c = \left( M\ddot{\omega}_c (\xi, t) + Mg \right) \delta (\xi - vt + i^{th}l)
\] (8)

where \(\delta (\xi - vt + i^{th}l)\) is the Dirac delta function. Substituting equations (4) and (8) into equation (7) and rearrangement of this equation gives as

\[
\left( [M]_e + [N_{u-z}]^T M [N_{u-z}] \right) \{\ddot{u}_i\} + [C]_e \{u_i\} + [K]_e \{u_i\} = -[N_{u-z}]^T Mg
\] (9)

By means of the finite element theory, the corresponding degrees of freedom of the stiffness matrix and the mass matrix of the beam element on nonlinear dynamic foundation are connected in the global coordinate. The equation of motion of the beam is defined as follows

\[
[M] \{\ddot{u}\} + [C] \{\dot{u}\} + [K] \{u\} = \{P(t)\}
\] (10)

where \([M]\), \([C]\) and \([K]\) are the overall mass, damping and stiffness matrices of the system, respectively; \(\{u\}\) is the displacement vector, while \(\{P(t)\}\) is the external force vector. It can be seen that the symbols \([M]\) and \([K]\) are called instantaneous matrices because they are time-dependent matrices which are due to the moving mass (matrix \([M]\)) and time histories of deflection at each point of the nonlinear dynamic foundation (matrix \([K]\)). The equation (10) is used for studying the dynamic response of the beam and is solved by Newmark algorithm.
3. Numerical results

3.1. Verified examples

In order to establish the accuracy of the above formulation, the results obtained from the present study are compared with available results in the literature. The first example considers an Euler-Bernoulli beam on the new foundation model without the influence of the nonlinear elastic spring and mass density of foundation with the dimensionless parameters defined as equation (11). The first frequencies of the beam are compared with results in Matsunaga (1999) [10], shown in Table 1.

\[
K_L = \frac{k_L L^4}{EI}, \quad K_{NL} = \frac{k_{NL} L^6}{EI}, \quad K_S = \frac{k_S L^2}{\pi^2 EI}, \quad \lambda = \omega L^2 \sqrt{\frac{\rho A}{EI}}
\]

(11)

Table 1. The first dimensionless natural frequencies $\lambda$ of the beam.

<table>
<thead>
<tr>
<th>L/h</th>
<th>$K_S$</th>
<th>$K_L$</th>
<th>$K_{NL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>10</td>
<td>$10^2$</td>
</tr>
<tr>
<td>Present study</td>
<td>13.9577</td>
<td>14.3115</td>
<td>17.1703</td>
</tr>
</tbody>
</table>

The next example is carried out to verify the present study for a simply supported beam without foundation subjected to a moving load and mass. The time history vertical displacements at the midpoint are plotted in Figure 3 with those of Nguyen and Le (2011) [16] with speed parameter of moving load $SP = v/\tau L$ and Stanisic and Hardin (1969) [17] with ratio moving mass $R = M/\rho AL$.

Through above examples, the numerical results from the program based on the suggested formulation of this paper show quite good agreement with those in the literature. Therefore, this program can be using to analyze the influence of many parameters to dynamic responses of the beams on the nonlinear dynamic foundation subjected to a moving mass in the next part.

3.2. Numerical investigations

Consider a simply supported beam resting on the new foundation model subjected to a moving mass with the following properties of the beam adopted as $L = 5$ m, $h = 0.1$ m, $\rho = 7860$ kg/m$^3$, $E = 206.10^9$ N/m$^2$, $R = 0.5$ and $v = 20$ m/s and foundation parameters as $c_f$, $K_L$, $K_{NL}$ and $K_S$. The Figure 4 shows the time histories of the beam for various $K_L$. The next investigation presents the influences of the foundation mass on a dynamic magnification factors (DMFs), defined as the ratio of maximum dynamic deflection to maximum static deflection at the midpoint.
of the beam, for dimensionless various parameters of the foundation: $K_L$ plotted in Figure 5, $K_{NL}$ as Figure 6, $K_S$ (Figure 7) and $c_f$ (Figure 8). Finally, Figure 9 studies the effects of the foundation mass on DMFs with ratio moving mass $R$ in both case moving load and moving mass.

SP=0.25

Fig. 4. The displacements of the midpoint for various linear stiffness with $K_{NL} = 10^7$, $K_S = 1$, $c_f = 10^2$ Ns/m: (a) $K_L = 25$, (b) $K_L = 50$, (c) $K_L = 100$.

Fig. 5. The influence of the foundation mass on the DMFs for various linear stiffness: (a) $K_L = 25$, (b) $K_L = 50$, (c) $K_L = 100$.

Fig. 6. The influence of the foundation mass on the DMFs for various nonlinear stiffness: (a) $K_{NL} = 10^5$, (b) $K_{NL} = 10^7$, (c) $K_{NL} = 10^9$
Fig. 7. The influence of the foundation mass on the DMFs for various shear layer stiffness: (a) $K_S = 1$, (b) $K_S = 3$, (c) $K_S = 5$.

Fig. 8. The influence of the foundation mass on the DMFs for various damping viscous: (a) $c_f = 0$ Ns/m, (b) $c_f = 5 \times 10^2$ Ns/m, (c) $c_f = 5 \times 10^3$ Ns/m.

Fig. 9. The influence of the foundation mass on the DMFs for various ratio moving mass with $K_s = 25$, $K_M = 10^7$, $K_S = 1$, $c_f = 10^2$ Ns/m: (a) $R = 0.25$, (b) $R = 0.75$, (c) $R = 1$ (--- moving load, ⬇️ moving mass).

As can be expected, the foundation mass parameter affects significantly and increases the DMFs of the beam. The comparisons show that the dynamic response of the beam on the new model and normal nonlinear foundation model (without mass density, $\beta = 0$) have significantly the difference in Figures 5, 6, 7, 8. It also can be seen that the influence of the foundation mass on the DMFs is clearly both case moving load and moving mass in the range of high velocity in Figure 9.

4. Conclusions

The new foundation model which fully describes dynamic parameters including the linear and nonlinear Winkler (normal) parameters, the linear Pasternak (shear) parameter, viscous damping parameter, and special consideration of the influence of mass density (the foundation mass parameter) for dynamic analysis of the beam on foundation to
moving mass has been formulated. The foundation mass parameter has a significant effect on the dynamic response of the beam and the displacements from the nonlinear dynamic foundation model are more increasing than those obtained from the nonlinear foundation model (without influence of the foundation mass).

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