Available online at www.sciencedirect.com
ScienceDirect
Procedia Engineering 119 (2015) 1241 – 1248
Procedia Engineering
www.elsevier.com/locate/procedia

13th Computer Control for Water Industry Conference, CCWI 2015

Graph-theoretic surrogate measures for analysing the resilience of water distribution networks

Manuel Herrera[a,*], Edo Abraham[a], Ivan Stoianov[a]

[a] *InfraSense Labs, Dept. of Civil and Environmental Engineering, Imperial College London, London, UK.*

Abstract

Hydraulic resilience can be formulated as a measure of the ability of a water distribution network to maintain a minimum level of service under operational and failure conditions. This paper explores a hybrid approach to bridge the gap between graph-theoretic and hydraulic measures of resilience. We extend the concept of geodesic distance of a pipeline by taking into account energy losses associated with flow. New random-walk algorithms evaluate hydraulically feasible routes and identify nodes with different levels of hydraulic resilience. The nodes with the lowest scores are further analysed by considering the availability and capacity of their supply routes.

Peer-review under responsibility of the Scientific Committee of CCWI 2015

Keywords: Water distribution systems; network resilience; graph theory; complex networks

1. Introduction

Resilience can be understood as the capacity of a system to maintain its performance level under alterations to its normal status. It measures a system's ability to deal with adverse scenarios by absorbing their undesirable effects on its operations and adapting itself to the new operational environment [1,2]. The analysis of network resilience is becoming increasingly important in ma domains. These range from safety-critical control systems for space shuttles and train networks to ubiquitous computing and communications systems like the Internet [3,4]. For non-engineered and natural systems, this same concept of resilience is also used to measure the ability of humans [5] and ecological systems [6] to effectively deal with the extremes of trauma, stress and disasters.

There are no universally accepted definitions for the concept of water distribution network (WDN) resilience. A common method is based on the work of Todini [7], where hydraulic resilience is formulated as a measure of the capability to get over failure conditions in supply. Todini [7] proposed a resilience index based on the steady state flow analysis of the WDN and the energy dissipated through its pipes. From this perspective, the resilience of a water network is a measure of the surplus energy available in the supply. More recent work builds upon this index, for example: Raad [8] approached surrogate measures of this index focused on network design, di Nardo et al. [9] included the use of this index in their methodology for sectorising a WDN, and Baños et al. [10] proposed an

*Corresponding author.
E-mail address: a.herrera-fernandez@imperial.ac.uk

Peer-review under responsibility of the Scientific Committee of CCWI 2015
doi:10.1016/j.proeng.2015.08.985

extension by considering failures under widely varying water demand scenarios. Other approaches based on steady state hydraulic analysis include Prasad and Park [11], who adapted Todini's index by incorporating the effects of both surplus pressure and reliability of loops assessed by the variability in the diameter of pipes connected to the same node. They also applied it in a multi-objective problem for optimal design and rehabilitation of a water distribution network. Jayaram and Srinivasan [12] extend the approach in [11] enabling it to also evaluate networks with multiple sources. Finally, Wright et al. [13] assess resilience of networks with dynamic topologies by how much demand can be supplied when disruptive events occur.

A common challenge for the above approaches is that the combination of possible failure scenarios grows exponentially as the network becomes larger [14]. In addition to the infeasible computational complexity of a high number of hydraulic simulations, there may also be inconsistencies in accurately modelling disruptive scenarios [15]. Trifunović [16] explores hydraulic properties of the network based on statistical analysis of the common parameters (eg. nodal pressures, demands, etc.) under regular operation and proposed them as indices to assess the resilience of a water network. However, these statistical parameters may not be valid under failure conditions [17]. As an alternative, some purely graph-theoretic indices have been directly related to water network resilience assessment; applying the emerging knowledge to a more resilient expansion of networks [18].

This paper assesses the resilience of WDNs from a hybrid hydraulic-graph-theory point of view by considering energy losses associated with flow as a distance measure between two different points in a WDN. This allows to identify nodes which require large dissipated energy for their supply. A novel measure on node closeness centrality (water-flow closeness, W-Fc) is proposed and developed to quantify this information. In addition to energy losses in supply, some nodes may also have poor connectivity to sources. Consequently, we also compute the availability and capacity of the supply routes to the demand nodes as an additional index. We are especially interested in identifying demand nodes that score low on both resilience measures since this will help in detecting nodes with larger head losses in supply that also have poorest topological connectivity to their water sources. Here we adapt a K-shortest path algorithm to compute connectivity to sources, thereby adding statistical robustness to the measure [19]. Figure 1 summarises this combined process and the way in which we can assess/classify the resilience level for each water network node.

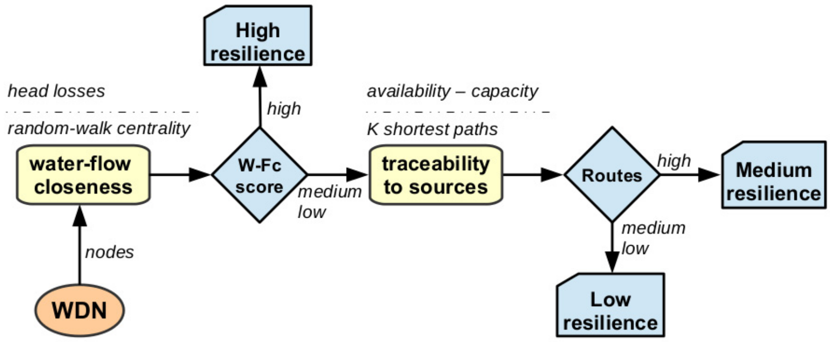

Fig. 1: Flowchart of the process to assess lower resilient for nodes in a WDN

The outline of the rest of the document is as follows: Section 2 introduces novel paradigms on the graph theory perspective for assessing water network resilience. Hybrid hydraulic and graph-theoretic measures based on closeness and also on K shortest paths are presented in this section. An experimental study in Section 3 is used to validate the performance of the new processes proposed based on both a theoretical water system and a real utility network model. Section 4 concludes the paper with a summary of the novel proposals and their advantages.

2. Graph-theoretic framework for assessing water network resilience

Throughout this paper, we consider the abstraction of a WDN as a graph $G = (V, E)$ that is simple, undirected and connected. Where values from steady state flow analysis are used to compute water-flow closeness, a directed graph is considered. The nodes of this graph are of special characteristics: junctions, consumption nodes, and water sources (reservoirs and storage tanks) have different roles in the network that should be taken into account in any further analysis. WDN links are essential in the distribution of water; they represent pipes, valves, and pumps. These links also have physical properties associated with them, indicating the resistance or capacity of each link in the supply of water. For instance, the Darcy-Weisbach friction factor f defines the proportional relationship between pressure gradient (head loss) across a link and the velocity of fluid flowing through it as follows

$$h_L = f \frac{L}{D} \frac{v^2}{2g}, \tag{1}$$

where h_L is the head loss, v velocity, L is the pipe length, D is the pipe diameter and g the acceleration of gravity. The semi-empirical Darcy-Weisbach equation in (1) relates the pressure loss due to frictional forces along a pipe with the average flow velocity. Replacing v in Equation (1) by Q/A, where the cross-sectional area is defined as $A = \frac{\pi D^2}{4}$, the expression can be written in terms of the flow rate, q, rather than velocity v:

$$h_L = rq^2, \tag{2}$$

where the coefficient $r = \frac{8f}{\pi^2 g} \frac{L}{D^5}$ is the pipe resistance.

To quantify resilience or explain the relative importance of demand nodes [20], we utilize physical conservation laws for WDNs. The first of these considers that water demands are distributed along pipes, these demands are lumped at junctions and defined as d_i with $i \in V$. For a junction node i, conservation of mass can be written as:

$$\sum_{j \in J_{in}} q_l - \sum_{j \in J_{out}} q_j = d_i \,, \quad \forall i \in V \tag{3}$$

where d_i is the external demand (withdrawal), J_{in} and J_{out} are the set of pipes supplying to and carrying flow away from node i, respectively. The other equation to be taken into consideration is the conservation of energy. Along the path between nodes A and B that is linked by a pipe, conservation of energy is written as:

$$H_A - H_B = \sum_{j \in I_{jpath}} h_{L,j} = \sum_{j \in I_{jpath}} \mathtt{sign}(q_j) r_j q_j^2, \tag{4}$$

where H_A and H_B are the total energy at nodes A and B; $h_{L,j}$, r_j, and q_j are respectively the head loss, head loss pipe resistance coefficient from Equation (2), and flow rate in pipe j. Equation (4) assumes the use of the Darcy-Weisbach expression to compute the head losses and $\mathtt{sign}(q_j)$ takes into account the flow direction for pipe j in the pressure head calculations.

The equations in (3) and (4) are taken into account in the computation of graph theoretic centrality measures to include both physical and hydraulic laws. This adaptation process is mainly based on weighting the edges of the associated graph of the WDN and will be explained in Subsection 2.1 and Subsection 2.2.

2.1. Water-flow closeness centrality (W-Fc)

The commonly used expression for closeness centrality is based on the length of shortest-paths. Thus, the more central one node is the lower its total proximity to all other nodes. Closeness can be regarded as a measure of the distance taken to pass a specific flow through a network to all other nodes sequentially [21–23]:

$$c_C(v) = \frac{n_C}{\sum_{t \neq v} g(t, v)}, \tag{5}$$

where $g(v, t)$ denotes the length of a shortest path between v and t, and $n_C = (n - 1)$ is a normalizing constant.

In this work we propose to use as the distance between two different points of a WDN the dissipated energy in the water flowing between these nodes shown in Equation (4). Then, W-Fc is defined by

$$c_{WC}(v) = \frac{n_C}{\sum_{t \neq v} \lambda(t, v)}, \tag{6}$$

in which $n_C = (n-1)$ and $\lambda(v,t)$ are related with the head losses by:

$$\lambda(t,v) = \min_{I(t,v)} \left\{ \sum_{j=1}^{m^*} \mathtt{sign}(q_j) r_j q_j^2, \right\} \tag{7}$$

where $I(t,v)$ represents the set of all the paths connecting the nodes t and v, m^* is the number of pipes in $I(v)$.

2.2. A Graph Theory measure of connectivity to sources

For a WDN, we propose to compute a measure of resilience for each node by identifying the number of water sources to which each node is connected and simultaneously assess the available supply for these water sources. This is done by automatically identifying the number of water sources to which each node is hydraulically connected and computing the energy loss associate with the supply. To accomplish this, we compute all routes connecting a node i to all tanks and reservoirs, $s_j(i)$, and quantify the capacity or ease of supply of the corresponding routes. Since computing all the routes linking nodes to sources is prohibitively expensive, our approach is enhanced by restricting the computations to K 'shortest' routes [24] to traverse the network going from any node i to a source $s_j(i)$. A limited number of K, between 5 – 10 % of the number of nodes [19], is enough to find stable and statistically robust measures. In the calculus of this index, every route should be weighted by the frictional resistance which the water transport along it has to overcome; a 'shorter' path is one with a lower resistance in supply or 'capacity' in general sense.

We consider energy loss across a link is proportional to L/D^α; where L is the length of the pipe and D represents its diameter, $\alpha = \{4, 5\}$. This ratio can be used to weight every path in the calculus of shortest paths, providing a measure of energy loss associated with each path connecting a node and its water source; as we can check in Equation (2). The proposed index for a node is

$$I_{GT}(i) = \sum_{s=1}^{S} \left(\frac{1}{K} \sum_{k=1}^{K} \frac{1}{g(k,s)} \right), \tag{8}$$

where S is the total number of sources in the network and $g(k,s)$ is a surrogate for the energy loss (the difficulty of transmission) associated with the k^{th} path to source s. One such measure of energy loss is given as:

$$g(k) = \sum_{m=1}^{M} \frac{L_m}{D_m^\alpha}, \tag{9}$$

where M is the number of pipes in path k.

For a given node i and a corresponding source s, the K-smallest $g(\cdot)$ values are calculated by Eppstein's weighted K-shortest path (KSP) algorithm [24,25]. Equation (9) represents the total energy lost for the k-th node-to-source path. Equation (8) develops a resilience measure for the i-th node, in which K weighted shortest routes are considered for each source (i.e. tanks and reservoirs) together with a weight that measures the capacity for each route. This weighted KSP algorithm returns infinity for (9) if there is no path of water transport connecting the given node with a source. The contribution of such a source to the resilience index for the node in (8) will be zero. Therefore, both the quality of paths to sources and the number of sources are explicitly taken into account by this index. In the following, we study the two proposed indices using water network models of varying size.

3. Experimental study

The experimental study shows the performance of the proposed process to identify low resilience nodes for two WDNs. The first example is the well known C-Town model [26], while the second one is a proprietary model from an operational water utility supply network of a medium size in an urban area in South-West England.

3.1. C-Town network

In order to validate this novel method for analysis networks resilience, we firstly use a well known network of C-Town, see Figure 2. This network consists on 333 nodes, 429 pipes, 4 valves, 5 pump stations, 7 elevated tanks, and 1 reservoir. Figure 2 also shows two areas where nodes with lower W-Fc are predominantly located: north of the layout (`A2` area) and near the reservoir (`A1` area).

- `A1` area: This zone is dependent on the reservoir and any tank or auxiliary water source can not easily reach to its nodes.
- `A2` area: This zone is poorly connected with the rest of the system and its supply is interrupted in case of disruptions on any part of the network.

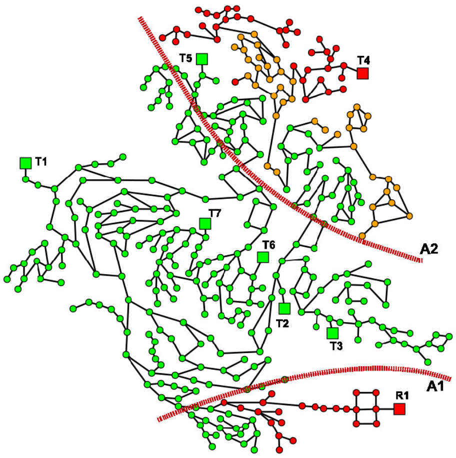

Fig. 2: C-Town layout with lower and medium W-Fc nodes marked in red and orange, respectively.

We also used a critical link analysis process (CLA), based on random pipe failure percolation and the subsequent hydraulic simulations, to obtain information about nodes that are worst affected in the form of insufficient pressures levels, after disruption events in the WDN. The CLA results indicate that 90 % of nodes affected more often by lower pressures, happen to be ranked as medium and lower W-Fc for this WDN, validating the measure.

In Figure 3, we plot the W-Fc and I_{GT} indices on different axis to add a connectivity analysis perspective to the closeness measure to improve the accuracy in identifying low resilient nodes. For example, Figure 3 shows in red all the nodes that have low W-Fc measures, these exist both in `A1` and `A2` sectors. Although the red nodes in both areas have similarly low centrality because they are the periphery of the WDN, the loss associated with their routes to sources may not always be the same. This is clearly demonstrated in Figure 3, where the low W-Fc points in `A1` (top-right) are shown to have very high I_{GT} indices because they nearest to the only water source in the network `R1`. On the other hand, the low W-Fc points in `A2` (bottom right of Figure 3) also have the lowest I_{GT} index as they are furthest away from the source `R1`. These nodes are classified as having the lowest resilience among all nodes of the network; their true resilience will, therefore, depend on the reliability and filling levels of the tanks near `A2`.

3.2. An operational network example

The area of the second case study, referred to as WDN-FL, is fed by 2 reservoirs which supplies water through 2,374 nodes and 2,434 pipes (1,945 pipes and 489 valves). The network is presented in Figure 4.

Critical link analysis process (CLA) was used to identify nodes that are worst affected (with regards to insufficient pressures levels) after disruption events in the WDN. It was observed again that the outcome of W-Fc is directly

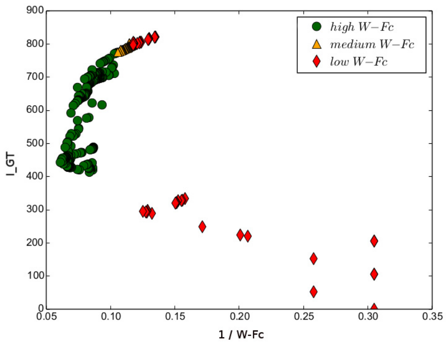

Fig. 3: Comparison of water-flow closeness (W-Fc) and connectivity to water sources (I_{GT}) for all the C-Town network nodes. The values of W-Fc decrease to the right on the x-axis.

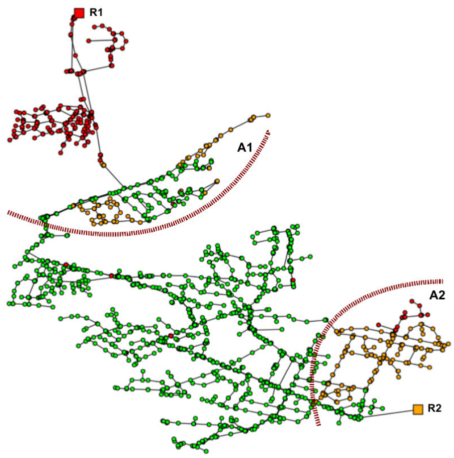

Fig. 4: WDN-FL layout with lower and medium W-Fc nodes marked in red and orange, respectively.

related with the number of times that a node is affected by lower pressure following a failure. For WDN-FL, the CLA results showed that 85 % of nodes affected more often by lower pressure, are ranked as medium and lower W-Fc for this WDN, validating the measure. These nodes are located in the WDN graph (see Figure 4) in two different areas:

- `A1` area (North part of the network near Reservoir `R1`): A major proportion of these nodes are apparently well connected to their reservoir. However, a large distance is traversed with high associated head losses to supply these nodes from the other parts of the network since they are at the extremities of the network.

- `A2` area (South-East extreme): there are nodes naturally supplied by Reservoir `R2` but are closer to the rest of the network compared to those in `A1`. A study of their topological connectivity with the two reservoirs is a must to accurately assess their resilience.

This case study offers an additional context to analyse the resilience measures since this example model has significantly different connectivity and serviceability compared to the first example.The network is a multi-feed feed zone with all its demand nodes hydraulically linked with two different reservoirs. Moreover, the connectivity is far less sparse than the previous example.

Figure 5 shows the W-Fc and I_{GT} indices on different axis for WDN-FL. Most of the red and orange points (i.e low and medium W-Fc nodes, respectively) have higher I_{GT} as they belong to the areas in A1 and A2, respectively, that are close to the sources. Some nodes in these areas but close to the centre of the network have medium levels of reachability to sources and have medium I_{GT} values. The small number of red and orange nodes with low I_{GT} indices are spread around the central part of the network. We can conclude that these nodes have the lowest resilience since they are neither close to the sources and nor have high water flow closeness. This is an advantage of the presented method since these nodes would have been characterized as having high centrality by purely graph-theoretic measures based on geodesic paths.

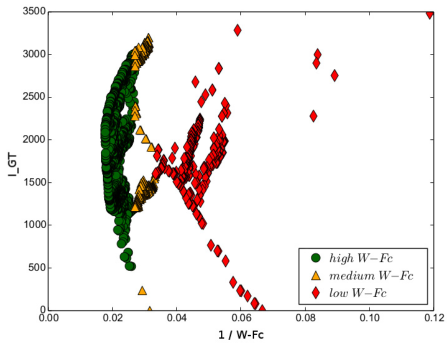

Fig. 5: Comparison of water-flow closeness (W-Fc) and connectivity to water sources (I_{GT}) for all the WDN-FL network nodes. The values of W-Fc decrease to the right on the x-axis.

4. Conclusions

This paper introduces a novel method to assess water network resilience. This method is based on an adaptation of graph theoretic concepts to a WDN that take into account all possible network paths together with hydraulic based information. The introduced measures take further typical graph theory indices that are based on geodesic shortest paths to make them more realistic in approximating true network resilience.

- Water-Flow closeness complements graph centrality analyses suggesting areas of low resilience nodes. Our computations are based on a random-walk closeness measure but using hydraulic head losses for weighting link distances. This measure is efficient and reliable for assessing water network resilience given that it just needs to solve a single steady state hydraulic equations, and does not suffer from the curse of dimensionality in simulating combinations of failures for large scale networks.

- Weighted K-level shortest paths from consumption nodes to water sources have been used to estimate the abundance and capacity of supply routes of nodes to sources. The K-shortest paths are weighted by quantities associated with energy losses of water transport routes, garnering from the flow nature of the derived graphs. The main difference with the single shortest path measures is that the use of K different routes for every node provides statistical robustness to our analysis; besides, water is mainly distributed by alternative pipelines to those just belonging to the shortest path (i.e. low energy loss routes).

The two measures provide complementary perspectives on the reliability of supply to WDN nodes. These measures can help water utilities to identify low resilience areas and to prioritize their operation and management plans (maintaining and rehabilitation plans or control-valves location, among others) which lead to the improvement of reliability in supply. Future work will investigate a comparison of the proposed indices with hydraulic-based methods and a large set of different benchmarking networks.

Acknowledgements

This work was done with the support of the NEC - Imperial research project on “Big data technologies for smart water systems”.

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