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Note on Seiberg duality in matrix model

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Abstract

In this note, a method to derive the Seiberg duality by the matrix model is given. The key fact we used is that effective actions given by matrix models should be identical for both electric and magnetic theories. We demonstrate our method for SQCD with $U(N)$, $SO(N)$ and $Sp(N)$ gauge groups.

Keywords: Matrix model; Seiberg duality

1. Introduction and motivation

The field theory vs. matrix model conjecture proposed by Dijkgraaf and Vafa [1–3] has intrigued a lot of works from various perspectives. The original idea comes from the relationship between field theories and string theories, but later the conjecture is proved by pure field theory methods in [5,6] for adjoint matter[1] and in [7] for massive fundamental flavors and adjoint matter. (The generalization to massless flavors has been given in [19] based on the work of Seiberg [7].) With these achievements, matrix model becomes another alternative way to investigate many interesting problems in fields theories, like the new duality demonstrated in [8] (the generalization to other cases in [9–11]) and related works in [12–14].

Besides these successes of matrix models, we also like to know the limit of the new method. The baryonic deformation has been addressed in [21–24] where it has been shown that although the baryonic deformation makes the boundary condition in matrix models very tricky, there is a way to sum up relative contributions for field theory in matrix model expansions. The multi-trace deformation was investigated in [18,32] where it was pointed out [18] that the direct integration in the multi-trace matrix model does not give back correct results in field theory. But by linearization we can reduce the multi-trace matrix model to the single-trace matrix model, thus the standard result can be applied. Except the adjoint and fundamental flavors, other matter contents, like the symmetric or anti-symmetric representation, have been considered in [25]. We also like to ask what is the correct matrix model description (if it exists) for chiral theories because of their role in phenomenology.

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[1] The method in [5] can be applied to more general situations like multi-adjoint fields and matter in the fundamental representation. We want to thank Cumrun Vafa for informing us these results. In fact, Gukov has checked these results explicitly in an un-published work.

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The question we like to address in the note is the Seiberg duality[2] in matrix model. Seiberg duality of $\mathcal{N}=1$ theories [26–29] is a very nontrivial statement above two different UV theories in IR. It states that these two theories (the electric theory and the magnetic theory) will flow to same (nontrivial) conformal fixed point in IR. With the new method provided by matrix models, it is natural to apply to the Seiberg duality. In [16,17], explicit calculations in matrix models have been done for both electric and magnetic theories of SQCD with mass deformations of quarks and it has been shown that effective actions are same for both theories, thus the Seiberg duality was checked. Generalizations to SO/Sp groups are given in [30,31].

However, as we emphasized in [17], these calculations serve as the check of Seiberg duality and we want to ask more profound question: *could we derive the Seiberg duality from the matrix model*? If we could, the matrix model will be another powerful tool to study the duality in field theory.

Let us analyze this question. The first idea to derive Seiberg duality in matrix model is to try to find a proper transformation of superpotentials in one matrix model to another matrix model. However, it seems this naive method does not work. There are several reasons. First, familiar transformations (like the Legendre transformation) change one theory into another equivalent theory while the dual pair are total different UV theories. This can be seen from another point of view. The dual pair will contribute to same effective action in IR, while the effective action in IR is not directly related to the free energy of matrix models, but through

$$W_{\mathrm{eff}}=N_c\frac{\partial\mathcal{F}_{\chi=2}(S,g)}{\partial S}+\mathcal{F}_{\chi=1}(S,g). \tag{1.1}$$

The relationship (1.1) shows that if $W_e=W_g$, with general different N_c for dual pair we will have $\mathcal{F}_e\neq\mathcal{F}_g$, i.e., they are two different matrix theories with total different free energies.

The second reason can also be seen from (1.1) that the matrix model does not have any memory about the rank of gauge groups. We recover the information of ranks only when we go from the free energy to the effective action where the rank N_c appears as a multiplier. *It tells us that we should not seek to derive the Seiberg duality at the level of free energy* (*or the superpotential of the matrix model*), *but at the level of effective action*. More concretely, starting with two matrix models with superpotential $W_{e,\mathrm{tree}}$ and $W_{g,\mathrm{tree}}$, we do the independent matrix model integrations and calculate effective actions $W_{e,\mathrm{eff}}$ and $W_{g,\mathrm{eff}}$. These effective actions will be functions of glueball field S and other fields as well as coupling constants. The idea is that if we require $W_{e,\mathrm{eff}}\equiv W_{g,\mathrm{eff}}$ as functions of all variables, we may derive the Seiberg duality. We will show that the idea works, at least for these examples we will discuss in this note.

2. The Seiberg dual theory of $U(N_c)$ group

The theory we want to discuss is the $U(N_c)$ gauge group with N_f flavors $Q_i,\tilde{Q}^i$[3] and arbitrary deformation $W_{\mathrm{tree}}=V(M)$ of meson fields $M_i^j=Q_i^\alpha\tilde{Q}_\alpha^j$ where α is color index. The matrix model integration of the prototype has been done in [15] by using the $\delta(M_i^j-Q_i^\alpha\tilde{Q}_\alpha^j)$ insertion and the result is

$$W_{\mathrm{eff}}(S,M)=(N_c-N_f)S\left[1-\log\frac{S}{\Lambda^3}\right]-S\log\left(\frac{\det(M)}{\Lambda^{2N_f}}\right)+V_{\mathrm{tree}}(M). \tag{2.1}$$

It is a very neat result because usually we cannot do the matrix model integration exactly.[4] For this simple example with arbitrary deformation of $V(M)$, (2.1) is exact. As a simple exercise we can take $V(M)=m^i_jQ_i^\alpha\tilde{Q}_\alpha^j=\mathrm{tr}(mM)$ which has been done explicitly in [16]. Eq. (2.1) gives

$$W=(N_c-N_f)S\left[1-\log\frac{S}{\Lambda^3}\right]-S\log\left(\frac{\det(M)}{\Lambda^{2N_f}}\right)+\mathrm{tr}(mM).$$

[2] Seiberg like duality in matrix model has been proved in [3,4]. We want to thank Cumrun Vafa for pointing out this point to us.

[3] Various results in the SQCD like $\mathcal{N}=1$ theory with $U(N)$ gauge group in matrix model can be found in [35].

[4] The matrix model integration of delta-function requires that the rank M of matrix is larger than the number N_f of flavors. Since we have kept N_f fixed while taking the large M limit in the matrix model integration, the condition is satisfied.

Integrating out M by

$$\frac{\partial W}{\partial M} = 0 = -SM^{-1} + m$$

we get

$$W = N_c S\left[1 - \log\frac{S}{\Lambda^3}\right] - S\log\left(\frac{\Lambda^{N_f}}{\det(m)}\right) \tag{2.2}$$

which matches the result in [16].

Now we will apply above general result given by Demasure and Janik to our Seiberg dual pair. First we need to guess the possible matter representations. Since the electric theory has the global $SU(N_f)$ symmetry where the meson fields M is at the adjoint representation, it is not unreasonable to assume that the dual magnetic theory $U(\tilde{N}_c)$ will have N_f flavors $q_i, \tilde{q}^i$, singlets M and proper superpotential $V(q, \tilde{q}, M)$. In principal, there could be other representations, like symmetric or anti-symmetric tensors. However, unlike the singlets M, these tensor fields will effect the integration in the matrix model a lot by interacting with flavors $q_i, \tilde{q}^i$. Because we require the dual pair match for *arbitrary deformation* $V(M)$, it is unlikely to have these tensor fields in the dual magnetic theory.

After constraining our scope of possible matter contents to only fundemantal representations and singlets, all we need to do is to integrate out the magnetic matrix model. Here we have fields $q_i, \tilde{q}^i$ and gauge singlets M. Should we integrate them all in the magnetic matrix model? The answer is no. *We should only integrate out fields $q_i, \tilde{q}^i$ in the matrix model while keeping M as parameters*. The reason is following. According to the field theory analysis in [5–7], because fields M are gauge singlets, we should leave M untouched at the level of free energy and add them back to the effective action directly by the prescription (1.1). This point has also been emphasized in [18, 19]. Using this new understanding, we redo the integration of magnetic matrix model in [16,17] at Appendix to show the consistence.

Since we do not need to integrate fields M, the integration of the magnetic matrix model is same prototype as discussed by Demasure and Janik and we can write down the effective superpotential directly as

$$\begin{aligned} W_{g,\text{eff}}(S, M, \tilde{M}) = (\tilde{N}_c - N_f)\tilde{S}\left[1 - \log\frac{\tilde{S}}{\tilde{\Lambda}^3}\right] \\ - \tilde{S}\log\left(\frac{\det(\tilde{M})}{\tilde{\Lambda}^{2N_f}}\right) + V(M, \tilde{M}), \end{aligned} \tag{2.3}$$

where to distinguish the magnetic theory from the electric theory, we use tilde for fields in the magnetic theory (for example, $\tilde{M}_i^j$ are magnetic meson fields given by $q_i \cdot \tilde{q}^j$). To compare with the electric theory (2.1) we need to integrate out magnetic meson fields $\tilde{M}$.

Now it comes to the key point. *Since we require $W_{e,\text{eff}} = W_{g,\text{eff}}$ for arbitrary deformation $V(M)$*, it is conceivable that we should have $V(M, \tilde{M}) = V(M) + f(M, \tilde{M})$ where $f(M, \tilde{M})$, which describes the interaction of M and $q_i \cdot \tilde{q}^j$, does not depend on the deformation $V(M)$. Because M are gauge singlets and adjoint under the flavor symmetry $SU(N_f)$, the interaction of M and $q_i \cdot \tilde{q}^j$ should be like $\sum \operatorname{tr}(M^{p_1}\tilde{M}^{q_1}M^{p_2}\tilde{M}^{q_2}\cdots)$. Integrating out the magnetic meson $\tilde{M}$, we have the equation

$$\frac{\partial W_g}{\partial \tilde{M}} = 0 = -\tilde{S}\tilde{M}^{-1} + \frac{\partial f(M, \tilde{M})}{\partial \tilde{M}}. \tag{2.4}$$

From (2.4) we suppose to solve $\tilde{M}$, put it back to $W_{g,\text{eff}}$ and compare with $W_{e,\text{eff}}$. Especially, we should have the term $S\log(\det(M))$ by putting $\tilde{M}$ back to the term $\tilde{S}\log(\det(\tilde{M}))$. It is hard to imagine we can have this result unless the solution is $\tilde{M}^{-1} \sim M^n$. In another word,

$$f(M, \tilde{M}) = \operatorname{tr}\left(\tilde{M}\frac{M^n}{\mu^{2n-1}}\right), \tag{2.5}$$

where μ is a scale constant. Under this assumption, we have

$$\tilde{M}^{-1} = \frac{M^n}{\tilde{S}\mu^{2n-1}}. \tag{2.6}$$

Putting it back to $W_{g,\text{eff}}$ and simplifying, we get

$$\begin{aligned} W_{g,\text{eff}} = n\tilde{S}\det(M) + \tilde{N}_c\tilde{S} - \tilde{N}_c\tilde{S}\log\tilde{S} \\ + \tilde{S}\log\frac{\tilde{\Lambda}^{3\tilde{N}_c - N_f}}{(\mu^{2n-1})^{N_f}}, \end{aligned} \tag{2.7}$$

where we have neglected the term $V(M)$ in $W_{g,\text{eff}}$ (we will neglect the same term in $W_{e,\text{eff}}$). The result should be compared with the effective action of the electric theory

$$W_{e,\text{eff}} = -S\det(M) + (N_c - N_f)S - (N_c - N_f)S\log S + S\log \Lambda^{3N_c - N_f} \tag{2.8}$$

which is just the rewritting of Eq. (2.1). Comparing the first term of (2.7) and (2.8) we get the first condition

$$-S = n\tilde{S}. \tag{2.9}$$

Using (2.9) to second and third terms we get

$$\tilde{N}_c = n(N_f - N_c). \tag{2.10}$$

From this we see that n must be positive integer. Comparing the last term we get

$$\Lambda^{3N_c - N_f}\left(\tilde{\Lambda}^{3\tilde{N}_c - N_f}\right)^{\frac{1}{n}} = (-n)^{\frac{-\tilde{N}_c}{n}}\left(\mu^{2n-1}\right)^{\frac{N_f}{n}}. \tag{2.11}$$

Now we need to determine the positive integer n. Using the fact that the dual theory of the dual theory will go back to the original theory

$$S \to \left[\tilde{S} = -\frac{S}{n}\right] \to \left[\tilde{\tilde{S}} = -\frac{\tilde{S}}{n} = \frac{S}{n^2}\right]$$

we should choose $n = 1$. Then Eqs. (2.9)–(2.11) are exactly these relationships connecting the Seiberg dual pair. In another word, under some minor assumptions, we do derive the Seiberg duality from the matrix model.

3. The Seiberg dual theory of $SO(N)$ and $Sp(N)$ groups

The checking of Seiberg duality in matrix models for $SO(N)$ gauge group with N_f flavors Q^j [5] has been done in [30]. The procedure to derive the Seiberg duality will be parallel to the $U(N)$ gauge groups. Using the delta-function technique, the general effective superpotential under arbitrary meson deformations $V(M)$ with $M = Q^j \cdot Q^j$ is given by [30]

$$W_{e,\text{eff}} = \frac{1}{2}(N_c - 2 - N_f)S\left[1 - \log\frac{S}{\Lambda^3}\right] - \frac{S}{2}\log\frac{\det(M)}{\Lambda^{2N_f}} + V(M). \tag{3.1}$$

[5] Other works of SO/Sp groups in matrix models can be found also in [36].

To see this, choosing $V(M) = \frac{1}{2}\operatorname{tr}(mM)$ and minimizing $W_{e,\text{eff}}$ in (3.1) regarding to M we get

$$\frac{\partial W_{e,\text{eff}}}{\partial M} = \frac{-S}{2}M^{-1} + \frac{m}{2} = 0.$$

Putting it back to $W_{e,\text{eff}}$ and simplifying we get

$$W_{e,\text{eff}} = \frac{S}{2}(N_c - 2)\left[1 - \log\frac{S}{\left(\Lambda^{3(N_c-2)-N_f}\det(m)\right)^{\frac{1}{N_c-2}}}\right]$$

which is the result got in [30]. Using similar arguments (i.e., (1) M should not be integrated in matrix model; (2) the matching for arbitrary deformation $V(M)$ and the term $S\log\det(M)$) for the magnetic theory we will have

$$W_{g,\text{eff}} = \frac{1}{2}(\tilde{N}_c - 2 - N_f)\tilde{S}\left[1 - \log\frac{\tilde{S}}{\tilde{\Lambda}^3}\right] - \frac{\tilde{S}}{2}\log\frac{\det(\tilde{M})}{\tilde{\Lambda}^{2N_f}} + V(M) + \frac{1}{2\mu^{2n-1}}\operatorname{tr}(M^n\tilde{M}). \tag{3.2}$$

Integrating out meson fields $\tilde{M}$ we have

$$\frac{\partial W_{g,\text{eff}}}{\partial \tilde{M}} = \frac{\tilde{S}}{2}\tilde{M}^{-1} + \frac{M^n}{2\mu^{2n-1}} = 0. \tag{3.3}$$

Solving $\tilde{M}$ and putting it back we simplify the effective action as (notice that we have neglected the term $V(M)$)

$$W_{g,\text{eff}} = \frac{n\tilde{S}}{2}\log\det(M) + \frac{\tilde{S}}{2}(\tilde{N}_c - 2)(1 - \log\tilde{S}) + \frac{\tilde{S}}{2}\log\frac{\tilde{\Lambda}^{3(\tilde{N}_c-2)-N_f}}{(\mu^{2n-1})^{N_f}} \tag{3.4}$$

which should be compared with

$$W_{e,\text{eff}} = -\frac{S}{2}\log\det(M) + \frac{S}{2}(N_c - N_f - 2)(1 - \log S) + \frac{S}{2}\log\Lambda^{3(N_c-2)-N_f}. \tag{3.5}$$

From the first three terms we get

$$-S = n\tilde{S}, \qquad \tilde{N}_c - 2 = n\big(N_f - (N_c - 2)\big) \tag{3.6}$$

and from the last term we get

$$\Lambda^{3(N_c-2)-N_f}\left(\Lambda^{3(\tilde{N}_c-2)-N_f}\right)^{\frac{1}{n}} = (-n)^{-\frac{\tilde{N}_c-2}{n}}\left(\mu^{2n-1}\right)^{\frac{N_f}{n}}. \tag{3.7}$$

Similar reason as in $U(N_c)$ case tells us to choose $n=1$. In this case, Eqs. (3.6) and (3.7) are exactly the dual relationships of Seiberg dual pair with $SO(N)$ gauge group. Notice that to compare (3.7) with the result in field theory [27], we need to set

$$\Lambda_{\text{matrix}}^{3(N_c-2)-N_f} = 16\Lambda_{\text{field}}^{3(N_c-2)-N_f} \tag{3.8}$$

as noticed in [30].

Comparing above calculation of $SO(N_c)$ with the one of $U(N_c)$, we see that they are same if we make the following replacement $N_c \to N_c-2$. When we discuss the gauge group $Sp(N)$ we just need to use the replacement $N_c \to N_c+2$. With this replacement we will simply write down results. Unlike the $SO(N)$ case where the meson fields $M = Q^i \cdot Q^j$ are symmetric, for $Sp(N)$ (the rank r of $Sp(N)$ is $N/2$) the meson fields $M = Q^i_a Q^j_b J^{ab}$ are anti-symmetric [31] where $J_{ab} = i\sigma_2 \otimes \mathbf{1}_{r\times r}$. The effective superpotential under general meson deformations is

$$W_{e,\text{eff}} = \frac{1}{2}(N_c+2-N_f)S\left[1-\log\frac{S}{\Lambda^3}\right] - \frac{S}{2}\log\frac{\det(M)}{\Lambda^{2N_f}} + V(M). \tag{3.9}$$

Similar reason constraints the effective superpotential for the dual magnetic theory to be

$$W_{g,\text{eff}} = \frac{1}{2}(\tilde{N}_c+2-N_f)\tilde{S}\left[1-\log\frac{\tilde{S}}{\tilde{\Lambda}^3}\right] - \frac{\tilde{S}}{2}\log\frac{\det(\tilde{M})}{\tilde{\Lambda}^{2N_f}} + V(M) + \frac{1}{2\mu^{2n-1}}\operatorname{tr}\left(M^n\tilde{M}\right). \tag{3.10}$$

Integrating out $\tilde{S}$ from (3.10) and comparing with (3.9), we get following dual relationships from matrix models for $Sp(N)$ gauge group

$$-S = n\tilde{S}, \qquad \tilde{N}_c + 2 = n\left(N_f-(N_c+2)\right), \tag{3.11}$$

$$\Lambda^{3(N_c+2)-N_f}\left(\Lambda^{3(\tilde{N}_c+2)-N_f}\right)^{\frac{1}{n}} = (-n)^{-\frac{\tilde{N}_c+2}{n}}\left(\mu^{2n-1}\right)^{\frac{N_f}{n}}. \tag{3.12}$$

The requirement of two time dualities going back to the original theory picks up $n=1$ solution.

These examples we discussed in this Letter are simple and standard. It will be interesting to generalize above method to other dual theories found in field theory, for example, the one discussed by Kutasov and Schwimmer in [33,34]. Unlike these did in this paper for which general effective actions are known by matrix models, we do not know results for generalized Seiberg dual theories at this moment. But if we manage to do it by matrix models, it should be possible to derive the dual theory by the matrix model method.

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Appendix A. Matrix integration in the magnetic theory

For the simplest magnetic theory with mass deformation

$$W_g = \operatorname{tr}(mM) + \frac{1}{\mu}q_i M^i_j \tilde{q}_j \tag{A.1}$$

the matrix integration has been done in [16,17], where we integrated all fields $q, \tilde{q}$ as well as the gauge singlet fields M. However, from the field theory analysis in [5–7] as well as emphasized in [18,19], we should only integrate fields $q, \tilde{q}$ in matrix model and leave terms which are gauge invariant to the effective superpotential. This method has been used to generalize the work of Seiberg [7] with massive flavors to the case of massless flavors in [19] where as a by-product, the original proposal of insertion of delta-function with fundamental flavors [15] has been explained (see also [20] from another point of view about the delta-function). With these new understandings, we should redo the matrix model integration for above magnetic superpotential (A.1). It is similar to the example given in [19], but we include following calculations for completeness which can also be considered as another example for the justification of the delta-function.

Now let us do the calculation. The matrix model integration for $q, \tilde{q}$ can be found in [16] where meson fields $\frac{M_i^j}{\mu}$ have been treated as mass parameters. The result is

$$W_{g,\mathrm{eff}} = \tilde{N}_c \left(\tilde{\Lambda}^{3\tilde{N}_c - N_f} \det\left(\frac{M}{\mu}\right)\right)^{\frac{1}{\tilde{N}_c}} + \mathrm{tr}(mM), \tag{A.2}$$

where the first term comes after integrating out the glueball field $\tilde{S}$ and the second term, from the original tree level superpotential without matrix model integration. The next step is to minimize meson fields M. From (A.2) we have

$$\frac{\partial W_{g,\mathrm{eff}}}{\partial M} = 0 = \left(\tilde{\Lambda}^{3\tilde{N}_c - N_f} \det\left(\frac{M}{\mu}\right)\right)^{\frac{1}{\tilde{N}_c}} M^{-1} + m \tag{A.3}$$

which gives us

$$\det(M)^{\frac{\tilde{N}_c - N_f}{\tilde{N}_c}} = (-)^{N_f} \left(\frac{\tilde{\Lambda}^{3\tilde{N}_c - N_f}}{\mu^{N_f}}\right)^{\frac{N_f}{\tilde{N}_c}} \big(\det(m)\big)^{-1}. \tag{A.4}$$

Putting them back we get

$$\begin{aligned} W_{g,\mathrm{eff}} &= (\tilde{N}_c - N_f)\left(\frac{\tilde{\Lambda}^{3\tilde{N}_c - N_f}}{\mu^{N_f}}\right)^{\frac{1}{\tilde{N}_c}} \det(M)^{\frac{1}{\tilde{N}_c}} \\ &= (\tilde{N}_c - N_f)\left(\frac{\tilde{\Lambda}^{3\tilde{N}_c - N_f}}{\det(-\mu m)}\right)^{\frac{1}{\tilde{N}_c - N_f}} \end{aligned}$$

which is exactly the correct effective superpotential of the magnetic theory.

References

[1] R. Dijkgraaf, C. Vafa, Nucl. Phys. B 644 (2002) 3, hep-th/0206255.
[2] R. Dijkgraaf, C. Vafa, Nucl. Phys. B 644 (2002) 21, hep-th/0207106.
[3] R. Dijkgraaf, C. Vafa, hep-th/0208048.
[4] R. Dijkgraaf, C. Vafa, hep-th/0302011.
[5] R. Dijkgraaf, M.T. Grisaru, C.S. Lam, C. Vafa, D. Zanon, hep-th/0211017.
[6] F. Cachazo, M.R. Douglas, N. Seiberg, E. Witten, hep-th/0211170.
[7] N. Seiberg, hep-th/0212225.
[8] F. Cachazo, N. Seiberg, E. Witten, JHEP 0302 (2003) 042, hep-th/0301006.
[9] C.H. Ahn, Y. Ookouchi, hep-th/0302150.
[10] A. Brandhuber, H. Ita, H. Nieder, Y. Oz, C. Romelsberger, hep-th/0303001.
[11] V. Balasubramanian, B. Feng, M.-x. Huang, A. Naqvi, hep-th/0303065.
[12] F. Ferrari, hep-th/0211069.
[13] F. Ferrari, hep-th/0301157.
[14] T. Friedmann, Nucl. Phys. B 635 (2002) 384, hep-th/0203256.
[15] Y. Demasure, R.A. Janik, hep-th/0211082.
[16] B. Feng, hep-th/0211202.
[17] B. Feng, Y.H. He, hep-th/0211234.
[18] V. Balasubramanian, J. de Boer, B. Feng, Y.H. He, M.-x. Huang, V. Jejjala, A. Naqvi, hep-th/0212082.
[19] B. Feng, hep-th/0212274.
[20] K. Ohta, hep-th/0212025.
[21] R. Argurio, V.L. Campos, G. Ferretti, R. Heise, hep-th/0211249.
[22] I. Bena, R. Roiban, R. Tatar, hep-th/0211271.
[23] H. Suzuki, hep-th/0212121.
[24] I. Bena, H. Murayama, R. Roiban, R. Tatar, hep-th/0303115.
[25] P. Kraus, M. Shigemori, hep-th/0303104.
[26] N. Seiberg, Nucl. Phys. B 435 (1995) 129, hep-th/9411149.
[27] K.A. Intriligator, N. Seiberg, Nucl. Phys. B 444 (1995) 125, hep-th/9503179.
[28] K.A. Intriligator, N. Seiberg, hep-th/9506084.
[29] K.A. Intriligator, N. Seiberg, Nucl. Phys. B (Proc. Suppl.) 45 (1996) 1, hep-th/9509066.
[30] C. Ahn, S. Nam, hep-th/0212231.
[31] C. Ahn, hep-th/0301011.
[32] M. Alishahiha, H. Yavartanoo, hep-th/0303074.
[33] D. Kutasov, Phys. Lett. B 351 (1995) 230, hep-th/9503086.
[34] D. Kutasov, A. Schwimmer, Nucl. Phys. B 442 (1995) 447, hep-th/9501024.
[35] N. Dorey, T.J. Hollowood, S.P. Kumar, A. Sinkovics, hep-th/0209099;
N. Dorey, T.J. Hollowood, S.P. Kumar, A. Sinkovics, hep-th/0209089;
N. Dorey, T.J. Hollowood, S.P. Kumar, hep-th/0210239;
F. Ferrari, hep-th/0210135;
R. Dijkgraaf, S. Gukov, V.A. Kazakov, C. Vafa, hep-th/0210238;
R. Argurio, V.L. Campos, G. Ferretti, R. Heise, hep-th/0210291;
J. McGreevy, hep-th/0211009;
H. Suzuki, hep-th/0211052;
I. Bena, R. Roiban, hep-th/0211075;
R. Gopakumar, hep-th/0211100;
S. Naculich, H. Schnitzer, N. Wyllard, hep-th/0211123;
S. Naculich, H. Schnitzer, N. Wyllard, hep-th/0211254;
R. Dijkgraaf, A. Neitzke, C. Vafa, hep-th/0211194;
H. Itoyama, A. Morozov, hep-th/0211259;
H. Itoyama, A. Morozov, hep-th/0212032;
T.J. Hollowood, hep-th/0212065;
S. Seki, hep-th/0212079;
I. Bena, S. de Haro, R. Roiban, hep-th/0212083;
C. Hofman, hep-th/0212095;
Y. Demasure, R.A. Janik, hep-th/0212212;
T. Mansson, hep-th/0302077;
C.I. Lazaroiu, hep-th/0303008;
D. Berenstein, hep-th/0210183;

D. Berenstein, hep-th/0303033.
[36] H. Fuji, Y. Ookouchi, hep-th/0210148;
H. Fuji, Y. Ookouchi, hep-th/0205301;
H. Ita, H. Nieder, Y. Oz, hep-th/0211261;
Y. Ookouchi, hep-th/0211287;
R.A. Janik, N.A. Obers, hep-th/0212069;
S.K. Ashok, R. Corrado, N. Halmagyi, K.D. Kennaway, C. Romelsberger, hep-th/0211291;
B. Feng, hep-th/0212010;
C. Ahn, S. Nam, hep-th/0301203;
Y. Ookouchi, Y. Watabiki, hep-th/0301226;
R. Abbaspur, A. Imaanpur, S. Parvizi, hep-th/0302083;
A. Klemm, K. Landsteiner, C.I. Lazaroiu, I. Runkel, hep-th/0303032.