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Impact factors for Reggeon-gluon transition in $N = 4$ SYM with large number of colours ☆

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ABSTRACT

We calculate impact factors for Reggeon-gluon transition in supersymmetric Yang–Mills theory with four supercharges at large number of colours N_c. In the next-to-leading order impact factors are not uniquely defined and must accord with BFKL kernels and energy scales. We obtain the impact factor corresponding to the kernel and the energy evolution parameter, which is invariant under Möbius transformation in momentum space, and show that it is also Möbius invariant up to terms taken into account in the BDS ansatz.

1. Introduction

In the BFKL approach [1], impact factors appear as an integral part. Scattering amplitudes of high energy processes are given in this approach by convolutions of Green functions of interacting Reggeized gluons with the impact factors of scattered particles, therefore the notion of these impact factors is well known. Less known are the impact factors for Reggeon-particle (in particular Reggeon-gluon) transitions, where for Reggeon here and in the following we mean Reggeized gluon. They appeared firstly [2] in the proof of the multi-Regge form of QCD amplitudes. An idea of this form is the basis of the BFKL approach. It appeared [3,1] from results of fixed order calculations. Later it was proved in the leading logarithmic approximation (LLA) [4] with the use of the s-channel unitarity. The proof of the multi-Regge form in the next-to-leading approximation (NLA) is based also on the s-channel unitarity [5]. Compatibility of the unitarity with the multi-Regge form leads to bootstrap relations connecting discontinuities of the amplitudes with products of their real parts and gluon trajectories. It turns out [2,5] that the fulfilment of an infinite set of these relations guarantees the multi-Regge form of scattering amplitudes. On the other hand, all bootstrap relations are fulfilled if several conditions imposed on the Reggeon vertices and the trajectory (bootstrap conditions) hold true. The most complicated condition, which includes the impact factors for Reggeon-gluon transition, was proved recently, both in QCD [6–8] and in its supersymmetric generalization [9].

Recently, the impact factors for Reggeon-gluon transition were used for the calculation of the high-energy behavior of the remainder function to the BDS ansatz [10] for multi-particle amplitudes with maximal helicity violation (MHV amplitudes) in Yang–Mills theory, with maximally extended supersymmetry ($N = 4$ SYM) in the limit of large number of colours. It was shown [11] that in the so called Mandelstam kinematical region the BDS amplitude $M^{BDS}_{2\to 4}$ should be multiplied by the factor containing the contribution of the Mandelstam cut, and this contribution for the 6-point scattering amplitude was found in the leading logarithmic approximation (LLA) [12] and in the next-to-leading one (NLA) [13–16].

In the BFKL approach this contribution is given by the convolution of the Green function of two interacting Reggeons with the impact factors for Reggeon-gluon transition. In the NLA the remainder function was calculated [16] assuming the existence of conformal invariant (in momentum space) representations of the modified (i.e. with the subtracted gluon trajectory depending on the total momentum transfer) BFKL kernel for the adjoint representation of the gauge group and impact factors for Reggeon-gluon transition. Later it was shown [17] that indeed the modified BFKL kernel has the conformal invariant representation. As for the impact factors, actually not the impact factors themselves, but the convolution of two impact factors (which was called for brevity also impact factor) was used. Moreover, the convolution used in Ref. [16] was not calculated in the framework of the BFKL approach, but was extracted [14] from the two-loop 6-point

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remainder function obtained in Ref. [18] by simplification of the results of Refs. [19] and [20]. In turn, in the derivation of these results it was supposed that the remainder function appears as expectation value of Wilson loops in $N=4$ SYM. All this makes the direct calculation of the impact factors for Reggeon-gluon transition in the BFKL framework and the investigation of their properties very important.

In this paper we calculate the impact factors for the Reggeon-gluon transition in the next-to-leading order (NLO) for $N=4$ SYM with a large number of colours, i.e. in the planar approximation. As it is well known, the NLO impact factors are not uniquely defined (they are scheme dependent) and must accord with BFKL kernels and energy scales (energy evolution parameters). Our aim is to find the impact factor which corresponds to the conformal invariant kernel found in Ref. [17] and to the energy scale used in Ref. [16]. Just this impact factor, with the deduction of terms contained in the BDS ansatz, is expected to be invariant under Möbius transformation in momentum space, according to the conjecture (not yet proved) about the dual conformal invariance of the remainder function. We reach this aim starting from the impact factor in the "bootstrap scheme", which was found in Refs. [6–9] in Yang–Mills theories with any number of fermions and scalars in arbitrary representations of the gauge group. Using these results and the known relation between the bootstrap scheme and the scheme defined in Ref. [5], which is called standard scheme, we obtain the impact factor for $N=4$ SYM in the last scheme. In this scheme, however, neither the BFKL kernel, nor the energy evolution parameter are Möbius invariant. Therefore, to obtain the impact factor, which is supposed to be Möbius invariant (after subtraction of terms included in the BDS ansatz), one has to transform the standard impact factor so as to accord it with the Möbius invariant kernel found in Ref. [17] and with the Möbius invariant evolution parameter. If the arguments for the dual conformal invariance of the remainder function are correct, the result should be Möbius invariant, up to terms kept in the BDS ansatz. Below we demonstrate that it is the case.

The paper is organized as follows. In the next section we calculate in the planar approximation the impact factor in $N=4$ SYM in the bootstrap scheme. In Section 3 this impact factor is transformed into the standard scheme. In Section 4 the result obtained in Section 3 is transformed into the scheme with conformal kernel and energy evolution parameter (we call this scheme Möbius scheme). Conclusions are drawn in Section 5.

2. The impact factor in the bootstrap scheme

In the Born approximation, with the denotations and state normalizations used in Refs. [5–9], the impact factor for the transition of a Reggeon R with transverse (to the plane of initial momenta p_A, p_B) momentum $\vec{q}_1$ into a gluon G with transverse momentum $\vec{k}$ and polarization vector $e(k)$ in interaction with two Reggeized gluons $\mathcal{G}_1$ and $\mathcal{G}_2$ is written as

$$\langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle^{(B)} = 2g^2\delta(\vec{q}_1-\vec{k}-\vec{r}_1-\vec{r}_2)\big(T^aT^b\big)_{c_1c_2}\vec{e}^{\,*}\vec{C}_1. \tag{1}$$

Here g is the coupling constant, T^i are the colour group generators, $\vec{r}_1$, $\vec{r}_2$ and c_1, c_2 are the transverse momenta and colour indices of the Reggeized gluons $\mathcal{G}_1$ and $\mathcal{G}_2$ correspondingly, a and b are the colour indices of the Reggeon R and the gluon G, $\vec{e}^{\,*}$ is the conjugated transverse part of the polarization vector $e(k)$ in the gauge $e(k)p_2=0$ with the lightcone vector p_2 close to the vector p_B, and

$$\vec{C}_1 = \vec{q}_1 - (\vec{q}_1-\vec{r}_1)\frac{\vec{q}_1^{\,2}}{(\vec{q}_1-\vec{r}_1)^2}. \tag{2}$$

In $N=4$ SYM the NLO impact factor contains gluon, fermion and scalar contributions. These contributions were found in Refs. [7–9] for Yang–Mills theories with any number of fermions and scalars in arbitrary representations of the gauge group.

In general, the impact factors contain two parts with different colour structure. In the planar limit, which we are interested in, only parts with the Born colour structure remain. They are given by Eq. (61) in Ref. [6], Eq. (61) in Ref. [8] and Eq. (123) in Ref. [9] for fermions, gluons and scalars correspondingly. Note however that these equations were derived using the dimensional regularization, which differs from the dimensional reduction used in supersymmetric theories. To take into account this difference we have to take the number n_S of the scalar fields equal to $6-2\epsilon$ (here and below $\epsilon=(D-4)/2$, D being the space–time dimension). With account of this, we obtain (details will be given elsewhere),

$$\langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle = g^2\delta(\vec{q}_1-\vec{k}-\vec{r}_1-\vec{r}_2)\big(T^aT^b\big)_{c_1c_2} \times \vec{e}^{\,*}\big[2\vec{C}_1+\bar{g}^2\vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1*}\big], \tag{3}$$

where

$$\begin{aligned}\vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1*} &= \vec{C}_1\Bigg(\ln\bigg(\frac{(\vec{q}_1-\vec{r}_1)^2}{\vec{k}^2}\bigg)\ln\bigg(\frac{\vec{r}_2^{\,2}}{\vec{k}^2}\bigg) \\ &\quad + \ln\bigg(\frac{(\vec{q}_1-\vec{r}_1)^2\vec{q}_1^{\,2}}{\vec{k}^4}\bigg)\ln\bigg(\frac{\vec{r}_1^{\,2}}{\vec{q}_1^{\,2}}\bigg) - 4\frac{(\vec{k}^2)^\epsilon}{\epsilon^2}+6\zeta(2)\Bigg) \\ &\quad + \vec{C}_2\Bigg(\ln\bigg(\frac{\vec{k}^2}{\vec{r}_2^{\,2}}\bigg)\ln\bigg(\frac{(\vec{q}_1-\vec{r}_1)^2}{\vec{r}_2^{\,2}}\bigg)+\ln\bigg(\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\bigg)\ln\bigg(\frac{\vec{k}^2}{\vec{q}_2^{\,2}}\bigg)\Bigg) \\ &\quad - 2\big[\vec{C}_1\times\big([\vec{k}\times\vec{r}_2]I_{\vec{k},\vec{r}_2}-[\vec{q}_1\times\vec{r}_1]I_{\vec{q}_1,-\vec{r}_1}\big)\big] \\ &\quad + 2\big[\vec{C}_2\times\big([\vec{k}\times\vec{r}_2]I_{\vec{k},\vec{r}_2}+[q_1\times\vec{k}]I_{\vec{q}_1,-\vec{k}}\big)\big].\end{aligned} \tag{4}$$

Here $\bar{g}^2=g^2\Gamma(1-\epsilon)/(4\pi)^{2+\epsilon}$ (note that in the expression (4) and in the following only terms not vanishing at $\epsilon\to 0$ should be kept),

$$\vec{C}_2 = \vec{q}_1 - \vec{k}\frac{\vec{q}_1^{\,2}}{\vec{k}^2}, \tag{5}$$

$\Gamma(x)$ is the Euler gamma-function, $\zeta(n)$ is the Riemann zeta-function ($\zeta(2)=\pi^2/6$), $[\vec{a}\times c[\vec{b}\times\vec{c}]]$ is a double vector product, and

$$I_{\vec{p},\vec{q}} = \int_0^1 \frac{dx}{(\vec{p}+x\vec{q})^2}\ln\bigg(\frac{\vec{p}^{\,2}}{x^2\vec{q}^{\,2}}\bigg),$$
$$I_{\vec{p},\vec{q}} = I_{-\vec{p},-\vec{q}} = I_{\vec{q},\vec{p}} = I_{\vec{p},-\vec{p}-\vec{q}}. \tag{6}$$

Note that the expression (4) is obtained after huge cancellations between gluon, fermion and scalar contributions. In particular, solely due to these cancellations only two vector structures ($\vec{C}_1$ and $\vec{C}_2$) remain; each of the contributions separately contains three independent vector structures.

As it was already mentioned, NLO corrections are scheme dependent. The scheme used in the derivation of $\langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle$ (given in Eqs. (3) and (4)) was adjusted simplifying the verification of the bootstrap conditions (we call it bootstrap scheme). It is different from the scheme defined in Ref. [5] (in turn, we call it standard scheme). The impact factors in these schemes are connected by the transformation [7]

$$\langle GR_1| = \langle GR_1|_s - \langle GR_1|^{(B)}\hat{\mathcal{U}}_k, \tag{7}$$

where the subscript s means the standard scheme and the operator $\hat{\mathcal{U}}_k$ is defined by the matrix elements

$$\langle \mathcal{G}_1'\mathcal{G}_2'|\hat{\mathcal{U}}_k|\mathcal{G}_1\mathcal{G}_2\rangle = \frac{1}{2}\ln\left(\frac{\vec{k}^2}{(\vec{r}_1-\vec{r}_1')^2}\right)\langle \mathcal{G}_1'\mathcal{G}_2'|\hat{\mathcal{K}}_r^B|\mathcal{G}_1\mathcal{G}_2\rangle. \tag{8}$$

Here $\hat{\mathcal{K}}_r^B$ is the part of the LO BFKL kernel related to the real gluon production:

$$\langle \mathcal{G}_1'\mathcal{G}_2'|\hat{\mathcal{K}}_r^B|\mathcal{G}_1\mathcal{G}_2\rangle = \delta(\vec{r}_1'+\vec{r}_2'-\vec{r}_1-\vec{r}_2)\frac{g^2}{(2\pi)^{D-1}}T^i_{c_1c_1'}T^i_{c_2'c_2} \times\left(\frac{\vec{r}_1^{\,2}\vec{r}_2^{\,\prime 2}+\vec{r}_2^{\,2}\vec{r}_1^{\,\prime 2}}{\vec{l}^{\,2}}-\vec{q}_2^{\,2}\right), \tag{9}$$

where $\vec{l}=\vec{r}_1-\vec{r}_1'=\vec{r}_2'-\vec{r}_2$, $\vec{q}_2=\vec{r}_1+\vec{r}_2=\vec{r}_1'+\vec{r}_2'$.

3. Transformation to the standard scheme

From Eqs. (1)–(3) and (7)–(9) it follows that at large number N_c of colours we can write

$$\vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1s} = \vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1*}+\vec{\mathcal{I}}_1,$$

$$\vec{\mathcal{I}}_1 = \int\frac{d\vec{l}}{\Gamma(1-\epsilon)\pi^{1+\epsilon}}\vec{C}_1'\frac{1}{\vec{r}_1^{\,\prime 2}\vec{r}_2^{\,\prime 2}}\left(\frac{\vec{r}_1^{\,2}\vec{r}_2^{\,\prime 2}+\vec{r}_2^{\,2}\vec{r}_1^{\,\prime 2}}{\vec{l}^{\,2}}-\vec{q}_2^{\,2}\right)\times\ln\left(\frac{\vec{k}^2}{\vec{l}^{\,2}}\right), \tag{10}$$

where

$$\vec{C}_1' = \vec{q}_1-\vec{q}_1^{\,2}\frac{(\vec{q}_1-\vec{r}_1')}{(\vec{q}_1-\vec{r}_1')^2}. \tag{11}$$

When $\epsilon\to 0$, the integral $\vec{\mathcal{I}}_1$ is infrared divergent at $\vec{l}=0$. To calculate this integral, it is convenient to use the decomposition

$$\vec{C}_1' = \vec{C}_1+\vec{\Delta}_1,\qquad \vec{\Delta}_1 = \vec{q}_1^{\,2}\left(\frac{(\vec{q}_1-\vec{r}_1)}{(\vec{q}_1-\vec{r}_1)^2}-\frac{(\vec{q}_1-\vec{r}_1')}{(\vec{q}_1-\vec{r}_1')^2}\right). \tag{12}$$

Then, the divergency will appear only in the term with $\vec{C}_1$, which does not depend on $\vec{l}$ and can be taken outside of the integral sign. After that, using the basic integrals

$$\int\frac{d\vec{l}}{\Gamma(1-\epsilon)\pi^{1+\epsilon}}\frac{1}{(\vec{q}-\vec{l})^2(\vec{p}+\vec{l})^2}\ln\left(\frac{\vec{l}^{\,2}}{\mu^2}\right) = \left((\vec{q}+\vec{p})^2\right)^{\epsilon-1}\left[\frac{1}{\epsilon}\ln\left(\frac{\vec{p}^{\,2}\vec{q}^{\,2}}{\mu^4}\right)+\frac{1}{2}\ln^2\left(\frac{\vec{p}^{\,2}}{\vec{q}^{\,2}}\right)\right]+\mathcal{O}(\epsilon),$$

$$\int\frac{d\vec{l}}{\Gamma(1-\epsilon)\pi^{1+\epsilon}}\frac{1}{\vec{l}^{\,2}(\vec{q}-\vec{l})^2}\ln\left(\frac{\vec{l}^{\,2}}{\mu^2}\right) = \left(\vec{q}^{\,2}\right)^{\epsilon-1}\left[-\frac{1}{\epsilon^2}+\zeta(2)+\frac{2}{\epsilon}\ln\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)\right]+\mathcal{O}(\epsilon), \tag{13}$$

we obtain

$$\int\frac{d\vec{l}}{\Gamma(1-\epsilon)\pi^{1+\epsilon}}\frac{1}{\vec{r}_1^{\,\prime 2}\vec{r}_2^{\,\prime 2}}\left(\frac{\vec{r}_1^{\,2}\vec{r}_2^{\,\prime 2}+\vec{r}_2^{\,2}\vec{r}_1^{\,\prime 2}}{\vec{l}^{\,2}}-\vec{q}_2^{\,2}\right)\ln\left(\frac{\vec{k}^2}{\vec{l}^{\,2}}\right) = 2\frac{(\vec{k}^2)^\epsilon}{\epsilon^2}-2\zeta(2)-\ln^2\left(\frac{(\vec{r}_1+\vec{r}_2)^2}{\vec{k}^2}\right) - \ln\left(\frac{\vec{r}_1^{\,2}\vec{r}_2^{\,2}}{(\vec{k}^2)^2}\right)\ln\left(\frac{\vec{r}_1^{\,2}\vec{r}_2^{\,2}}{(\vec{r}_1+\vec{r}_2)^4}\right) + \ln\left(\frac{\vec{r}_1^{\,2}}{(\vec{r}_1+\vec{r}_2)^2}\right)\ln\left(\frac{\vec{r}_2^{\,2}}{(\vec{r}_1+\vec{r}_2)^2}\right). \tag{14}$$

The integral with $\vec{\Delta}_1$ is infrared finite and can be calculated at $\epsilon=0$. It is convenient to calculate it using "helical" vector components $\pm$ instead of the Cartesian ones x, y ($a^\pm=a_x\pm ia_y$) and the decomposition

$$\frac{1}{\vec{q}_1^{\,2}}\Delta_1^+\frac{1}{\vec{r}_1^{\,\prime 2}\vec{r}_2^{\,\prime 2}}\left(\frac{\vec{r}_1^{\,2}\vec{r}_2^{\,\prime 2}+\vec{r}_2^{\,2}\vec{r}_1^{\,\prime 2}}{\vec{l}^{\,2}}-\vec{q}_2^{\,2}\right) = \frac{1}{(q_1-r_1)^-}\times\left[\frac{r_2^-}{k^-}\left(\frac{1}{(r_1-l)^+}+\frac{1}{l^+}\right)\left(\frac{1}{(r_2+l)^-}-\frac{1}{(q_1-r_1+l)^-}\right)+\frac{r_1^-}{q_1^-}\left(\frac{1}{l^+}-\frac{1}{(r_2+l)^+}\right)\left(\frac{1}{(r_1-l)^-}+\frac{1}{(q_1-r_1+l)^-}\right)\right]. \tag{15}$$

Note that each term in this decomposition gives an ultraviolet divergent contribution to the integral (10) (of course, the total integral is ultraviolet convergent). Therefore, we introduce the ultraviolet cut-off $\Lambda\to\infty$. Integrals with separate terms in the decomposition (15) are calculated using the basic integral

$$\int\frac{d\vec{l}}{\pi}\frac{1}{(a-l)^+}\frac{1}{(b-l)^-}\ln\left(\frac{\vec{l}^{\,2}}{\mu^2}\right)\theta\left(\Lambda^2-\vec{l}^{\,2}\right) = \frac{1}{2}\ln\left(\frac{\Lambda^2}{(\vec{a}-\vec{b})^2}\right)\ln\left(\frac{\Lambda^2(\vec{a}-\vec{b})^2}{\mu^4}\right)+\frac{1}{2}\ln\left(\frac{(\vec{a}-\vec{b})^2}{\vec{b}^{\,2}}\right)\ln\left(\frac{(\vec{a}-\vec{b})^2}{\vec{a}^{\,2}}\right)+\frac{a^+b^--a^-b^+}{2}I_{\vec{a},-\vec{b}}, \tag{16}$$

where $I_{\vec{a},\vec{b}}$ is defined in Eq. (6). With the help of this integral, one has

$$\int\frac{d\vec{l}}{\pi}\vec{\Delta}_1\frac{1}{\vec{r}_1^{\,\prime 2}\vec{r}_2^{\,\prime 2}}\left(\frac{\vec{r}_1^{\,2}\vec{r}_2^{\,\prime 2}+\vec{r}_2^{\,2}\vec{r}_1^{\,\prime 2}}{\vec{l}^{\,2}}-\vec{q}_2^{\,2}\right)\ln\left(\frac{\vec{k}^2}{\vec{l}^{\,2}}\right) = \frac{1}{2}\vec{C}_1\left(\ln\frac{\vec{q}_2^{\,2}}{\vec{r}_1^{\,2}}\ln\frac{\vec{k}^4}{\vec{r}_1^{\,2}\vec{r}_2^{\,2}}+\ln\frac{(\vec{q}_1-\vec{r}_1)^2}{\vec{k}^2}\ln\frac{\vec{k}^4}{(\vec{q}_1-\vec{r}_1)^2\vec{r}_2^{\,2}}\right)+\frac{1}{2}(\vec{C}_1-\vec{C}_2)\times\left(\ln\frac{\vec{q}_2^{\,2}}{\vec{r}_2^{\,2}}\ln\frac{\vec{k}^4}{\vec{r}_1^{\,2}\vec{r}_2^{\,2}}+\ln\frac{(\vec{q}_1-\vec{r}_1)^2}{\vec{q}_1^{\,2}}\ln\frac{\vec{k}^4}{\vec{r}_1^{\,2}(\vec{q}_1-\vec{r}_1)^2}\right)+\left[\vec{C}_1\times[\vec{k}\times\vec{r}_2]\right]I_{\vec{k},\vec{r}_2}+\left[\vec{C}_2\times[\vec{r}_1\times\vec{r}_2]\right]I_{\vec{r}_1,\vec{r}_2}-\left[(\vec{C}_1-\vec{C}_2)\times[\vec{q}_1\times\vec{r}_1]\right]I_{\vec{q}_1,-\vec{r}_1}. \tag{17}$$

Using Eqs. (14) and (17) we obtain

$$\vec{\mathcal{I}}_1 = \frac{1}{2}\vec{C}_1\left[\ln\left(\frac{\vec{r}_2^{\,2}}{\vec{k}^2}\right)\ln\left(\frac{\vec{k}^4}{(\vec{q}_1-\vec{r}_1)^2\vec{r}_2^{\,2}}\right)+\ln\left(\frac{\vec{r}_1^{\,2}}{\vec{k}^2}\right)\ln\left(\frac{\vec{k}^2\vec{q}_1^{\,2}}{(\vec{q}_1-\vec{r}_1)^2\vec{r}_1^{\,2}}\right)-\ln\left(\frac{\vec{k}^2}{(\vec{q}_1-\vec{r}_1)^2}\right)\ln\left(\frac{\vec{k}^2\vec{q}_1^{\,2}}{(\vec{q}_1-\vec{r}_1)^4}\right)+4\frac{(\vec{k}^2)^\epsilon}{\epsilon^2}-4\zeta(2)\right] - \frac{1}{2}\vec{C}_2\left[\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{r}_2^{\,2}}\right)\ln\left(\frac{\vec{k}^4}{\vec{r}_1^{\,2}\vec{r}_2^{\,2}}\right)+\ln\left(\frac{(\vec{q}_1-\vec{r}_1)^2}{\vec{q}_1^{\,2}}\right)\ln\left(\frac{\vec{k}^4}{\vec{r}_1^{\,2}(\vec{q}_1-\vec{r}_1)^2}\right)\right]$$

$$+\big[\vec C_1\times\big([\vec k\times\vec r_2]I_{\vec k,\vec r_2}-[\vec q_1\times\vec r_1]I_{\vec q_1,-\vec r_1}\big)\big]$$
$$+\big[\vec C_2\times\big([\vec r_1\times\vec r_2]I_{\vec r_1,\vec r_2}+[\vec q_1\times\vec r_1]I_{\vec q_1,-\vec r_1}\big)\big]. \tag{18}$$

The one-loop correction to the impact factor in the standard scheme is given by Eqs. (10), (4) and (18) and reads

$$\begin{aligned}\vec\Phi^{\mathcal{G}_1\mathcal{G}_2}_{GR_1s}=\frac{1}{2}\vec C_1\Bigg[&\ln\left(\frac{\vec q_1^{\,2}}{\vec r_1^{\,2}}\right)\ln\left(\frac{\vec k^{\,2}\vec r_1^{\,2}}{\vec q_1^{\,4}}\right)\\&+\ln\left(\frac{(\vec q_1-\vec r_1)^2}{\vec k^{\,2}}\right)\ln\left(\frac{\vec k^{\,4}\vec r_1^{\,2}}{(\vec q_1-\vec r_1)^4\vec q_1^{\,2}}\right)\\&+\ln\left(\frac{\vec r_2^{\,2}}{\vec k^{\,2}}\right)\ln\left(\frac{(\vec q_1-\vec r_1)^2}{\vec r_2^{\,2}}\right)-4\frac{(\vec k^{\,2})^\epsilon}{\epsilon^2}+8\zeta(2)\Bigg]\\&+\big[\vec C_1\times\big([\vec q_1\times\vec r_1]I_{\vec q_1,-\vec r_1}-[\vec k\times r_2]I_{\vec k,\vec r_2}\big)\big]\\&+\frac{1}{2}\vec C_2\Bigg[\ln\left(\frac{\vec q_2^{\,2}}{\vec q_1^{\,2}}\right)\ln\left(\frac{\vec r_1^{\,2}\vec r_2^{\,2}}{\vec q_2^{\,4}}\right)\\&+\ln\left(\frac{\vec r_2^{\,2}}{(\vec q_1-\vec r_1)^2}\right)\ln\left(\frac{\vec r_2^{\,2}\vec q_1^{\,2}}{\vec r_1^{\,2}(\vec q_1-\vec r_1)^2}\right)\Bigg]\\&+\big[\vec C_2\times\big([\vec r_1\times\vec r_2]I_{\vec r_1,\vec r_2}+[\vec q_1\times\vec r_1]I_{\vec q_1,-\vec r_1}\\&+2[\vec k\times\vec r_2]I_{\vec k,\vec r_2}+2[\vec q_1\times\vec k]I_{\vec q_1,-\vec k}\big)\big].\end{aligned}\tag{19}$$

The correction (19) fit the standard kernel [21] and the energy scale $|\vec k_1||\vec k_2|$, where $\vec k_{1,2}$ are the transverse momenta of produced gluons in the two impact factors connected by the Green function of the two interacting Reggeons (BFKL ladder).

4. The impact factor in the Möbius scheme

The impact factor in the Möbius scheme means the impact factor for Reggeon-gluon transition which can be used for the calculation of the remainder function with conformal invariant kernel and energy evolution parameter. Let us remind here that the kernel used for the calculation of the remainder function [11–16] (which is called modified kernel) is the BFKL kernel in $N=4$ SYM for the adjoint representation of the gauge group with the subtracted gluon trajectory depending on the total momentum transfer (the subtraction is made to avoid double counting of terms included in the BDS ansatz).

To obtain the impact factor in the Möbius scheme from the correction (19) we have to perform two transformations, to reconcile the impact factor with the kernel and the energy scale. As it was shown in Ref. [17], the conformal invariant $\hat{\mathcal{K}}_c$ and the standard $\hat{\mathcal{K}}_m$ forms of the modified kernel are connected by the similarity transformation

$$\hat{\mathcal{K}}_c=\hat{\mathcal{K}}_m-\frac{1}{4}\big[\hat{\mathcal{K}}^B,\big[\ln\big(\hat{\vec q}_1^{\,2}\hat{\vec q}_2^{\,2}\big),\hat{\mathcal{K}}^B\big]\big], \tag{20}$$

where $\hat{\mathcal{K}}^B$ is the usual LO kernel and $\hat{\vec q}_{1,2}$ are the operators of the Reggeon momenta. Note that in the commutator there is no difference between the usual and modified kernels, so that $\hat{\mathcal{K}}^B$ is taken instead of $\hat{\mathcal{K}}^B_m$. The corresponding transformation for the impact factor is

$$\langle GR_1|_t=\langle GR_1|_s-\frac{1}{4}\langle GR_1|^{(B)}\big[\ln\big(\hat{\vec q}_1^{\,2}\hat{\vec q}_2^{\,2}\big),\hat{\mathcal{K}}^{(B)}\big], \tag{21}$$

where the subscript t means transformed to fit the conformal kernel. For the NLO correction we obtain

$$\vec\Phi^{\mathcal{G}\mathcal{G}_2}_{GR_1t}=\vec\Phi^{\mathcal{G}\mathcal{G}_2}_{GR_1s}+\vec{\mathcal{I}}_2,$$

$$\vec{\mathcal{I}}_2=\frac{1}{2}\int\frac{d\vec l}{\Gamma(1-\epsilon)\pi^{1+\epsilon}}\vec C_1'\frac{1}{\vec r_1^{\,\prime 2}\vec r_2^{\,\prime 2}}\left(\frac{\vec r_1^{\,2}\vec r_2^{\,\prime 2}+\vec r_2^{\,2}\vec r_1^{\,\prime 2}}{\vec l^{\,2}}-\vec q_2^{\,2}\right)\times\ln\left(\frac{\vec r_1^{\,2}\vec r_2^{\,2}}{\vec r_1^{\,\prime 2}\vec r_2^{\,\prime 2}}\right). \tag{22}$$

This integral is infrared finite and can be calculated in two-dimensional space, with the help of the decomposition (15), the decomposition

$$\begin{aligned}&\frac{1}{\vec r_1^{\,\prime 2}\vec r_2^{\,\prime 2}}\left(\frac{\vec r_1^{\,2}\vec r_2^{\,\prime 2}+\vec r_2^{\,2}\vec r_1^{\,\prime 2}}{\vec l^{\,2}}-\vec q_2^{\,2}\right)\\&=-\left(\frac{1}{(r_1-l)^+}+\frac{1}{l^+}\right)\left(\frac{1}{(r_2+l)^-}-\frac{1}{l^-}\right)\\&\quad-\left(\frac{1}{(r_1-l)^-}+\frac{1}{l^-}\right)\left(\frac{1}{(r_2+l)^+}-\frac{1}{l^+}\right)\end{aligned}\tag{23}$$

and the integral (16). Using the result of integration,

$$\begin{aligned}\vec{\mathcal{I}}_2=\frac{1}{4}\vec C_2\Bigg[&\ln\left(\frac{\vec q_2^{\,2}}{\vec r_2^{\,2}}\right)\ln\left(\frac{\vec q_2^{\,4}}{\vec r_1^{\,2}\vec r_2^{\,2}}\right)+\ln\left(\frac{\vec q_1^{\,2}}{\vec k^{\,2}}\right)\ln\left(\frac{\vec r_2^{\,2}}{\vec q_2^{\,2}}\right)\\&+\ln\left(\frac{(\vec q_1-\vec r_1)^2}{\vec q_1^{\,2}}\right)\ln\left(\frac{\vec k^{\,2}\vec q_1^{\,2}}{\vec r_1^{\,2}\vec r_2^{\,2}}\right)\Bigg]\\-\frac{1}{4}\vec C_1\Bigg[&\ln\left(\frac{\vec r_1^{\,2}}{\vec r_2^{\,2}}\right)\ln\left(\frac{\vec k^{\,2}\vec r_1^{\,2}}{\vec q_1^{\,2}\vec r_2^{\,2}}\right)\\&+\ln\left(\frac{\vec k^{\,2}\vec q_1^{\,2}}{\vec r_1^{\,2}\vec r_2^{\,2}}\right)\ln\left(\frac{(\vec q_1-\vec r_1)^4}{\vec k^{\,2}\vec q_1^{\,2}}\right)\Bigg]\\&+\big[\vec C_1\times\big([\vec k\times\vec r_2]I_{\vec k,\vec r_2}-[\vec q_1\times\vec r_1]I_{\vec q_1,-\vec r_1}\big)\big]\\&+\frac{1}{2}\big[\vec C_2\times\big([\vec r_1\times\vec r_2]I_{\vec r_1,\vec r_2}+[\vec q_1\times\vec r_1]I_{\vec q_1,-\vec r_1}\\&-[\vec k\times\vec r_2]I_{\vec k,\vec r_2}-[\vec q_1\times\vec k]I_{\vec q_1,-\vec k}\big)\big],\end{aligned}\tag{24}$$

we obtain the correction to the transformed impact factor:

$$\begin{aligned}\vec\Phi^{\mathcal{G}_1\mathcal{G}_2}_{GR_1t}=\vec C_1\Bigg[&\ln\left(\frac{\vec q_2^{\,2}}{\vec q_1^{\,2}}\right)\ln\left(\frac{\vec q_2^{\,4}(\vec q_1-\vec r_1)^4}{\vec q_1^{\,2}\vec r_2^{\,2}\vec k^{\,2}\vec r_1^{\,2}}\right)\\&-\ln\left(\frac{\vec q_2^{\,2}(\vec q_1-\vec r_1)^2}{\vec q_1^{\,2}\vec r_2^{\,2}}\right)\ln\left(\frac{\vec q_2^{\,2}(\vec q_1-\vec r_1)^2}{\vec k^{\,2}\vec r_1^{\,2}}\right)\\&-\frac{3}{4}\ln^2\left(\frac{\vec k^{\,2}\vec r_1^{\,2}}{\vec q_1^{\,2}\vec r_2^{\,2}}\right)-\ln^2\left(\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}}\right)-2\frac{(\vec k^{\,2})^\epsilon}{\epsilon^2}+4\zeta(2)\Bigg]\\&+\frac{1}{4}\vec C_2\ln\left(\frac{\vec q_2^{\,2}(\vec q_1-\vec r_1)^2}{\vec q_1^{\,2}\vec r_2^{\,2}}\right)\ln\left(\frac{(\vec q_1-\vec r_1)^4\vec q_1^{\,2}\vec k^{\,2}\vec r_1^{\,2}}{\vec r_2^{\,6}\vec q_2^{\,4}}\right)\\&+\frac{3}{2}\big[\vec C_2\times\big([\vec r_1\times\vec r_2]I_{\vec r_1,\vec r_2}+[\vec q_1\times\vec r_1]I_{\vec q_1,-\vec r_1}\\&+[\vec k\times\vec r_2]I_{\vec k,\vec r_2}+[\vec q_1\times\vec k]I_{\vec q_1,-\vec k}\big)\big].\end{aligned}\tag{25}$$

The Möbius invariant kernel was used for the calculation of the NLO remainder function in Ref. [16] with the Möbius invariant convolution of the NLO BFKL impact factor (which was called for brevity simply impact factor) obtained in Ref. [13] from direct two-loop calculations and with the energy scale s_0 chosen in such a way that the ratio (energy evolution parameter) $s/s_0=s\vec q_2^{\,2}/\sqrt{\vec q_1^{\,2}\vec q_3^{\,2}\vec k_1^{\,2}\vec k_2^{\,2}}$ is Möbius invariant. This energy scale differs from the energy scale used in the correction (19) of the impact factor (see, for instance, Ref. [5]) which is equal to $|\vec k_1||\vec k_2|$. To adjust the correction (19) to the energy scale used in Ref. [16], we need to perform an additional transformation:

$$\langle GR_1|_t \to \langle GR_1|_c = \langle GR_1|_t - \frac{1}{2}\ln\left(\frac{\vec q_2^{\,2}}{\vec q_1^{\,2}}\right)\langle GR_1|^{(B)}\hat{\mathcal{K}}_m^{(B)}|\mathcal{G}_1\mathcal{G}_2\rangle, \tag{26}$$

where the subscript c means transformed to fit the conformal energy scale and $\hat{\mathcal{K}}_m^{(B)}$ is the modified LO kernel. Let us put, in a way similar to Eqs. (10) and (22),

$$\vec\Phi_{GR_1c}^{\mathcal{G}\mathcal{G}_2} = \vec\Phi_{GR_1t}^{\mathcal{G}\mathcal{G}_2} + \vec{\mathcal{I}}_3, \tag{27}$$

then the integral for $\vec{\mathcal{I}}_3$ can be written as

$$\vec{\mathcal{I}}_3 = -\ln\left(\frac{\vec q_2^{\,2}}{\vec q_1^{\,2}}\right)\int\frac{d\vec l}{\pi}(\vec C_1' - \vec C_1) \times \frac{1}{\vec r_1'^{\,2}\vec r_2'^{\,2}}\left(\frac{\vec r_1^{\,2}\vec r_2'^{\,2} + \vec r_2^{\,2}\vec r_1'^{\,2}}{\vec l^{\,2}} - \vec q_2^{\,2}\right). \tag{28}$$

Here instead of $\vec C_1'$ the difference $(\vec C_1' - \vec C_1)$ is taken and instead of the full modified kernel only its part related to real gluon production is kept. Moreover, the integral is written in two-dimensional transverse space. Indeed, due to gluon Reggeization the BFKL kernel for the adjoint representation of the colour group has the eigenvalue which is equal to the gluon trajectory, and the corresponding eigenfunction in the LO is a constant. It means that for the modified kernel the same eigenfunction corresponds to zero eigenvalue. Therefore, in the initial integral with $\vec C_1'$ and the modified kernel we can change in the integrand $(\vec C_1' - \vec C_1)$ with $\vec C_1'$ without change of the integral. After that, the virtual part of the kernel, which conserves Reggeon momenta, can be omitted, and we come to the integral (28) which is infrared finite and can be calculated in two-dimensional space. Integration can be done using the same decomposition as in Eq. (15) and the basic integral (16), and we get

$$\vec{\mathcal{I}}_3 = -\ln\left(\frac{\vec q_2^{\,2}}{\vec q_1^{\,2}}\right)\left[\vec C_1\ln\left(\frac{\vec q_2^{\,4}(\vec q_1 - \vec r_1)^4}{\vec q_1^{\,2}\vec r_2^{\,2}\vec k^{\,2}\vec r_1^{\,2}}\right) - \vec C_2\ln\left(\frac{\vec q_2^{\,2}(\vec q_1 - \vec r_1)^2}{\vec q_1^{\,2}\vec r_2^{\,2}}\right)\right]. \tag{29}$$

This result, together with the transformation (27) and the correction (25) gives

$$\begin{aligned}\vec\Phi_{GR_1c}^{\mathcal{G}_1\mathcal{G}_2} = {}& \vec C_1\left[-\ln\left(\frac{\vec q_2^{\,2}(\vec q_1 - \vec r_1)^2}{\vec r_1^{\,2}\vec k^{\,2}}\right)\ln\left(\frac{\vec q_2^{\,2}(\vec q_1 - \vec r_1)^2}{\vec q_1^{\,2}\vec r_2^{\,2}}\right)\right. \\ & \left. - \ln^2\left(\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}}\right) - \frac{3}{4}\ln^2\left(\frac{\vec k^{\,2}\vec r_1^{\,2}}{\vec q_1^{\,2}\vec r_2^{\,2}}\right) - 2\frac{(\vec k^{\,2})^\epsilon}{\epsilon^2} + 4\zeta(2)\right] \\ & + \frac{1}{4}\vec C_2\ln\left(\frac{\vec q_2^{\,2}(\vec q_1 - \vec r_1)^2}{\vec q_1^{\,2}\vec r_2^{\,2}}\right)\ln\left(\frac{\vec q_2^{\,4}(\vec q_1 - \vec r_1)^4\vec r_1^{\,2}\vec k^{\,2}}{\vec r_2^{\,6}\vec q_1^{\,6}}\right) \\ & + \frac{3}{2}\big[\vec C_2 \times \big([\vec r_1 \times \vec r_2]I_{\vec r_1,\vec r_2} + [\vec q_1 \times \vec r_1]I_{\vec q_1,-\vec r_1} \\ & + [\vec k \times \vec r_2]I_{\vec k,\vec r_2} + [\vec q_1 \times \vec k]I_{\vec q_1,-\vec k}\big)\big].\end{aligned} \tag{30}$$

This expression gives us the NLO correction to the impact factor for Reggeon-gluon transition in the scheme with conformal kernel and energy evolution parameter, which were used for the calculation of the remainder function. However, it is the impact factor for the full amplitude, not for the remainder function. To obtain the impact factor for the remainder function we have to take the impact factor (3) with $\Phi_{GR_1c}^{\mathcal{G}_1\mathcal{G}_2}$ instead of $\Phi_{GR_1*}^{\mathcal{G}_1\mathcal{G}_2}$ and with the polarization vector $\vec e^{\,*}$ of definite helicity, and to extract from it the piece included in the BDS ansatz.

Let us consider, for definiteness, the production of a gluon with positive helicity, $\vec e^{\,*} = (\vec e_x - i\vec e_y)/\sqrt{2}$. Then,

$$\vec e^{\,*}\vec C_1 = -\frac{q_1^- r_1^+}{\sqrt{2}(q_1 - r_1)^+}, \qquad \vec e^{\,*}\vec C_2 = -\frac{q_1^- q_2^+}{\sqrt{2}k^+}, \qquad \frac{\vec e^{\,*}\vec C_2}{\vec e^{\,*}\vec C_1} = 1 - z, \tag{31}$$

where $z = -q_1^+ r_2^+/(k^+ r_1^+)$ is the conformal invariant ratio (invariant with respect to Möbius transformations of complex variables p_i such that $r_1^+ = p_1 - p_2, r_2^+ = p_2 - p_3, -q_1^+ = p_3 - p_4, k_1^+ = p_4 - p_1$). Chiral components of the vector $\vec\Phi_{GR_1c}^{\mathcal{G}_1\mathcal{G}_2}$ (30) can be rewritten using the relations

$$\big[\vec c \times [\vec a \times \vec b]\big]^- = \frac{1}{2}c^-[a,b], \tag{32}$$

where $[a,b] = a^-b^+ - a^+b^-$, and

$$\begin{aligned}\int_0^1 \frac{dx}{|x - z|^2}\ln\frac{|z|^2}{x^2} &= \frac{1}{z^+ - z^-}\left(2\int_0^1\frac{dx}{x}\ln\frac{1 - xz^-}{1 - xz^+} - \ln|z|^2\ln\frac{1 - z^-}{1 - z^+}\right) \\ &= \frac{1}{z^+ - z^-}\left(-2\int_0^1\frac{dx}{x}\ln\frac{1 - x/z^-}{1 - x/z^+} - \ln|z|^2\ln\frac{(1 - z^-)z^+}{(1 - z^+)z^-}\right).\end{aligned} \tag{33}$$

Taking into account these relations and Eq. (6) we have

$$\begin{aligned}\big[\vec c \times [\vec a \times \vec b]\big]^- I_{\vec a,\vec b} &= \frac{c^-}{2}\left(2\int_0^1\frac{dx}{x}\ln\frac{(1 + xa^-/b^-)}{(1 + xa^+/b^+)} - \ln\left(\frac{\vec a^{\,2}}{\vec b^{\,2}}\right)\ln\frac{(a + b)^- b^+}{(a + b)^+ b^-}\right) \\ &= \frac{c^-}{2}\left(-2\int_0^1\frac{dx}{x}\ln\frac{(1 + xb^-/a^-)}{(1 + xb^+/a^+)} - \ln\left(\frac{\vec a^{\,2}}{\vec b^{\,2}}\right)\ln\frac{(a + b)^- a^+}{(a + b)^+ a^-}\right).\end{aligned} \tag{34}$$

Then, we transform the sum of dilogarithms which are obtained from the correction (30) with the help of the relation (34) using the identity

$$Li_2\left(-\frac{b}{a}\right) + Li_2\left(-\frac{c}{a}\right) + Li_2\left(-\frac{b}{d}\right) + Li_2\left(-\frac{c}{d}\right) = Li_2\left(\frac{bc}{ad}\right) - \frac{1}{2}\ln^2\left(\frac{a}{d}\right), \tag{35}$$

where $a + b + c + d = 0$. As result, after some algebra and with account of Eq. (3) we obtain

$$\langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle = \langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle^{(B)} \times \left\{1 + \frac{\bar g^2}{8}\left[(1 - z)\left(\ln\left(\frac{|1 - z|^2}{|z|^2}\right)\ln\left(\frac{|1 - z|^4}{|z|^6}\right)\right.\right.\right.$$

$$- 6Li_2(z) + 6Li_2(z^*) - 3\ln|z|^2 \ln\frac{1-z}{1-z^*}\Big)$$
$$- 4\ln|1-z|^2 \ln\frac{|1-z|^2}{|z|^2} - 3\ln^2|z|^2$$
$$- 4\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right) - 8\frac{(\vec{k}^{\,2})^\epsilon}{\epsilon^2} + 16\zeta(2)\Big]\Big\}. \tag{36}$$

Finally, in order to move to the impact factor for the calculation of the remainder function, one has to discard the terms $\bar{g}^2(-(1/2)\ln^2(\vec{q}_1^{\,2}/\vec{q}_2^{\,2}) - (\vec{k}^{\,2})^\epsilon/\epsilon^2 + 2\zeta(2))$ in the impact factor (36), since they are already taken into account in the BDS ansatz.

5. Conclusion

In this paper, we have calculated in the next-to-leading order the impact factor for Reggeon-gluon transition in the maximally extended supersymmetric Yang–Mills theory ($N = 4$ SYM) with large number of colours. Our final goal was the impact factor for the calculation of the high energy behavior of the remainder function for the BDS ansatz. On the way to this goal we have obtained several noteworthy intermediate results.

In the next-to-leading order impact factors are scheme dependent. First, we have found the impact factor in the bootstrap scheme, which was used in Refs. [6–9] for the check of validity of the bootstrap condition, the last and the most complicate in the set of the conditions, the fulfilment of which provides the multi-Regge form of production amplitudes. Starting from rather cumbersome results of Refs. [6–9] for Yang–Mills theories with any number of fermions and scalars in arbitrary representations of the gauge group, after great simplifications we have obtained a simple expression for the impact factor in the bootstrap scheme for $N = 4$ SYM with large number of colours. Then, we have transformed it in the standard scheme. To reach our goal, we needed to have the impact factor in the scheme with conformal invariant kernel and energy evolution parameter (Möbius scheme). The impact factor in the Möbius scheme was obtained by the transformation from the standard scheme. Finally, the impact factor for the calculation of the remainder function was obtained from the impact factor in the Möbius scheme by subtraction of the terms contained in the BDS ansatz. It turns out that this impact factor is invariant with respect to Möbius transformations in momentum space. Definitely, it is the reaffirmation of justice of the conjecture about dual conformal invariance of the remainder function. From the other side, it can be considered as a cross-check of a large number of calculations in the BFKL theory.

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