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Solar System constraints on a cosmologically viable $f(R)$ theory

Yousef Bisabr

Department of Physics, Shahid Rajaei Teacher Training University, Lavizan, Tehran 16788, Iran

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ABSTRACT

Recently, a model $f(R)$ theory is proposed (Miranda et al. (2009)) [1] which is cosmologically viable and distinguishable from Λ CDM. We use chameleon mechanism to investigate viability of the model in terms of Solar System experiments.

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1. Introduction

There are strong observational evidences that the expansion of the universe is accelerating. These observations are based on type Ia supernova [2], cosmic microwave background radiation [3], large scale structure formation [4], weak lensing [5], etc. The standard explanation invokes an unknown component, usually referred to as dark energy. It contributes to energy density of the universe with $\Omega_d = 0.7$ where Ω_d is the corresponding density parameter, see, e.g., [6] and references therein. The simplest dark energy scenario which seems to be both natural and consistent with observations is the Λ CDM model in which dark energy is identified as a cosmological constant [6–8]. However, in order to avoid theoretical problems [7], other scenarios have been investigated. Among different scenarios there are modified gravity models [9], in which one modifies the laws of gravity whereby a late time acceleration is produced without recourse to a dark energy component. One family of these modified gravity models is obtained by replacing the Ricci scalar R in the usual Einstein–Hilbert Lagrangian density for some function $f(R)$. These models are cosmologically acceptable if they meet some certain conditions. The most important ones is that they should follow a usual matter-dominated era preceding a late-time accelerated stage. Several models are proposed that admit cosmological solutions with accelerated expansion of the universe at late times [10]. However, among all cosmologically viable $f(R)$ theories there is still an important issue to be pursued, they must be probed at Solar System scale. In fact, changing gravity Lagrangian has consequences not only in cosmological scales but also in galactic and Solar System ones so that it seems to be necessary to investigate the low energy limit of such $f(R)$ theories.

E-mail address: y-bisabr@srstu.edu.

Early works on the weak field limit of $f(R)$ theories led to negative results. Using the equivalence of $f(R)$ and scalar-tensor theories [11–13], it is originally suggested that all $f(R)$ theories should be ruled out [14] since they violate the weak field constraints coming from Solar System experiments. This claim was based on the fact that $f(R)$ theories (in the metric formalism) are equivalent to Brans–Dicke theory with $\omega = 0$ while observations set the constraint $\omega > 40000$ [15]. In this case the post-Newtonian parameter satisfies $\gamma = \frac{1}{2}$ instead of being equal to unity as required by observations. Later, it was noted by many authors that for scalar fields with sufficiently large mass it is possible to drive γ close to unity even for null Brans–Dicke parameter. In this case the scalar field becomes short-ranged and has no effect at Solar System scales. Recently, it is shown that there exists an important possibility that the effective mass of the scalar field be scale dependent [16]. In this chameleon mechanism, the scalar field may acquire a large effective mass in Solar System scale so that it hides local experiments while at cosmological scales it is effectively light and may provide an appropriate cosmological behavior.

In the present work we intend to use this criterion to investigate constraints set by local experiments on a cosmologically viable model recently proposed by Miranda et al. [1]. It is a two parameter $f(R)$ model which is introduced in the form

$$f(R) = R - \alpha R_1 \ln\left(1 + \frac{R}{R_1}\right) \quad (1)$$

where α and R_1 are positive parameters. This model is reduced to general relativity for $\alpha = 0$. A cosmologically viable $f(R)$ model must start with a radiation-dominated era and have a saddle point matter-dominated phase followed by an accelerated epoch as a final attractor. This is formally stated by introducing the parameters $m = R \frac{d^2 f/dR^2}{dR^2}$ and $r = -R \frac{df/dR}{f}$ [17]. The cosmological dynamics of $f(R)$ models can then be understood by considering $m(r)$ curves in the (r, m) plane. Using this criteria, the authors of [1] showed that the model (1) satisfies all the cosmological requirements for

$\alpha > 1$ and regardless of R_1 .¹ Here we will focus on viability of the model in terms of local gravity experiments. We will show that these local experiments rule out the model as an explanation for the current accelerated expansion of the universe.

2. Chameleon mechanism

In this section we offer a brief review of the chameleon mechanism. We consider the following action²

$$S = \frac{1}{2} \int d^4x \sqrt{-g} f(R) + S_m(g_{\mu\nu}, \psi) \quad (2)$$

where g is the determinant of $g_{\mu\nu}$, $f(R)$ is an unknown function of the scalar curvature R and S_m is the matter action depending on the metric $g_{\mu\nu}$ and some matter field ψ . We may use a new set of variables

$$\bar{g}_{\mu\nu} = p g_{\mu\nu} \quad (3)$$

$$\phi = \frac{1}{2\beta} \ln p \quad (4)$$

where $p \equiv \frac{df}{dR} = f'(R)$ and $\beta = \sqrt{\frac{1}{6}}$. This is indeed a conformal transformation which transforms the above action in the Jordan frame to the Einstein frame [11–13]

$$S = \frac{1}{2} \int d^4x \sqrt{-\bar{g}} \{ \bar{R} - \bar{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2V(\phi) \} + S_m(\bar{g}_{\mu\nu} e^{2\beta\phi}, \psi) \quad (5)$$

In the Einstein frame, ϕ is a scalar field with a self-interacting potential which is given by

$$V(\phi) = \frac{1}{2} e^{-2\beta\phi} \{ r[p(\phi)] - e^{-2\beta\phi} f(r[p(\phi)]) \} \quad (6)$$

where $r(p)$ is a solution of the equation $f'[r(p)] - p = 0$ [11]. Note that conformal transformation induces the coupling of the scalar field ϕ with the matter sector. The strength of this coupling β , is fixed to be $\sqrt{\frac{1}{6}}$ and is the same for all types of matter fields. In the case of such a strong matter coupling, the role of the potential of the scalar field is important for consistency with local gravity experiments. When the potential satisfies certain conditions it is possible to attribute an effective mass to the scalar field which has a strong dependence on ambient density of matter. A theory in which such a dependence is realized is said to be a chameleon theory [16]. In such a theory the scalar field ϕ can be heavy enough in the environment of the laboratory tests so that the local gravity constraints suppressed even if β is of the order of unity. Meanwhile, it can be light enough in the low-density cosmological environment to be considered as a candidate for dark energy.

Variation of the action (2) with respect to $\bar{g}_{\mu\nu}$ and ϕ , gives the field equations

$$\bar{G}_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \bar{g}_{\mu\nu} \partial_\gamma \phi \partial^\gamma \phi - V(\phi) \bar{g}_{\mu\nu} + \bar{T}_{\mu\nu} \quad (7)$$

$$\bar{\square} \phi - \frac{dV}{d\phi} = -\beta \bar{T} \quad (8)$$

¹ In the model proposed in [1], it is stated that the relation (1) is a special case of a general parametrization unifying the models of [18] and [19]. In the latter models there is a parameter with the same dimension of R_1 which is taken to be of the same order of the presently observed cosmological constant. The relation between this parameter and R_1 and also the relevance of R_1 with the time of the beginning of the acceleration phase are important issues that are not explicitly addressed in [1].

² We use the unit $(8\pi G)^{-1} = 1$.

where

$$\bar{T}_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S_m}{\delta \bar{g}^{\mu\nu}} \quad (9)$$

and $\bar{T} = \bar{g}^{\mu\nu} \bar{T}_{\mu\nu}$. Covariant differentiation of (7) and the Bianchi identities give

$$\bar{\nabla}^\mu \bar{T}_{\mu\nu} = \beta \bar{T} \partial_\nu \phi \quad (10)$$

which implies that the matter field is not generally conserved and feels a new force due to gradient of the scalar field. Let us consider $\bar{T}_{\mu\nu}$ as the stress-tensor of dust with energy density $\bar{\rho}$ in the Einstein frame. In a static and spherically symmetric spacetime Eq. (8) gives

$$\frac{d^2\phi}{d\bar{r}^2} + \frac{2}{\bar{r}} \frac{d\phi}{d\bar{r}} = \frac{dV_{\text{eff}}(\phi)}{d\phi} \quad (11)$$

where \bar{r} is distance from center of the symmetry in the Einstein frame and

$$V_{\text{eff}}(\phi) = V(\phi) - \frac{1}{4} \rho e^{-4\beta\phi} \quad (12)$$

Here we have used the relation $\bar{\rho} = e^{-4\beta\phi} \rho$ that relates the energy densities in the Jordan and the Einstein frames. We consider a spherically symmetric body with a radius \bar{r}_c and a constant energy density $\bar{\rho}_{\text{in}}$ ($\bar{r} < \bar{r}_c$). We also assume that the energy density outside the body ($\bar{r} > \bar{r}_c$) is given by $\bar{\rho}_{\text{out}}$. We will denote by ϕ_{in} and ϕ_{out} the field values at two minima of the effective potential $V_{\text{eff}}(\phi)$ inside and outside the object, respectively. They must clearly satisfy $V'_{\text{eff}}(\phi_{\text{in}}) = 0$ and $V'_{\text{eff}}(\phi_{\text{out}}) = 0$ where prime indicates differentiation of $V_{\text{eff}}(\phi)$ with respect to the argument. As usual, masses of small fluctuations about these minima are given by $m_{\text{in}} = [V''_{\text{eff}}(\phi_{\text{in}})]^{\frac{1}{2}}$ and $m_{\text{out}} = [V''_{\text{eff}}(\phi_{\text{out}})]^{\frac{1}{2}}$ which depend on ambient matter density. A region with large mass density corresponds to a heavy mass field while regions with low mass density corresponds to a field with lighter mass. In this way it is possible for the mass field to take sufficiently large values near massive objects in the Solar System scale and to hide the local tests. For a spherically symmetric body there is a thin-shell condition

$$\frac{\Delta \bar{r}_c}{\bar{r}_c} = \frac{\phi_{\text{out}} - \phi_{\text{in}}}{6\beta\Phi_c} \ll 1 \quad (13)$$

where $\Phi_c = M_c/8\pi\bar{r}_c$ is the Newtonian potential at $\bar{r} = \bar{r}_c$ with M_c being the mass of the body. In this case, Eq. (11) with some appropriate boundary conditions gives the field profile outside the object [16]

$$\phi(\bar{r}) = -\frac{\beta}{4\pi} \frac{3\Delta \bar{r}_c}{\bar{r}_c} \frac{M_c e^{-m_{\text{out}}(\bar{r}-\bar{r}_c)}}{\bar{r}} + \phi_{\text{out}} \quad (14)$$

3. The model

The function $f(R)$ in the Jordan frame is closely related to the potential function of the scalar degree of freedom of the theory in the Einstein frame. Any functional form for the potential function corresponds to a particular class of $f(R)$ theories. To find a viable function $f(R)$ passing Solar System tests one can equivalently work with its corresponding potential function in the Einstein frame and put constraints on the relevant parameters via chameleon mechanism. Taking this as our criterion, we write potential function of the model (1)

$$V(\phi) = \frac{1}{2} R_1 e^{-4\beta\phi} \left\{ \alpha \ln \left(\frac{\alpha}{1 - e^{2\beta\phi}} \right) - e^{2\beta\phi} - (\alpha - 1) \right\} \quad (15)$$

Assuming that $\phi \ll 1$, one can find the solution of $V'_{\text{eff}}(\phi) = 0$ by substituting (15) into (12)

$$\phi = \frac{1}{2A} \{-C \pm \sqrt{C^2 - 4AB}\} \quad (16)$$

where

$$A = 2\beta^2 R_1 (1 - 2\alpha) \quad (17)$$

$$B = -\frac{1}{2} \alpha R_1 \quad (18)$$

$$C = \beta[\rho + R_1(\alpha - 1) - 2\alpha R_1 \ln \alpha] \quad (19)$$

In the following we shall consider thin-shell condition together with constraints set by equivalence principle and fifth force experiments.

3.1. Thin-shell condition

In the chameleon mechanism, the chameleon field is trapped inside large and massive bodies and its influence on the other bodies is only due to a thin-shell near the surface of the body. The criterion for this thin-shell condition is given by (13). If we combine (13) and (16) we obtain

$$\frac{\Delta \bar{r}_c}{\bar{r}_c} = \frac{1}{12\Phi_c A} \{(\rho_{\text{in}} - \rho_{\text{out}}) \pm \sqrt{(\rho_{\text{out}} + a)^2 - b} \mp \sqrt{(\rho_{\text{in}} + a)^2 - b}\} \quad (20)$$

where ρ_{in} and ρ_{out} are energy densities inside and outside of the body in the Jordan frame. Here $a = \frac{A}{\beta} - \rho$ and $b = 4\alpha R_1^2(2\alpha - 1)$. In weak field approximation, the spherically symmetric metric in the Jordan frame is given by

$$ds^2 = -[1 - 2X(r)]dt^2 + [1 + 2Y(r)]dr^2 + r^2 d\Omega^2 \quad (21)$$

where $X(r)$ and $Y(r)$ are some functions of r . There is a relation between r and \bar{r} so that $\bar{r} = p^{1/2}r$. If we consider this relation under the assumption $m_{\text{out}}r \ll 1$, namely that the Compton wavelength m_{out}^{-1} is much larger than Solar System scales, then we have $\bar{r} \approx r$. In this case, the chameleon mechanism gives for the post-Newtonian parameter γ [20]

$$\gamma = \frac{3 - \frac{\Delta r_c}{r_c}}{3 + \frac{\Delta r_c}{r_c}} \simeq 1 - \frac{2}{3} \frac{\Delta r_c}{r_c} \quad (22)$$

We can now apply (20) on the Earth and obtain the condition that the Earth has a thin-shell. To do this, we assume that the Earth is a solid sphere of radius $R_e = 6.4 \times 10^8$ cm and mean density $\rho_e \sim 10$ g/cm³. We also assume that the Earth is surrounded by an atmosphere with homogeneous density $\rho_a \sim 10^{-3}$ g/cm³ and thickness 100 km. For simplifying equation (20), we proceed under the assumption $\rho_{\text{in}}, \rho_{\text{out}} \ll R_1 \alpha$.³ We will return to this issue later. In this case, Eq. (20) simplifies to

$$\frac{\Delta R_e}{R_e} \approx \frac{1}{4\Phi_e} \frac{(\rho_{\text{in}} - \rho_{\text{out}})}{R_1(1 - 2\alpha)} \quad (23)$$

where $\Phi_e = 6.95 \times 10^{-10}$ [21] is Newtonian potential on surface of the Earth. The tightest Solar System constraint on γ comes from Cassini tracking which gives $|\gamma - 1| < 2.3 \times 10^{-5}$ [15]. This together with (22) and (23) yields

$$(1 - 2\alpha) > 10^{13} \left(\frac{\rho_{\text{in}}}{R_1} \right) \quad (24)$$

³ It can be easily checked that our main results do not change when $\rho_{\text{in}}, \rho_{\text{out}} \gg R_1 \alpha$.

With $\rho_{\text{in}} = \rho_e = 7 \times 10^{-28}$ cm⁻², this is equivalent to $R_1(1 - 2\alpha) > 10^{-15}$ cm⁻².

3.2. Equivalence principle

We now consider constraints coming from possible violation of weak equivalence principle. We assume that the Earth, together with its surrounding atmosphere, is an isolated body and neglect the effect of the other compact objects such as the Sun, the Moon and the other planets. Far away the Earth, matter density is modeled by a homogeneous gas with energy density $\rho_G \sim 10^{-24}$ g/cm³. To proceed further, we first consider the condition that the atmosphere of the Earth satisfies the thin-shell condition [16]. If the atmosphere has a thin-shell the thickness of the shell (ΔR_a) must be clearly smaller than that of the atmosphere itself, namely $\Delta R_a < R_a$, where R_a is the outer radius of the atmosphere. If we take thickness of the shell equal to that of the atmosphere itself $\Delta R_a \sim 10^2$ km we obtain $\frac{\Delta R_a}{R_a} < 1.5 \times 10^{-2}$. It is then possible to relate $\frac{\Delta R_e}{R_e} = \frac{\Phi_a - \Phi_e}{6\beta\Phi_e}$ and $\frac{\Delta R_a}{R_a} = \frac{\Phi_G - \Phi_a}{6\beta\Phi_a}$ where Φ_e , Φ_a and Φ_G are the field values at the local minimum of the effective potential in the regions $r < R_e$, $R_a > r > R_e$ and $r > R_a$ respectively. Using the fact that Newtonian potential inside a spherically symmetric object with mass density ρ is $\Phi \propto \rho R^2$, one can write $\Phi_e = 10^4 \Phi_a$ where Φ_e and Φ_a are Newtonian potentials on the surface of the Earth and the atmosphere, respectively. This gives $\Delta R_e/R_e \approx 10^{-4} \Delta R_a/R_a$. With these results, the condition for the atmosphere to have a thin-shell is

$$\frac{\Delta R_e}{R_e} < 1.5 \times 10^{-6} \quad (25)$$

The tests of equivalence principle measure the difference of free-fall acceleration of the Moon and the Earth towards the Sun. The constraint on the difference of the two acceleration is given by [15]

$$\frac{|a_m - a_e|}{a_N} < 10^{-13} \quad (26)$$

where a_m and a_e are acceleration of the Moon and the Earth respectively and a_N is the Newtonian acceleration. The Sun and the Moon are all subject to the thin-shell condition [16] and the field profile outside the spheres are given by (14) with replacement of corresponding quantities. The accelerations a_m and a_e are then given by [16]

$$a_e \approx a_N \left\{ 1 + 18\beta^2 \left(\frac{\Delta R_e}{R_e} \right)^2 \frac{\Phi_e}{\Phi_s} \right\} \quad (27)$$

$$a_m \approx a_N \left\{ 1 + 18\beta^2 \left(\frac{\Delta R_e}{R_e} \right)^2 \frac{\Phi_e^2}{\Phi_s \Phi_m} \right\} \quad (28)$$

where $\Phi_e = 6.95 \times 10^{-10}$, $\Phi_m = 3.14 \times 10^{-11}$ and $\Phi_s = 2.12 \times 10^{-6}$ are Newtonian potentials on the surfaces of the Earth, the Moon and the Sun, respectively [21]. This gives a difference of free-fall acceleration

$$\frac{|a_m - a_e|}{a_N} = (0.13)\beta^2 \left(\frac{\Delta R_e}{R_e} \right)^2 \quad (29)$$

Combining this with (26) results in

$$\frac{\Delta R_e}{R_e} < 6.74 \times 10^{-6} \quad (30)$$

which is of the same order of the condition (25) that the atmosphere has a thin-shell. Taking this as the constraint coming from

violation of equivalence principle and combining with (23), we obtain

$$(1 - 2\alpha) > (10^{14}) \left(\frac{\rho_{\text{in}}}{R_1} \right) \quad (31)$$

which is not much different from (24).

3.3. Fifth force

The potential energy associated with a fifth force interaction is parameterized by a Yukawa potential

$$U(r) = -\varepsilon \frac{m_1 m_2}{8\pi} \frac{e^{-r/\lambda}}{r} \quad (32)$$

where m_1 and m_2 are masses of the two test bodies separating by distance r , ε is strength of the interaction and λ is the range. Thus fifth force experiment constrains regions of (ε, λ) parameter space. These experiments are usually carried out in a vacuum chamber in which the range of the interaction inside it is of the order of the size of the chamber [16], namely $\lambda \sim R_{\text{vac}}$. The tightest bound on the strength of the interaction is $\varepsilon < 10^{-3}$ [22]. Inside the chamber we consider two identical bodies with uniform densities ρ_c , radii r_c and masses m_c . If the two bodies satisfy the thin-shell condition, their field profile outside the bodies are given by

$$\phi(r) = -\frac{\beta}{4\pi} \frac{3\Delta r_c}{r_c} \frac{m_c e^{-r/R_{\text{vac}}}}{r} + \varphi_{\text{vac}} \quad (33)$$

Then the corresponding potential energy of the interaction is

$$V(r) = -2\beta^2 \left(\frac{3\Delta r_c}{r_c} \right)^2 \frac{m_c^2}{8\pi} \frac{e^{-r/R_{\text{vac}}}}{r} \quad (34)$$

The bound on the strength of the interaction translates into

$$2\beta^2 \left(\frac{3\Delta r_c}{r_c} \right)^2 < 10^{-3} \quad (35)$$

One can write for each of the test bodies

$$\frac{\Delta r_c}{r_c} \approx \frac{1}{4\Phi_c} \frac{(\rho_c - \rho_{\text{vac}})}{R_1(1 - 2\alpha)} \quad (36)$$

where ρ_{vac} is energy density of the vacuum inside the chamber. In the experiment carried out in [22], one used a typical test body with mass $m_c \approx 40$ g and radius $r_c \approx 1$ cm. These correspond to $\rho_c \approx 9.5$ g/cm³ and $\Phi_c \sim 10^{-27}$. Moreover, the pressure in the vacuum chamber was reported to be 3×10^{-8} Torr which is equivalent to $\rho_{\text{vac}} \approx 4.8 \times 10^{-14}$ g/cm³. Substituting these into (36) and combining the result with (35) gives the bound

$$(1 - 2\alpha) > (10^{27}) \left(\frac{\rho_c}{R_1} \right) \quad (37)$$

which is equivalent to $R_1(1 - 2\alpha) > 10^{-1}$ cm⁻².

4. Discussion

We have discussed viability of the $f(R)$ model proposed in [1] in terms of local gravity constraints. We have used the correspondence between a general $f(R)$ theory with scalar field theories. In general, in the scalar field representation of an $f(R)$ theory there is a strong coupling of the scalar field with the matter sector. We have considered the conditions that this coupling is suppressed by chameleon mechanism. We have found that in order that the model (1) be consistent both with fifth force and equivalence principle experiments, the two parameters α and R_1 together should satisfy the condition $R_1(1 - 2\alpha) > 10^{-1}$ cm⁻². To have a bound on

the parameter α , one should attribute a physical meaning to the dimensional quantity R_1 . Following the models proposed by Hu and Sawicki [18] and Starobinski [19], if we take it as the same order of the observed cosmological constant $\Lambda_{\text{obs}} \sim 10^{-58}$ cm⁻² we obtain $|2\alpha - 1| > 10^{57}$.

It should be pointed out that this result is obtained under the assumption $\rho_{\text{in}}, \rho_{\text{out}} \ll R_1\alpha$. To understand the relevance of this assumption, let us consider the case that $\rho_{\text{in}}, \rho_{\text{out}} \sim R_1\alpha$. Taking energy density inside the Earth as a typical energy density in the Solar System, we obtain $R_1\alpha \sim 10^{-28}$ cm⁻². In the model (1), if the coefficient $R_1\alpha$ is so small then it would be hardly distinguishable from general relativity. As the point of view of local gravity experiments, a viable $f(R)$ model should simultaneously satisfy Solar System bounds as well as exhibit an appropriate deviation from general relativity. This requires that $R_1\alpha \gg 10^{-28}$ cm⁻², which confirms the assumption that our results are based on.

The last point we wish to remark is that, as reported by the authors of [1], the condition that the universe pass through a matter-dominated epoch and finally reach a late-time accelerated phase is that $\alpha > 1$ regardless of R_1 . This seems not to be consistent with our results in the context of the chameleon mechanism. Although our analysis do not place any experimental bound on the parameter R_1 , however for $R_1 > 0$ the relation (37) implies that $\alpha \in (-\infty, \frac{1}{2}]$ which is out of the range reported in [1].

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