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Nonlinear dynamical systems of trajectory design for 3D horizontal well and their optimal controls☆

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Abstract

The trajectory design of horizontal well is a optimal control problem of nonlinear multistage dynamical system. It is often sought using trial-and-error methods, but these methods depend on experience of designers and workers. In this paper, we create new optimal control model of nonlinear dynamical system for the trajectory design of horizontal well. Several properties are discussed. Uniform design method is used to choose the initial points in the feasible region. We demonstrate how to decompose the feasible region into finite subregions in which improved Hook–Jeeves algorithm is employed to search optimal solution. Finally, the feasible optimization algorithm is constructed to find the optimal solution of the system. Several results show the validity of our algorithm. This is preferable, since our method is independent of the experience.

MSC: 37M05

Keywords: Trajectory design; Nonlinear multistage dynamical system; Optimal control; Uniform design algorithm; Hooke–Jeeves algorithm

1. Introduction

Nonlinear multistage dynamical system is a problem of multistage decision process, belonging to dynamic programming problem. Currently, only inverse order method and order method are applied to solve this problem [6]. Designing the trajectory of horizontal well is a optimal control problem of nonlinear multistage dynamical system. In current works, the methods of designing the trajectory of horizontal well are often sought using trial-and-error methods such as cylindrical spiral method, slant-plane method and spiral-like method [2,9,5,7], etc. These methods depend on experience of designers and workers. So the results designed cannot be ensured optimization. In general, these methods belong to heuristic methods of man–computer interaction. As the number of segments of horizontal well increases, it is more difficult to design the trajectory of horizontal well [4].

In this paper, piecewise smoothing dynamic system and optimal control model are not only nonlinear about control variables and state variables but also combinatorial optimization and topological optimization. So the trajectory design of horizontal well has been classified as an NP-complete problem [8]. To address this problem, we apply uniform

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design method to choose initial points. Then we decompose the feasible region into several subregions such that every subregion includes a local minimizer at most. Modified Hooke–Jeeves method is applied to search optimum solution on every subregion. Our improved Hooke–Jeeves method has advantage of applying on local convex optimal problem with constraints. This might be preferable, since our method does not take into account the experience of designers and workers.

In the following sections, in Section 2, we present the nonlinear multistage dynamical system and prove several properties of system for the trajectory design of horizontal well. Section 3 is devoted to show the optimal control model of nonlinear multistage dynamic system and to give several properties of model. In Section 4, we describe uniform design algorithm and improved Hooke–Jeeves method for the trajectory design of 3D horizontal well, and demonstrate how to decompose the feasible region. Finally, the experimental results of ci-16 slant-plant $146^\sharp$ horizontal well are shown in Section 5.

2. Nonlinear multistage dynamical system

Suppose that the trajectory of horizontal well consists of n pieces of smoothing curves $\{d_1, d_2, \ldots, d_n\}$. For the ith curve, well oblique angle is denoted by $x_{i1} \in (0, \pi/2]$, and azimuthal angle is x_{i2}. State variables is written as $x_i = (x_{i1}, x_{i2})^\top \in R^2$. $z_0 = (z_{01}, z_{02}, z_{03}) \in R^3$ acts as initial coordinate of horizontal well. $z_t = (z_{t1}, z_{t2}, z_{t3}) \in R^3$ which is fixed presents the coordinate of objective point. x_{t1} and x_{t2} are well oblique angle and azimuthal angle, respectively, of objective point. We regard terminal coordinates of the ith curve as $z_i = (z_{i1}, z_{i2}, z_{i3}) \in R^3$. According to the design rule of the trajectory of horizontal well, state equation of the ith curve can be given as follows:

$$\begin{cases} \dfrac{\mathrm{d}x_{i1}}{\mathrm{d}s} = u_{i1}\cos(u_{i2}), \\ \dfrac{\mathrm{d}x_{i2}}{\mathrm{d}s} = u_{i1}\sin(u_{i2})/\sin(x_{i1}), \end{cases} \qquad s \in (u_{i-1,3}, u_{i3}), \tag{1}$$

$$\begin{cases} x_{ij}(u_{i-1,3}) = x_{i-1,j}u_{i-1,3}, \\ x_{ij}(0) = x_{0j}, \end{cases} \qquad i \in \{2, 3, \ldots, n\}, \quad j \in I_2, \tag{2}$$

where unit of length is meter, unit of angle is radian. Variable s is arc length of the trajectory of horizontal well. Control variables u_{i1}, u_{i2} and u_{i3} act as curvature, implement face angle and arc length to terminal point of trajectory, respectively. $u_{ik} \in [a_{ik}, b_{ik}]$, where a_{ik} and b_{ik} are known, $i \in I_n$, $k \in \{1, 2, 3\}$. x_{01} and x_{02} are well oblique angle and azimuthal angle of initial point of trajectory, respectively. It is easy to find that the solution of system (1) and (2) is existential and unique for fixed $u_{ik} \in [a_{ik}, b_{ik}]$, because of continuity of right formula of (1). The solution of above system is denoted by $x_i(s, u_i) = (x_{i1}(s, u_i), x_{i2}(s, u_i))$, $(i \in I_n)$ in which $u_i = (u_{i1}, u_{i2}, u_{i3})^\top \in R^3$.

For nonlinear multistage dynamic system (1) and (2), we should make the following explanations:

(1) Terminal coordinate $z_i = (z_{i1}, z_{i2}, z_{i3}) \in R^3$ of the ith curve can be represented by $u_i \in R^3$ and $x_i \in R^2$ [5].
(2) For overall system, control variables can be represented by $u = (u_1, u_2, \ldots, u_n) \in R^{3n}$ in which $u_i = (u_{i1}, u_{i2}, u_{i3})$. State variable is $x(s, u) = (x_1(s, u_1), x_2(s, u_2), \ldots, x_n(s, u_n)) \in R^{2n}$. $x_n(s, u_n)$ acts as terminal state variable of overall system. $z_n = (z_{n1}, z_{n2}, z_{n3}) \in R^3$ is terminal coordinate of overall system.
(3) The feasible region can be written as $U_{ad} = \{u = (u_1, u_2, \ldots, u_n) \in R^{3n} \mid u_i = (u_{i1}, u_{i2}, u_{i3}), u_{ik} \in [a_{ik}, b_{ik}], k \in I_3, i \in I_n\}$. $d_i = [u_{i-1,3}, u_{i3})$ $(i \in I_n)$ is arc length of the ith curve. We refer to $d_0 = [0, u_{n3}] = \bigcup_{i=1}^{n} d_i$ as arc length of overall horizontal well and to the set of all solutions satisfying system (1) and (2) as $V_x(d_0, U_{ad})$.

To translate system (1) and (2) into normal control system, we define functions $y_{ij}(s, u_j)$ and $y(s, u)$ as follows:

$$y_{ij}(s, u_i) = \begin{cases} x_{ij}(s, u_i), & s \in d_i, \\ 0, & s \in d_0 \backslash d_i, \end{cases} \quad i \in I_n, j \in I_2, \tag{3}$$

$$y(s, u) = \sum_{i=1}^{n} (y_{i1}(s, u_i), y_{i2}(s, u_i)), \quad s \in d_0. \tag{4}$$

We find $y(s,u) \in C(d_0, R^2)$ about s by (3) and (4). In terms of system (1), we present the following function:

$$f_{ij}(s,u_i) = \begin{cases} u_{i1}\cos(u_{i2}), & s \in d_i,\ j=1, \\ u_{i1}\sin(u_{i2})/\sin(y_{i1}), & s \in d_i,\ j=2, \\ 0, & s \in d_0 \backslash d_i,\ j=1,2, \end{cases} \tag{5}$$

$$f(y(s),u(s),s) = \sum_{i=1}^{n} (f_{i1}(s,u_i), f_{i2}(s,u_i)), \quad s \in d_0. \tag{6}$$

Via functions $y(s,u)$ and $f(y(s),u(s),s)$, system (1) and (2) can be written as the following normal form:

$$\begin{cases} \dot{y}(s,u) = f(y(s),u(s),s), \\ y(0,u) = (x_{01}, x_{02}), \end{cases} \quad s \in d_0. \tag{7}$$

Similarly the set of all solutions satisfying (7) is denoted by $V_y(d_0, U_{ad})$.

Property 1. *$\forall u \in U_{ad}$, the solution $x(s,u) = (x_1(s,u_1), x_2(s,u_2), \ldots, x_n(s,u_n))$ of system* (1) *and* (2) *is existential and unique, and mapping $x_i(s,u_i)$: $d_i \times [a_{i1}, b_{i1}] \times [a_{i2}, b_{i2}] \times [a_{i3}, b_{i3}] \to R^2$ ($i \in I_n$) is continuous. Similarly $y : d_0 \times U_{ad} \to R^2$ is also continuous.*

Property 2. *$V_y(U_{ad})$ is compact set on $C(d_0, R^3)$ which is continuous function space.*

Proof. The mapping $u \in U_{ad} \to x(s,u) \in V_x$ is continuous according to Property 1. By system (1), Eqs. (3) and (4), we find that the mapping $u \in U_{ad} \to y(s,u) \in V_y$ is also continuous. Here U_{ad} is bounded closed set. Then V_y is compact set on $C(d_0, R^3)$. □

3. Optimal control model of nonlinear multistage dynamic system

There are two purposes for designing the trajectory of horizontal well, (a) terminal coordinate $z_n = (z_{n1}, z_{n2}, z_{n2}) \in R^3$ and terminal state variable $x_n(s,u_n)$ of system (1) and (2) are sufficiently close with objective coordinate $z_t = (z_{t1}, z_{t2}, z_{t3}) \in R^3$ and objective state variable $x_t = (x_{t1}, x_{t2})$, respectively. (b) the total length of the trajectory of horizontal well $\sum_{i=1}^{n}(u_{i3} - u_{i-1,3}) = u_{n3}$ is the shortest, that is, the cost is minimum. To implement above two purpose, firstly, we define positive deviations and negative deviations of of system (7) (or system (1) and (2)).

$$d_{1k}^{+} = d_{1k}^{+}(z_{nk}, z_{tk}) = \begin{cases} z_{tk} - z_{nk}, & z_{tk} > z_{nk}, \\ 0 & \text{otherwise}, \end{cases} \quad k \in I_3, \tag{8}$$

$$d_{1k}^{-} = d_{1k}^{-}(z_{nk}, z_{tk}) = \begin{cases} z_{nk} - z_{tk}, & z_{nk} > z_{tk}, \\ 0 & \text{otherwise}, \end{cases} \quad k \in I_3, \tag{9}$$

$$d_{2j}^{+} = d_{2j}^{+}(x_{nj}(u_{n3}), x_{tj}) = \begin{cases} x_{tj} - x_{nj}(u_{n3}), & x_{tj} > x_{nj}(u_{n3}), \\ 0 & \text{otherwise}, \end{cases} \quad j \in I_2, \tag{10}$$

$$d_{2j}^{-} = d_{2j}^{-}(x_{nj}(u_{n3}), x_{tj}) = \begin{cases} x_{nj}(u_{n3}) - x_{tj}, & x_{nj}(u_{n3}) > x_{tj}, \\ 0 & \text{otherwise}, \end{cases} \quad j \in I_2. \tag{11}$$

The objective function is given via these deviations

$$J(y(s,u)) = c_0 u_{n3} + \sum_{k=1}^{3} c_{1k}(d_{1k}^{+} + d_{1k}^{-}) + \sum_{j=1}^{2} c_{2j}(d_{2j}^{+} + d_{2j}^{-}), \tag{12}$$

where $c_0, c_{11}, c_{12}, c_{13}$ c_{21} and c_{22} are weighted coefficients.

To optimize the trajectory of horizontal well, we establish the optimal control model of system (7)

$$\begin{aligned} \text{CP:} \quad & \min \quad J(y(s,u)) \\ & \text{s.t.} \quad y(s,u) \in V_y(d_0, U_{ad}). \end{aligned} \tag{13}$$

Property 3. *$J(y(s,y))$ is continuous functional on $V_y(d_0, U_{ad})$.*

Above property is obviously according to the definition of positive and negative deviation (8)–(11).

Property 4. *For $\forall\ y \in V_y(d_0, U_{ad})$, exists the optimum solution $y^* \in V_y(d_0, U_{ad})$ such that $J(y^*(s,u)) \leqslant J(y(s,u))$.*

Proof. From Property 2, we know that $V_y(d_0, U_{ad})$ is compact set on $C(d_0, R^3)$, and $J(y(s,u))$ is continuous functional on $V_y(d_0, U_{ad})$ by Property 3. Then according to the existence theorem of continuous function, we can find $y^* \in V_y(d_0, U_{ad})$ such that $J(y^*(s,u)) \leqslant J(y(s,u))$. $\square$

4. An optimization algorithm

Since system (1) (or (7)) are nonlinear and multistage, and function $J(y(s,u))$ is not convex, optimal control problem CP is not only NP-complete but also topological optimization. Hence usual algorithms cannot solve it. However, a large number of experimental results indicate that $J(y(s,u))$ is multimodal, and both the solution of system (1) and (2) x_i and terminal coordinate z_i are continuous on state variable u. So we decompose the problem into several subproblems to find optimal solution.

4.1. Uniform design algorithm

To achieve the global optimum solution, at first, we apply uniform design algorithm to find **m** initial points $\{u^1, u^2, \ldots, u^m\}$, $u^j \in R^3$. Yuan and Kaitai [1] presented the uniform design algorithm which distributes the points on feasible region uniformly and searches feasible region effectively to explore information and to find maximum statistical probability points on **s**-dimensional space. In this paper, let $s = 3n$, on the trajectory design of horizontal well, the uniform design algorithm can be simply described as follows:

Step 1: Generate the point set $A = \{a \in Z^+ \mid a < m, a^{t+1} = 1 \bmod (m), t \in Z^+, t \geqslant s-1\} = \{a_1, a_2, \ldots, a_l\} \subset Z^+$.

Step 2: For $\forall a_k \in A$ $(k \in I_l)$, define new vector $h_k = (a_k^0, a_k^1, \ldots, a_k^{s-1}) \bmod (m)$, labelled as $(h_{k1}, h_{k2}, \ldots, h_{ks}) \in Z^s$. Element generated by uniform design is $x_j^k = (v_{1j}^k, v_{2j}^k, \ldots, v_{sj}^k) \in Z^s$, $j \in I_m, k \in I_l$ in which $v_{ij}^k = (jh_{ki}) \bmod (m)$, $i \in I_s$. Let $P^k = \{x_1^k, x_2^k, \ldots, x_m^k\} \subset Z^s$, $k \in I_l$.

Step 3: Compute the deviation of P^k according to the rule

$$D(P^k) = \max_{j \in I_m} \left\{ \frac{N(P^k, [0, x_j^k))}{m} - V_0 \mid [0, x_j^k) \right\},$$

where $[0, x_j^k) = [0, v_{1j}^k) \times [0, v_{2j}^k) \times \cdots \times [0, x_{sj}^k)$, and $V_0 \mid [0, x_j^k)$ act as the volume of $[0, x_j^k)$. $N(P^k, [0, x_j^k))$ represents the number of points which are in $P^k \bigcap [0, x_j^k)$, $k \in I_l$.

Step 4: Search the minimum $P^* = \{x_1^*, x_2^*, \ldots, x_m^*\} \subset R^{s \times m}$ in which $x_j^* = (v_{ij}^*, v_{2j}^*, \ldots, v_{sj}^*) \in Z^s$, $j \in I_m$, such that $D(P^*) = \min_{k \in I_l} D(P^k)$.

Step 5: Obtain the initial points in U_{ad} by the following equality:

$$u_{ik}^j = a_{ik} + \frac{v_{3(i-1)+k,j}}{m}(b_{ik} - a_{ik}), \quad k \in I_3, i \in I_n, j \in I_m \tag{14}$$

that is, $u^j = (u_{11}^j, u_{12}^j, u_{13}^j, u_{21}^j, \ldots, u_{n1}^j, u_{n2}^j, u_{n3}^j) \in U_{ad} \subset R^{3n}$, $j \in I_m$.

4.2. Domain decomposition

Set $P_m = \{u^1, u^2, \ldots, u^m\} \subset U_{ad} \subset R^{3n}$ which is obtained by above algorithm. Let $V_{ik} = b_{ik} - a_{ik}$, $i \in I_m, k \in I_3$.

Property 5. *If let* $\delta = c/(\sqrt[6n]{m} - 1)$, *in which* $c = \sqrt{\sum_{i=1}^{n}\sum_{k=1}^{2} V_{ik}^2}$, *then* $U_{ad} \subset \bigcup_{j=1}^{m} B_j$ *where* $B_j = B(u^j, \delta)$, *and exists convex bounded close set* $D_j = B_j \cap U_{ad}$, $j \in I_m$, *such that* $U_{ad} = \cup_{j=1}^{m} D_j$.

Proof. For $\forall x = (x_{11}, x_{12}, x_{13}, \ldots, x_{n1}, x_{n2}, x_{n3}) \in U_{ad}$, $\exists w_{ik} \in [0, m]$, such that

$$x_{ik} = a_{ik} + w_{ik} V_{ik}/m, \quad i \in I_n, k \in I_3. \tag{15}$$

We decompose $[a_{ik}, b_{ik}]$ into γ segments and set $\gamma = [\sqrt[6n]{m}]$ which is maximum integer and less than $\sqrt[6n]{m}$. Let $\lambda = \gamma^{3n}$, it is obviously that $\lambda < [\sqrt{m}] < m$. Then U_{ad} can be decomposed into λ subregions, denoted by $U_{ad} = \bigcup_{j=1}^{\lambda} E_j$, where $E_j = \{y = (y_{11}, y_{12}, y_{13}, \ldots, y_{n3}) \in U_{ad} \mid y_{ik} \in [c_{ik}^j, d_{ik}^j], i \in I_n, k \in I_3\}$ in which

$$d_{ik}^j - c_{ik}^j = \frac{V_{ik}}{\gamma} \tag{16}$$

and $a_{ik} \leqslant c_{ik}^j \leqslant d_{ik}^j \leqslant b_{ik}$. Based on uniformly distributed character of selected points, for $\forall j \in I_\lambda$, we show that there is a uniformly distributed point at less in E_j. If not so, $j \in I_\lambda$ exists such that the number of uniformly distributed points in E_j is 0. But $m > \lambda$. So $\exists k \in I_\lambda$ and $k \neq j$, such that the number of uniformly distributed points which are in E_k is maximum (or probability is maximum), that is, the uniformly distributed points are dense. This violates the rule of uniform design algorithm. It is also contradictory with $D(P_m) = \min_{j \in I_\lambda} D(P^j)$.

Suppose that $x \in E_j \subset U_{ad}$ $j \in I_\lambda$, it is inevitable that $\exists u^j \in E_j \bigcap P_m$ such that $\|x - u^j\| = \min\{\|x - u^t\| \mid u^t \in E_j \bigcap P_m\}$. Because of (14)–(16), we know that $c_{ik}^j \leqslant a_{ik} + (w_{ik}/m)V_{ik} \leqslant d_{ik}^j$ which may be written in other form $(c_{ik}^j - a_{ik}/V_{ik})m \leqslant w_{ik} \leqslant (d_{ik}^j - a_{ik}/V_{ik})m$. Similarly, there is

$$v_{3(i-1)+k,j} \in \left[\frac{c_{ik}^j - a_{ik}}{V_{ik}} m, \frac{d_{ik}^j - a_{ik}}{V_{ik}} m\right] \bigcap Z^+.$$

So we find

$$\|w_{ik} - v_{3(i-1)+k,j}\| < \frac{m}{V_{ik}}(d_{ik}^j - a_{ik} - c_{ik}^j + a_{ik}) = \frac{m}{V_{ik}}(d_{ik}^j - c_{ik}^j) = \frac{m}{\gamma}.$$

From (14) and (15), we find

$$\|x - u^j\|^2 = \sum_{i=1}^{n}\sum_{k=1}^{3} \frac{V_{ik}^2}{m^2}(w_{ik} - v_{3(i-1)+k,j})^2 \leqslant c^2\gamma^{-2}.$$

Above equation becomes $\|x - u^j\| \leqslant c\gamma^{-1} < \delta$ by using evolution. Because of arbitrary property of $x \in B_j$, $U_{ad} \subset \bigcup_{j=1}^{m} B_j$, that is inevitable where $B_j = B(u^j, \delta)$. We set $D_j = B_j \bigcap U_{ad}$, both U_{ad} and B_j are convex bounded close set, so D_j is also convex bounded close set and $U_{ad} = \bigcup_{j=1}^{m} D_j$. □

To solve the problem CP, we decompose $U_{ad} \subset R^{3n}$ into m subregions $\{D_1, D_2, \ldots, D_m\}$. As local optimal solution is not unique, we increase the value of m until a local optimal solution is on D_j at most subject to

$$u^j \in \operatorname{int} D_j \subset U_{ad} = \bigcup_{j=1}^{m} D_j, \quad j \in I_m. \tag{17}$$

The problem CP can be written as m subproblems CP$_j$ ($j \in I_m$) via above decomposition. Because there is an optimal solution on D_j for CP$_j$, and $J_j(u)$ is continuous on D_j, $J_j(u)$ is local convex on D_j. As $D_j \neq \emptyset$, if the feasible region of CP$_j$ is not empty, u^{j*} is used to represent the optimal solution of CP$_j$, else, we let $J(u^{j*}) = +\infty$. At last, the optimal solution of CP is obtained by $J(u^*) = \min_{j \in I_m}\{J_j(u^{j*})\}$.

4.3. Modified Hooke–Jeeves algorithm

In 1962, Hooke and Jeeves presented Hooke–Jeeves algorithm which belongs to direct method of multivariable function and need not compute derivative [3]. It applies for unconstrained problems. At any case, it does not require the regularity continuity and existence of derivation for objective function. In this paper, to solve constrained minimization problem CP_j on bounded subregion $D_j,\ j \in I_m$, we analyze and adjust the location of iteration points until all constraint conditions are satisfied. And descent tendency must be kept. Simultaneously, descent velocity and efficiency are improved via adjusting the accelerated factors. Since $J_j(u)$ is convex on D_j and Hooke–Jeeves algorithm is convergent for convex function, our modified Hooke–Jeeves method is also convergent for every subproblem CP_j $j \in I_m$.

The modified Hooke–Jeeves method can be simply described as follows:

Step 1: Given the initial step lengths $\{\delta_1, \delta_2, \delta_3\} \subset R$, acceleration factor α, the acceptance deviation $\varepsilon > 0$ and sufficiently large total arc length S; product m initial points $u^k = \{u^k_{ij},\ i \in I_n,\ j \in I_3\} \subset R^{n\times 3}$, $k \in I_m$ by uniform design algorithm; let $x^k = u^k$, $k \in I_m$ and the superscript of optimal solution $k^* = 0$; compute the upper bounds and lower bounds of all variables $L_{u^k_{ij}}$ and $U_{u^k_{ij}}$, $i \in I_m$, $j \in I_3$, $k \in I_m$ by above region partition; let $k = 1$.

Step 2: Compute objective function value $e = J(x^k)$ and the corresponding total arc length s.
If $e = -1$, then let $k = k + 1$, if $k \leqslant m$, return to step 2, else, go to step 18.
If $0 < e < \varepsilon$, then go to step 16.
If $e > \varepsilon$, then let $E = e$, go to step 3.

Step 3: $i = 1,\ j = 1$.

Step 4: Let $x^k_{ij} = x^k_{ij} + \delta_j$. If $x^k_{ij} > U_{u^k_{ij}}$, then $x^k_{ij} = U_{u^k_{ij}}$.

Step 5: Compute $e = J(x^k_{ij})$ and the corresponding total arc length s.
If $e = -1$, then let $k = k + 1$, if $k \leqslant m$, return to step 2, else, go to step 18.
If $0 < e < \varepsilon$, then go to step 16.
If $\varepsilon < e < E$, then let $E = e$, go to step 8.
If $e > E$, then let $x^k_{ij} = x^k_{ij} - \delta_j$, go to step 6.

Step 6: Let $x^k_{ij} = x^k_{ij} - \delta_j$. If $x^k_{ij} < L_{u^k_{ij}}$, then $x^k_{ij} = L_{u^k_{ij}}$.

Step 7: Compute $e = J(x^k_{ij})$ and the corresponding total arc length s.
If $e = -1$, then let $k = k + 1$, if $k \leqslant m$, return to step 2, else, go to step 18.
If $0 < e < \varepsilon$, then go to step 16.
If $\varepsilon < e < E$, then let $E = e$, go to step 8.
If $e > E$, then let $x^k_{ij} = x^k_{ij} + \delta_j$, go to step 8.

Step 8: Let $j = j + 1$. If $j \leqslant 3$, return to step 4, else, go to step 9.

Step 9: Compute the down-ladder operators $d_j = x^k_{ij} - u^k_{ij}$, $j = 1, 2, 3$.

Step 10: Let $x^k_{ij} = x^k_{ij} + \alpha d_j$, $j = 1, 2, 3$. If $L_{u^k_{ij}} < x^k_{ij} < U_{u^k_{ij}}$, go to step 12, else, let $x^k_{ij} = x^k_{ij} - \alpha d_j$, $j = 1, 2, 3$ and $\alpha = 0.9\alpha$, then go to step 11.

Step 11: If $\alpha > 0.01$, return to step 10, else, go to step 14.

Step 12: Compute $e = J(x^k_{ij})$ and the corresponding total arc length s.
If $e = -1$, then let $k = k + 1$, if $k \leqslant m$, return to step 2, else, go to step 18.
If $0 < e < \varepsilon$, then go to step 16.
If $\varepsilon < e < E$, then let $E = e$, return to step 10.
If $e > E$, then let $x^k_{ij} = x^k_{ij} - \alpha d_j$ and $\alpha = 0.9\alpha$, then go to step 13.

Step 13: If $\alpha > 0.01$, return to step 10, else, go to step 14.

Step 14: Let $y^k_{ij} = x^k_{ij}$, $j = 1, 2, 3$ and $i = i + 1$. If $i \leqslant n$, then let $j = 1$, return to step 4, else, go to step 15.

Step 15: Let $\delta_j = 0.9\delta_j$, $j = 1, 2, 3$. If $\delta_1^2 + \delta_2^2 + \delta_3^2 > 0.01$, return to step 3, else, go to step 16.

Step 16: If $s < S$, then let $S = s$ and $k^* = k$.
Step 17: Let $k = k + 1$, If $k \leqslant m$, return to step 2, else, go to step 18.
Step 18: Output the results k^* and $x^{k^*} = \{x_{ij}^{k^*},\ i \in I_n,\ j \in I_3\}$.

5. Results and discussion

The purpose of this article is to explore the most reasonable optimization model and strategy such that the cost is minimum for designing the trajectory of horizontal well. Our method and model are applied for several horizontal wells. Not surprisingly, the results show that the model and strategy are reasonable.

In this paper, we illustrate the software to compute the trajectory of ci-16 slant-plant 146$^\sharp$ well. Oblique angle, azimuthal angle and space coordinates of initial points and objective points are shown in Table 1. Intervals of implement face angle, curvature and arc length are in Table 2. We select 100 initial points ($m = 100$) on U_{ad} using uniform design method, then decompose U_{ad} into 100 subregions D_i ($i \in I_{100}$) in which there are 96 nonempty feasible regions. In our test, let $n = 3$, Table 3 shows five local optimal schemes selected from overall schemes.

For the trajectory design of horizontal well, our method improves computing precision and increases the optimization schemes than [8] about 37%. Unsurprisingly our method and model work better for the trajectory.

Table 1
The based data of ci-16 slant-plant-146$^\sharp$

	Oblique-angle	Azimuthal-angle	X	Y	Z
Well-bottom	10.4	228.18	102.69	−156.39	1673.15
Target-point	89.5	205.5	62.5	−192.9	1718

Table 2
The interval of control variables

	Implement face angle	Radius of curvature	Arc length
First segment	[−50,50]	[40,60]	[10,100]
Second segment	[−50,50]	[40,60]	[10,100]
Third segment	[−50,50]	[40,60]	[10,100]

Table 3
The optimal results of ci-16 slant-plant-146$^\sharp$

		Result 1	Result 2	Result 3	Result 4	Result 5
Implement face angle	First segment	−1.44	−7.46	−1.24	−4.93	3.29
	Second segment	−12.11	23.83	−9.01	2.51	−15.38
	Third segment	8.52	−12.60	7.04	−5.62	23.17
Radius of curvature	First segment	55.98	54.51	54.07	57.08	59.98
	Second segment	50.58	51.47	55.86	51.22	47.92
	Third segment	57.89	53.22	56.21	52.50	56.38
Length of trajectory	First segment	29.54	28.38	26.41	29.97	10.00
	Second segment	22.80	16.36	38.57	33.60	27.30
	Third segment	23.65	31.29	10.83	12.50	38.61
	Total length	75.99	76.03	75.81	76.07	75.91
	Error	0.37	0.32	0.26	0.29	0.49

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