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On vacuum structures of $N = 2$ LSUSY QED equivalent to $N = 2$ NLSUSY model

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Dedicated to the late Professor Julius Wess

Abstract

The vacuum structure of $N = 2$ linear supersymmetry (LSUSY) invariant QED, which is equivalent to $N = 2$ nonlinear supersymmetry (NLSUSY) model, is studied explicitly in two-dimensional space–time ($d = 2$). Two different isometries $SO(1,3)$ and $SO(3,1)$ appear for the vacuum field configuration corresponding to the various parameter regions. Two different field configurations of $SO(3,1)$ isometry describe the two different physical vacua, i.e. one breaks spontaneously both $U(1)$ and SUSY and the other breaks spontaneously SUSY alone.

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Supersymmetry (SUSY) in particle field theory [1,2] is a profound notions related to space–time symmetry. Therefore, the evidences of SUSY and its spontaneous breakdown [3–5] should be studied not only in (low energy) particle physics but also in cosmology, i.e. in a framework necessarily accommodating graviton.

In a series of works along these viewpoints, we have found group theoretically that the $SO(10)$ super-Poicaré (SP) group may be a unique and minimal group among all $SO(N)$ SP groups, which possesses a single irreducible linear (L) SUSY representation accommodating graviton and the standard model (SM) with just three generations of quarks and leptons [6].

The advocated difficulty for constructing non-trivial $N > 8$ SUSY (gravity) theory, the so-called no-go theorem based on S-matrix argument [7,8] can be circumvented by adopting the *nonlinear* (*NL*) *representation* of SUSY [9], i.e. the vacuum degeneracy of the fundamental action. Volkov–Akulov (VA) model [2] gives the NL representation of SUSY describing the dynamics of spin $1/2$ Nambu–Goldstone (NG) fermion accompanying the spontaneous SUSY breaking for $N = 1$.

The fundamental action (called nonlinear supersymmetric general relativity (NLSUSY GR)) of empty Einstein–Hilbert (EH) type for $N > 8$ SUSY (gravity) theory with $N > 8$ supercharges, i.e. the NLSUSY invariant interaction of N NG fermion with spin 2 graviton, has been constructed by extending the geometric arguments of Einstein general relativity (EGR) on Riemann space–time to a new space–time just inspired by NLSUSY, where tangent space–time is specified not only by x_a for $SO(1,3)$ but also by the Grassmanian ψ_α for isomorphic $SL(2C)$ of NLSUSY [10,11]. The compact isomorphic groups $SU(2)$ and $SO(3)$ for the gauge symmetry of 't Hooft–Polyakov monopole are generalized to the noncompact isomorphic groups $SO(1,3)$ and $SL(2C)$ for space–time symmetry and the consequent NLSUSY GR action possesses promising large symmetries isomorphic to $SO(10)$ SP [12,13]. These results mean that the no-go theorem is overcome (circumvented) in the sense that the non-trivial N-extended

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SUSY gravity theory with $N > 8$ has been constructed and graviton and N NG fermions with the spin difference $3/2$ can be coupled in a SUSY invariant way. We think that the geometric arguments of EGR principle has been generalized naturally, which accommodates *geometrically* spin $1/2$ matter as NG fermion accompanying spontaneous SUSY breaking encoded on tangent space–time as NLSUSY.

NLSUSY GR (called superon–graviton model (SGM) from composite viewpoint) on new empty space–time written in the form of the *vacuum* EH type is unstable due to NLSUSY structure of tangent space–time and decays (called *Big Decay* [13]) spontaneously into ordinary EH action with the cosmological constant Λ, NLSUSY action for N NG fermions (called *superons* as hypothetical spin $1/2$ objects) and their gravitational interactions on ordinary Riemann space–time, which ignites the Big Bang of the present universe. We have shown qualitatively that NLSUSY GR may potentially describe a new paradigm (SGM) for the SUSY unification of space–time and matter, where particular SUSY compositeness composed of superons for all (observed) particles except the graviton emerges as an ultimate feature of nature behind the familiar LSUSY models (MSSM, SUSY GUTs) [11,14] and SM as well. That is, all (observed) low energy particles may be eigenstates of $SO(N)$ SP expressed uniquely as the SUSY composites of N superons.

Due to the high nonlinearity of the SGM action we have not yet succeeded in extracting directly the evidence of such (low energy) physical meanings of SGM on curved Riemann space–time.

However, considering that SGM action reduces to the N-extended NLSUSY action in asymptotic Riemann-flat ($e^a{}_\mu \to \delta^a_\mu$) space–time after the Big Decay, it is interesting from the low energy physics viewpoint to construct the N-extended LSUSY theory equivalent to the N-extended NLSUSY model. The relation between $N = 1$ LSUSY representations and $N = 1$ NLSUSY representations in flat (Minkowski) space–time is well understood by using the superfield method [9,15]. The equivalence between $N = 1$ LSUSY *free* theory for LSUSY supermultiplet and $N = 1$ NLSUSY VA model for NG fermion is demonstrated by many authors [16–18] and $N = 2$ case as well [19], where each field of LSUSY supermultiplet is expressed uniquely as the composite of NG fermions of NLSUSY called SUSY invariant relations. Consequently we are tempted to imagine some composite structure (far) behind the SM and the familiar LSUSY models, e.g. MSSM and SUSY GUT.

Recently, we have shown explicitly by the heuristic arguments for simplicity in two space–time dimensions ($d = 2$) [20,21] that $N = 2$ LSUSY interacting QED is equivalent to $N = 2$ NLSUSY model. (Note that the minimal realistic SUSY QED in SGM composite scenario is described by $N = 2$ SUSY [19].)

In this Letter we study explicitly the vacuum structure of $N = 2$ LSUSY QED in the SGM scenario in $d = 2$ [21].

The $N = 2$ NLSUSY action for two superons (NG fermions) ψ^i $(i = 1, 2)$ in $d = 2$ is written as follows,

$$\begin{aligned} S_{N=2\text{NLSUSY}} &= -\frac{1}{2\kappa^2}\int d^2x\,|w| \\ &= -\frac{1}{2\kappa^2}\int d^2x\left\{1 + t^a{}_a + \frac{1}{2!}\left(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a\right)\right\} \\ &= -\frac{1}{2\kappa^2}\int d^2x\left\{1 - i\kappa^2\bar\psi^i \partial\!\!\!/ \psi^i - \frac{1}{2}\kappa^4\left(\bar\psi^i \partial\!\!\!/ \psi^i \bar\psi^j \partial\!\!\!/ \psi^j - \bar\psi^i\gamma^a\partial_b\psi^i\bar\psi^j\gamma^b\partial_a\psi^j\right)\right\} \\ &= -\frac{1}{2\kappa^2}\int d^2x\left\{1 - i\kappa^2\bar\psi^i \partial\!\!\!/ \psi^i - \frac{1}{2}\kappa^4\epsilon^{ab}\left(\bar\psi^i\psi^j\partial_a\bar\psi^i\gamma_5\partial_b\psi^j + \bar\psi^i\gamma_5\psi^j\partial_a\bar\psi^i\partial_b\psi^j\right)\right\}, \end{aligned} \tag{1}$$

where $\kappa^2 = (\frac{c^4\Lambda}{8\pi G})^{-1}$ in the SGM scenario and

$$|w| = \det\left(w^a{}_b\right) = \det\left(\delta^a_b + t^a{}_b\right), \qquad t^a{}_b = -i\kappa^2\bar\psi^i\gamma^a\partial_b\psi^i. \tag{2}$$

While, the helicity states contained in $d = 2$ $N = 2$ LSUSY QED are the vector supermultiplet containing $U(1)$ gauge field

$$\begin{pmatrix} +1 \\ +\frac{1}{2},\ +\frac{1}{2} \\ 0 \end{pmatrix} + [\text{CPT conjugate}],$$

and the scalar supermultiplet for matter fields

$$\begin{pmatrix} +\frac{1}{2} \\ 0,\ 0 \\ -\frac{1}{2} \end{pmatrix} + [\text{CPT conjugate}].$$

The $N = 2$ LSUSY QED action in $d = 2$ for the massless case is written as follows [21],

$$\begin{aligned} S_{N=2\text{SUSYQED}} = \int d^2x\bigg[&-\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar\lambda^i \partial\!\!\!/ \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a\phi)^2 + \frac{1}{2}D^2 - \frac{1}{\kappa}\xi D \\ &+ \frac{i}{2}\bar\chi \partial\!\!\!/ \chi + \frac{1}{2}\left(\partial_a B^i\right)^2 + \frac{i}{2}\bar\nu \partial\!\!\!/ \nu + \frac{1}{2}\left(F^i\right)^2 \end{aligned}$$

$$+ f\big(A\bar{\lambda}^i\lambda^i + \epsilon^{ij}\phi\bar{\lambda}^i\gamma_5\lambda^j + A^2 D - \phi^2 D - \epsilon^{ab}A\phi F_{ab}\big)$$
$$+ e\bigg\{iv_a\bar{\chi}\gamma^a\nu - \epsilon^{ij}v^a B^i\partial_a B^j + \bar{\lambda}^i\chi B^i + \epsilon^{ij}\bar{\lambda}^i\nu B^j - \frac{1}{2}D\big(B^i\big)^2$$
$$+ \frac{1}{2}(\bar{\chi}\chi + \bar{\nu}\nu)A - \bar{\chi}\gamma_5\nu\phi\bigg\} + \frac{1}{2}e^2\big({v_a}^2 - A^2 - \phi^2\big)\big(B^i\big)^2\bigg], \tag{3}$$

where $(v^a, \lambda^i, A, \phi, D)$ $(F_{ab} = \partial_a v_b - \partial_b v_a)$ is the off-shell vector supermultiplet containing v^a for a $U(1)$ vector field, λ^i for doublet (Majorana) fermions and A for a scalar field in addition to ϕ for another scalar field and D for an auxiliary scalar field, while (χ, B^i, ν, F^i) is off-shell scalar supermultiplet containing (χ, ν) for two (Majorana) fermions, B^i for doublet scalar fields and F^i for auxiliary scalar fields. Also ξ is an arbitrary dimensionless parameter giving a magnitude of SUSY breaking mass, and f and e are Yukawa and gauge coupling constants with the dimension $(\text{mass})^1$, respectively.

The equivalence of LSUSY QED action (3) to NLSUSY action (1) up to surface terms was shown explicitly by substituting the (generalized) SUSY invariant relations for the component fields of supermultiplet into the LSUSY QED action (3) [21]. $N = 2$ LSUSY QED action (3) can be rewritten as the familiar manifestly covariant form in terms of the complex quantities defined by

$$\chi_D = \frac{1}{\sqrt{2}}(\chi + i\nu), \qquad B = \frac{1}{\sqrt{2}}\big(B^1 + iB^2\big), \qquad F = \frac{1}{\sqrt{2}}\big(F^1 - iF^2\big). \tag{4}$$

The resulting action is manifestly invariant under the local $U(1)$ transformation

$$(\chi_D, B, F) \to \big(\chi'_D, B', F'\big)(x) = e^{i\Omega(x)}(\chi_D, B, F)(x),$$
$$v_a \to v'_a(x) = v_a(x) + \frac{1}{e}\partial_a\Omega(x). \tag{5}$$

(For further details see Ref. [21].)

For extracting the low energy particle physics contents of $N = 2$ SGM (NLSUSY GR) we consider in Riemann-flat asymptotic space–time, where $N = 2$ SGM reduces to essentially $N = 2$ NLSUSY action equivalent to $N = 2$ SUSY QED action, i.e.

$$L_{N=2\text{SGM}} \overset{{e^a}_\mu \to \delta^a_\mu}{\longrightarrow} L_{N=2\text{NLSUSY}} + [\text{surface terms}] = L_{N=2\text{SUSYQED}}. \tag{6}$$

Now we study the vacuum structure of $N = 2$ SUSY QED action (3). The vacuum is determined by the minimum of the potential $V(A, \phi, B^i, D)$,

$$V\big(A, \phi, B^i, D\big) = -\frac{1}{2}D^2 + \left\{\frac{\xi}{\kappa} - f\big(A^2 - \phi^2\big) + \frac{1}{2}e\big(B^i\big)^2\right\}D. \tag{7}$$

Substituting the solution of the equation of motion for the auxiliary field D we obtain

$$V\big(A, \phi, B^i\big) = \frac{1}{2}f^2\left\{A^2 - \phi^2 - \frac{e}{2f}\big(B^i\big)^2 - \frac{\xi}{f\kappa}\right\}^2 \geqslant 0. \tag{8}$$

The configurations of the fields corresponding to the vacua in (A, ϕ, B^i)-space, which are $SO(1,3)$ or $SO(3,1)$ invariant, are classified according to the signatures of the parameters e, f, ξ, κ as follows:

(I) For $ef > 0$, $\frac{\xi}{f\kappa} > 0$ case,

$$A^2 - \phi^2 - \big(\tilde{B}^i\big)^2 = k^2 \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}}B^i,\ k^2 = \frac{\xi}{f\kappa}\right). \tag{9}$$

(II) For $ef < 0$, $\frac{\xi}{f\kappa} > 0$ case,

$$A^2 - \phi^2 + \big(\tilde{B}^i\big)^2 = k^2 \quad \left(\tilde{B}^i = \sqrt{-\frac{e}{2f}}B^i,\ k^2 = \frac{\xi}{f\kappa}\right). \tag{10}$$

(III) For $ef > 0$, $\frac{\xi}{f\kappa} < 0$ case,

$$-A^2 + \phi^2 + \big(\tilde{B}^i\big)^2 = k^2 \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}}B^i,\ k^2 = -\frac{\xi}{f\kappa}\right). \tag{11}$$

(IV) For $ef < 0$, $\frac{\xi}{f\kappa} < 0$ case,

$$-A^2 + \phi^2 - \big(\tilde{B}^i\big)^2 = k^2 \quad \left(\tilde{B}^i = \sqrt{-\frac{e}{2f}}B^i,\ k^2 = -\frac{\xi}{f\kappa}\right). \tag{12}$$

We find that the vacua (I) and (IV) with $SO(1,3)$ isometry in (A, ϕ, B^i)-space are unphysical, for they produce pathological wrong sign kinetic terms for the fields expanded around the vacuum.

As for the cases (II) and (III) we perform similar arguments as shown below and find that two different physical vacua appear. The physical particle spectrum is obtained by expanding the field (A, ϕ, B^i) around the vacuum with $SO(3,1)$ isometry.

For case (II), the following two expressions (IIa) and (IIb) are considered:

(IIa)

$$A = (k+\rho)\sin\theta\cosh\omega, \qquad \phi = (k+\rho)\sinh\omega,$$
$$\tilde{B}^1 = (k+\rho)\cos\theta\cos\varphi\cosh\omega, \qquad \tilde{B}^2 = (k+\rho)\cos\theta\sin\varphi\cosh\omega$$

and

(IIb)

$$A = -(k+\rho)\cos\theta\cos\varphi\cosh\omega, \qquad \phi = (k+\rho)\sinh\omega,$$
$$\tilde{B}^1 = (k+\rho)\sin\theta\cosh\omega, \qquad \tilde{B}^2 = (k+\rho)\cos\theta\sin\varphi\cosh\omega.$$

Note that for the case (III) the arguments are the same by exchanging A and ϕ, which we call (IIIa) and (IIIb).

Substituting these expressions into $L_{N=2\mathrm{SUSYQED}}(A, \phi, B^i)$ and expanding the action around the vacuum configuration we obtain the physical particle contents. For the cases (IIa) and (IIIa) we obtain

$$\begin{aligned} L_{N=2\mathrm{SUSYQED}} = {} & \frac{1}{2}\left\{(\partial_a\rho)^2 - 2(-ef)k^2\rho^2\right\} + \frac{1}{2}\left\{(\partial_a\theta)^2 + (\partial_a\omega)^2 - 2(-ef)k^2\left(\theta^2+\omega^2\right)\right\} + \frac{1}{2}(\partial_a\varphi)^2 \\ & - \frac{1}{4}(F_{ab})^2 + (-ef)k^2 v_a^2 + \frac{i}{2}\bar{\lambda}^i\partial\!\!\!/\lambda^i + \frac{i}{2}\bar{\chi}\partial\!\!\!/\chi + \frac{i}{2}\bar{\nu}\partial\!\!\!/\nu + \sqrt{-2ef}\left(\bar{\lambda}^1\chi - \bar{\lambda}^2\nu\right) + \cdots, \end{aligned} \tag{13}$$

and the consequent mass generation

$$m_\rho^2 = m_\theta^2 = m_\omega^2 = m_{v_a}^2 = 2(-ef)k^2 = -\frac{2\xi e}{\kappa}, \qquad m_{\lambda^i} = m_\chi = m_\nu = m_\varphi = 0. \tag{14}$$

(Note that φ is the NG boson for the spontaneous breaking of $U(1)$ symmetry, i.e. the $U(1)$ phase of B, and totally gauged away by the Higgs–Kibble mechanism with $\Omega(x) = \sqrt{e\kappa/2}\varphi(x)$ for the $U(1)$ gauge (5).) The vacuum breaks both SUSY and the local $U(1)$ spontaneously. All bosons have the same mass which is different from the cases (IIb) and (IIIb) and remarkably all fermions remain massless. The physical origin of the off-diagonal mass terms $\sqrt{-2ef}(\bar{\lambda}^1\chi - \bar{\lambda}^2\nu) = \sqrt{-2ef}(\bar{\chi}_D\lambda + \bar{\lambda}\chi_D)$ $(\lambda \sim \lambda^1 - i\lambda^2)$ for fermions is unclear, which would induce mixings of fermions and/or the lepton (baryon) number violations.

By similar computations for (IIb) and (IIIb) we obtain

$$\begin{aligned} L_{N=2\mathrm{SUSYQED}} = {} & \frac{1}{2}\left\{(\partial_a\rho)^2 - 4f^2k^2\rho^2\right\} + \frac{1}{2}\left\{(\partial_a\theta)^2 + (\partial_a\varphi)^2 - e^2k^2\left(\theta^2+\varphi^2\right)\right\} + \frac{1}{2}(\partial_a\omega)^2 \\ & - \frac{1}{4}(F_{ab})^2 + \frac{1}{2}\left(i\bar{\lambda}^i\partial\!\!\!/\lambda^i - 2fk\bar{\lambda}^i\lambda^i\right) + \frac{1}{2}\left\{i(\bar{\chi}\partial\!\!\!/\chi + \bar{\nu}\partial\!\!\!/\nu) - ek(\bar{\chi}\chi + \bar{\nu}\nu)\right\} + \cdots \end{aligned} \tag{15}$$

and the following mass spectrum which indicates that SUSY is broken spontaneously as expected;

$$m_\rho^2 = m_{\lambda^i}^2 = 4f^2k^2 = \frac{4\xi f}{\kappa}, \qquad m_\theta^2 = m_\varphi^2 = m_\chi^2 = m_\nu^2 = e^2k^2 = \frac{\xi e^2}{\kappa f}, \qquad m_{v_a} = m_\omega = 0, \tag{16}$$

which can produce mass hierarchy by the factor $\frac{e}{f}$. The local $U(1)$ gauge symmetry is not broken. The massless scalar ω is an NG boson for the degeneracy of the vacuum in $(A, \tilde{B}_2)$-space, which is gauged away provided the gauge symmetry between the vector and the scalar multiplet is introduced.

From these arguments we conclude that $N=2$ SUSY QED is equivalent to $N=2$ NLSUSY action, i.e. the matter sector (in asymptotic flat space) of $N=2$ SGM produced by Big Decay (phase transition) of $N=2$ NLSUSY GR (new space–time), possesses two different vacua, the type (a) and (b) in the $SO(3,1)$ isometry of (II) and (III). The resulting models describe two charged chiral fermions, two neutral chiral fermions, one massive vector, one charged massive scalar and one massless scalar: $({\psi_L}^{cj} \sim (\tilde{\chi}_{DL}, \tilde{\nu}_{DL}), {\lambda_L}^j \sim \tilde{\lambda}^j_{DL}, v_a, \phi^c \sim \theta + i\omega, \phi^0 \sim \rho;\ j=1,2)$ for type (a) where (χ, ν, λ) are written by left-handed Dirac fields, and one charged Dirac fermion, one neutral (Dirac) fermion, a photon, one charged scalar and one neutral complex scalar (two neutral scalars): $({\psi_D}^c \sim \chi + i\nu, {\lambda_D}^0 \sim \lambda^1 - i\lambda^2, v_a, \phi^c \sim \theta + i\varphi, \phi^{0c} \sim \rho + i\omega)$ for type (b), which are the composites of superons.

As for cosmological significances of $N=2$ SUSY QED in the SGM scenario, the vacuum of the cases (IIb) and (IIIb) produces the same interesting predictions as already pointed out in $N=2$ pure SUSY QED in the SGM scenario [13], which may simply explain the observed mysterious (numerical) relations and give a new insight into the origin of mass

$$\big(\text{(dark) energy density of the universe}\big)_{\text{obs}} \sim 10^{-12} \sim (m_\nu)_{\text{obs}}{}^4 \sim \frac{\Lambda}{G} \sim g_{\text{sv}}{}^2,$$

provided $f\xi \sim O(1)$ and λ^i is identified with neutrino. (Λ, G and g_{sv} are the cosmological constant of NLSUSY GR (SGM) on *empty* new space–time for *everything*, the Newton gravitational constant and the superon–vacuum coupling constant via the supercurrent, respectively [10,13].) While the vacua of the cases (IIa) and (IIIa), equipped with automatic mixings of fermions in $d = 2$ so far, give new features characteristic of $N = 2$. They may be generic for $N > 2$ and deserve further investigations.

The similar investigation in $d = 4$ is urgent and the extension to large N, especially to $N = 5$ is important for *superon quintet hypothesis* in SGM scenario with $N = \underline{10} = \underline{5} + \underline{5^*}$ [11] and to $N = 4$ is suggestive for analyzing the anomaly free nontrivial $d = 4$ field theory. Also NLSUSY GR in extra space–time dimensions is an interesting problem, which can describe all observed particles as elementary *à la* Kaluza–Klein.

Our analysis shows that the vacua of the N-extended NLSUSY GR action in the SGM scenario possess rich structures promising for the unified description of nature, where N-extended LSUSY theory appears as the vacuum field configurations of N-extended NLSUSY theory on Minkowski tangent space–time.

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