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Study on interval intuitionistic fuzzy multi-attribute group decision making method based on Choquet integral

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Abstract

In this paper, a method based on Choquet integral is proposed to solve the interval intuitionistic fuzzy multiple attribute group decision making problems. Firstly, some concepts about interval intuitionistic fuzzy measure are defined, through the strict mathematical reasoning to prove the measure we proposed satisfying the axiomatic system of fuzzy measure. Then, on the basis of fuzzy measure and game theory, we propose two models to determine fuzzy measure based on interval intuitionistic fuzzy entropy and weight information matrix. By calculating the Shapely value to determine expert weights, we establish a linear programming model based on relative entropy to determine the fuzzy measure of attribute weights to reflect the interactive characteristics among the criteria, using the Choquet integral to aggregate the decision-making information. Finally, we give the process of decision making in details.

Keywords: Choquet integral; fuzzy measure; interval intuitionistic fuzzy entropy; group decision making

1. Introduction

Since it was introduced by Zadeh [1,2] in 1965, fuzzy sets theory has been the most important tool for dealing with uncertainty complex systems. It has been widely used in economics [3], management [4], artificial intelligence [5], image processing [6], processing control [7], pattern recognition [8,9], decision making [10,11] and some other fields. In 1986, Atanassov [12,13] extended traditional fuzzy sets theory with the concept of intuitionistic fuzzy sets. Intuitionistic fuzzy sets are characterized by three important indexes: membership degree, non-membership degree and hesitancy degree. From three different dimensions to describe the characteristics of complex systems that is more accurately and effectively to deal with vagueness and uncertainty. With a development of intuitionistic fuzzy sets theory and its application in the past few years, intuitionistic fuzzy sets has become an important branch, and attracted more and more researchers’ attention. Xu [18,24,25] has made a large number of excellent researches in the field of intuitionistic fuzzy sets, especially in the aspect of aggregation operator, proposed a series of intuitionistic fuzzy information aggregation operators and applied these theoretical results successfully in multiple attribute decision-making problems. Wu and F.Chiclana [14] proposed non-dominance and attitudinal prioritisation methods for intuitionistic fuzzy environment, defined a new score function to compute the interval-valued intuitionistic fuzzy sets. In addition, Yang and F. Chiclana [15,16] extended consistency of 2D and 3D distances of intuitionistic fuzzy sets for dealing with fuzzy preference information in GDM.

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As the theoretical foundation of fuzzy mathematics, fuzzy measure and fuzzy integral were originally defined by Choquet [17] in 1954. Different from classic integral, fuzzy integral focus on non-additive cases. Based on the theory of classic mathematics, we can prove the fact that when Choquet integral with the condition of additive, Choquet integral is reduced to Lebesgue integral. Compared with sugeno integral, Choquet integral has better mathematical properties and its theoretical basis is more complete. Hence, Choquet integral is more suited to deal with the problem of fuzzy quantitative. In recent years, many scholars have done a lot of good researches in this field and applied Choquet integral in many aspects, especially in multiple attribute decision-making problems. Xu [18] used Choquet integral to propose some intuitionistic fuzzy aggregation operators, extended interval value fuzzy operators under various fuzzy environment, and then applied them in intuitionistic fuzzy decision making. Tan [19] used Choquet integral to determine attribute weight and applied it in decision making problems under interval intuitionistic environment. Grabisch and Raufatse [20] investigated the statistical properties of Choquet and Sugeno integrals to deal with some sets of learning data through some procedure. Marichal [21] proposed an axiomatic approach of discrete Choquet to aggregate interacting criteria and extended the weighted arithmetic mean by considering the interaction. Huy et al.[22] proposed a fuzzy decision support method for customer preferences analysis based on Choquet integral and applied it to plan business management and marketing strategies. Rong et al.[23] presented a classification method of heterogeneous fuzzy data by Choquet integral with fuzzy-valued integrand and used this method in fuzzy decision trees and fuzzy-neuro networks. On the basis of theoretical analysis, some extensions were also discussed. When using Choquet integral to solve multiple attribute decision-making problems, the most difficulty is how to determine fuzzy measure in a reasonable method. To date, in current literatures, fuzzy measure is given by decision makers in advance, which makes it difficult to avoid the subjective preferences of decision makers. Hence, the reliability of the decision-making result is not so convincing. Therefore, how to determine the fuzzy measure based on expert decision-making information has become a very important issue in theory and practice.

The remainder of this paper is organized as follows. In Section 2, we briefly introduce some basic concepts of interval intuitionistic fuzzy sets and Choquet integral. In Section 3, we propose the axiomatic definition of interval fuzzy measure and get some useful propositions. In Section 4, we present a new fuzzy decision-making method based on interval intuitionistic Choquet integral with fuzzy information entropy, and by establishing interval intuitionistic fuzzy measure to determine the measure of expert weights and attribute weights. Finally, we end the paper with some conclusions and point out future researches in Section 5.

2. Preliminaries

**Definition 1** ([12,13]). Let $X$ be a universe of discourse, then

$$A = \{x, (\mu_A^L, \mu_A^R), (\nu_A^L, \nu_A^R)|x \in X\}$$

is called an interval intuitionistic set, where $\mu_A(x) = [\mu_A^L, \mu_A^R]$, $\nu_A(x) = [\nu_A^L, \nu_A^R]$ are called as interval membership degree and non-membership degree of $x$ which is belong to $X$, respectively. The parameters should satisfy the following conditions:

1. $\forall x \in X, [\mu_A^L(x), \mu_A^R(x)] \subseteq [0, 1]$, $[\nu_A^L(x), \nu_A^R(x)] \subseteq [0, 1]$;
2. $\forall x \in X, 0 \leq \mu_A^R(x) + \nu_A^R(x) \leq 1$.

Especially, when $\mu_A^L(x) = \mu_A^R(x), \nu_A^L(x) = \nu_A^R(x)$, for each $x \in X$. Then the interval intuitionistic set reduces to a common fuzzy set.

**Definition 2** ([24,25]). Let $\alpha = ([a^L, a^R], [b^L, b^R]), \beta = ([c^L, c^R], [d^L, d^R])$ are two interval intuitionistic numbers, the operation laws are shown as follows:

1. $\alpha \cap \beta = ([\min(a^L, c^L), \min(a^R, c^R)], [\max(b^L, d^L), \min(b^R, d^R)])$;
2. $\alpha \cup \beta = ([\max(a^L, c^L), \min(a^R, c^R)], [\max(b^L, d^L), \min(b^R, d^R)])$;
3. $\alpha \circ \beta = ([a^L+c^L-b^L-c^R, a^R+c^L-b^R-c^R], [b^L+d^L-b^R-d^R])$;
4. $\alpha \otimes \beta = ([a^Lc^L, d^Lc^R], [b^L+d^L-b^R-d^R])$;
(5) \( \alpha^C = ([b_l^C, b_r^C], [a_l^C, a_r^C]) \);

(6) \( \alpha^1 = ([a_l^1, a_r^1], [1 - (1 - b_l^1), 1 - (1 - b_r^1)]) \), \( \lambda > 0 \).

**Definition 3** ([24,25]). Let \( \alpha = ([a_l^i, a_r^i], [b_l^i, b_r^i]) \) be an interval intuitionistic number, the score function of interval intuitionistic number \( \alpha \) is defined as follows:

\[
s(\alpha) = \frac{a_l^i + a_r^i - b_l^i - b_r^i}{2}
\]

Where \( s(\alpha) \in [-1, 1] \). Obviously, the greater \( s(\alpha) \) is, the larger interval intuitionistic number \( \alpha \) will be.

**Definition 4** ([24,25]). Let \( \alpha_i = ([a_l^i, a_r^i], [b_l^i, b_r^i]) \) \( (i = 1, 2, \cdots n) \) be a collection of interval intuitionistic numbers, and let \( \text{IIFGA}: \Theta^o \rightarrow \Theta \)

\[
\text{IIFGA}_w(a_1, a_2, \cdots a_n) = \left( \prod_{i=1}^{n} (a_l^i)^{w_i}, \prod_{i=1}^{n} (a_r^i)^{w_i}, [1 - \prod_{i=1}^{n} (1 - b_l^i)^{w_i}, 1 - \prod_{i=1}^{n} (1 - b_r^i)^{w_i}] \right)
\]

then the function IIFGA is called an interval intuitionistic fuzzy weighted geometric averaging operator, where \( w = (w_1, w_2, \cdots w_n)^T \) is the weight of \( \alpha_i (i = 1, 2, \cdots, n) \), with \( w_i \in [0, 1] \) and \( \sum_{i=1}^{n} w_i = 1, w_i \geq 0 \).

**Definition 5.** Let \( N = \{1, 2, \cdots, n\} \) be a positive integer set, for any subset \( M \subseteq A \), there exists a real-value function such that:

\[
\begin{align*}
R(\phi) &= 0 \\
R(M_1 \cup M_2) &\geq R(M_1) + R(M_2), M_1 \cap M_2 = \phi
\end{align*}
\]

Then \([N, R]\) is called a multi-user cooperative game theory, \( R \) is defined as eigen function in union set, \( R(M) \) represents the revenue of the Union. Let \( \sigma = (\sigma_1(v), \sigma_2(v), \cdots \sigma_n(v)) \) be an allocation vector of the alliance, where \( \sigma_i(v) \) denotes the shapely value of member \( i \). The formula is defined as follows:

\[
\sigma_i(v) = \frac{(n - |M|)!(|M| - 1)!}{n!} (V(S) - V(S - \{i\}))
\]

Where \( V(S - \{i\}) \) denotes the revenue after member \( i (i = 1, 2, \cdots, n) \) is removed from the union set. Therefore, \( V(S) - V(S - \{i\}) \) can be regarded as the contribution of member \( i \), \( |M| \) represents the score of union.

**Definition 6** ([26]). Let \( X \) be a universe of discourse, \( P(X) \) be the power set of \( X \). A fuzzy measure on \( X \) be a set function \( \mu: P(X) \rightarrow [0, 1] \) satisfying the following properties:

(1) \( \mu(\phi) = 0, \mu(X) = 1 \) (boundary conditions);

(2) If \( A \subseteq B \) implies \( \mu(A) \leq \mu(B) \), for all \( A, B \subseteq X \) (monotonicity);

(3) \( \mu(A + B) = \mu(A) + \mu(B) + \mu(A)\mu(B) \), where \( A, B \subseteq X \) and \( A \cap B = \phi, \lambda \geq -1 \);

(4) \( \mu(\bigcap_{n=1}^{\infty} A_n) = \lim_{n \to \infty} \mu(A_n) \), whenever \( A_n \subseteq A_{n+1}, A_n \subseteq X, n \in N \) (continuity from below);

(5) \( \mu(\bigcup_{n=1}^{\infty} A_n) = \lim_{n \to \infty} \mu(A_n) \), whenever \( A_n \supseteq A_{n+1}, A_n \subseteq X, n \in N \) (continuity from above).

**Definition 7** ([27]). Let \( X \) be a universe of discourse, and \( X = \bigcup_{i=1}^{n} x_i \), then \( \lambda - \text{fuzzy measure} \mu \) satisfies following equation

\[
\mu(X) = \begin{cases} 
\frac{1}{n} \left( \prod_{i=1}^{n} [1 + \lambda \mu(x_i)] - 1 \right) & \text{if } \lambda \neq 0 \\
\sum_{i=1}^{n} \mu(x_i) & \text{if } \lambda = 0
\end{cases}
\]

Based on Eq.(6), the value of \( \lambda \) can be uniquely determined from \( \mu(X) = 1 \), which is equivalent by solving the following equation

\[
\lambda + 1 = \prod_{i=1}^{n} (1 + \lambda \mu(x_i))
\]
Definition 8 ([28]). Let μ be a fuzzy measure on X, f is a nonnegative, real-valued, measurable function, then the Choquet integral can be defined as:

\[
(c) \int f \, d\mu = \int_0^\infty \mu(F_a) \, da
\]  

(8)

where \( F_a = \{ x | f(x) \geq a \} \) denotes the \( \alpha \) - cut of the function \( f \). When \( X = \{ x_1, x_2, \cdots, x_n \} \) be a discrete set, the discrete form of Choquet integral with respect to a fuzzy measure \( \mu \) is defined as:

\[
(c) \int f \, d\mu = \sum_{i=1}^{n} (f(x^*_i) - f(x^*_{i+1})) \mu([x^*_i, x^*_{i+1}, \cdots, x^*_n])
\]  

(9)

Where \( \{ x^*_1, x^*_2, \cdots, x^*_n \} \) indicates a permutation of \( \{ x_1, x_2, \cdots, x_n \} \) such that \( f(x^*_1) \leq f(x^*_2) \leq \cdots \leq f(x^*_n) \).

3. Interval intuitionistic fuzzy measure

In this section, we will give the definition of interval intuitionistic fuzzy measure and propose two useful propositions, a new interval intuitionistic fuzzy entropy based on tangent trigonometric function is also proposed.

Definition 9. Let \( \mu \) be an interval intuitionistic fuzzy measure, \( \mu(A) = ([m_1(A), m_2(A)], [m_3(A), m_4(A)]) \) on \( P(X) \), the mapping \( f \) is measurable in the sense of Lebesgue, satisfying the following properties:

1. \( m(\phi) = ([m_1(\phi), m_2(\phi)], [m_3(\phi), m_4(\phi)]) = ([0, 0], [1, 1]) \);
2. \( m(X) = ([m_1(X), m_2(X)], [m_3(X), m_4(X)]) = ([1, 1], [0, 0]) \);
3. If \( E, F \subset X, E \subseteq F \), then \( m_1(E) \leq m_1(F), m_2(E) \leq m_2(F) \) and \( m_3(E) \geq m_3(F), m_4(E) \geq m_4(F) \);
4. For any \( E \subset X, m_2(E) + m_4(E) \leq 1 \).

Based on the decomposition theorem, the interval intuitionistic fuzzy Choquet integral is defined as:

\[
(c) \int f \circ dm = (\int [a_1^R dm_1, \int a_1^L dm_2], [\int b_1^R dm_3, \int b_1^L dm_4])
\]  

(10)

where \( f = ([a_1^L, a_1^R], [b_1^L, b_1^R]), m = ([m_1, m_2], [m_3, m_4]). \) For any \( A \subset X \), we have \( m_1(A) \leq m_2(A), m_3(A) \leq m_4(A), m_2(A) + m_4(A) \leq 1, \) “∗” denotes the dual operation, i.e., \( \overline{f(x)} = 1 - f(x), \overline{m(A)} = 1 - m(A) \).

Then the discrete form of interval intuitionistic fuzzy Choquet integral can be expressed as follows:

\[
(c) \int f \circ dm = ([\sum_{i=1}^n a_i^L(m_iA_i) - m_1(A_{i+1}), \sum_{i=1}^n a_i^R(m_iA_i) - m_2(A_{i+1})],
\]

\[
[\sum_{i=1}^n b_i^L(m_iB_i) - m_3(B_i), \sum_{i=1}^n b_i^R(m_iB_i) - m_4(B_i)]
\]  

(11)

where \( A_i = \{ x_i, x_{i+1}, \cdots, x_n \} f(x_i) \geq f(x_j) \), \( B_i = \{ x_i, x_{i+1}, \cdots, x_n \} f(x_i) \leq f(x_j) \).

According to Definition 9, it is easy to obtain the following two propositions.

Proposition 1. Let \( X \) be a universe of discourse, \( f = ([f_1, f_2], [f_3, f_4]) \) be an interval intuitionistic fuzzy-valued function, \( \overline{0} = ([0, 0], [1, 1]), \overline{1} = ([1, 1], [0, 0]) \) are the smallest and largest interval intuitionistic fuzzy function, then

1. (Idempotency) \( c(\overline{0}, \overline{0}, \cdots, \overline{0}) = \overline{0}, c(\overline{1}, \overline{1}, \cdots, \overline{1}) = \overline{1}, c(f, f, \cdots, f) = f; \)
2. (Boundary) \( \min(f^{(1)}, f^{(2)}, \cdots, f^{(n)}) \leq c(f^{(1)}, f^{(2)}, \cdots, f^{(n)}) \leq \max(f^{(1)}, f^{(2)}, \cdots, f^{(n)}); \)
3. (Monotonicity) If \( f^{(k)} \leq f^{(k)} \), then \( c(f^{(1)}, f^{(2)}, \cdots, f^{(k)}, \cdots, f^{(n)}) \leq c(f^{(1)}, f^{(2)}, \cdots, f^{(k)}), \cdots, f^{(n)}). \)

Proof. (1) Based on the definition of Choquet integral, we have \( c(\overline{0}, \overline{0}, \cdots, \overline{0}) = \overline{0} \times \sum_{i=1}^n (m(x_i) - m(x_{i+1})) = \overline{0} \times 1 = \overline{0}, \)

\( c(\overline{1}, \overline{1}, \cdots, \overline{1}) = \overline{1} \times \sum_{i=1}^n (m(x_i) - m(x_{i+1})) = \overline{1} \times 1 = \overline{1}, c(f, f, \cdots, f) = f \times \sum_{i=1}^n (m(x_i) - m(x_{i+1})) = f \times 1 = f. \)

(2) \( c(\min f^{(i)}, \cdots, \min f^{(i)}) \leq c(f^{(1)}, f^{(2)}, \cdots, f^{(n)}) \leq c(\max f^{(i)}, \cdots, \max f^{(i)}), c(\min f^{(i)}, \cdots, \min f^{(i)}) = \min f^{(i)}; \)

(3) Based on the monotonic of fuzzy Choquet integral and Eq.(9), the conclusion is obvious. \( \square \)
Proposition 2. Let $X$ be a universe of discourse, $f = ([f_1, f_2], [f_3, f_4])$ be an interval intuitionistic fuzzy-valued function, $\bar{0} = ([0, 0], [1, 1]), \bar{1} = ([1, 1], [0, 0])$ are the smallest and largest interval intuitionistic fuzzy function, then

(1) $\int f \circ \bar{1} = \max f$;
(2) $\int f \circ \bar{0} = \min f$;
(3) For any fuzzy measure $\mu$ on $X$, $\min f \leq (c) \int f \circ \mu = \max f$.

Based on Proposition 1 and Definition 8, the conclusion is obvious.

Definition 10. Let $A = \{x_i, ([a^l_i, a^R_i], [b^l_i, b^R_i])|x_i \in A\}$, the entropy of interval intuitionistic fuzzy set is defined as:

$$E(A) = \frac{1}{n} \sum_{i=1}^{n} (1 - \tan \frac{|b^l_i - a^l_i| + |b^R_i - a^R_i|}{4(\pi^R_i + \pi^l_i + 2)\pi})$$

(12)

The interval intuitionistic fuzzy entropy satisfies the following properties:

(1) $E(A) = 0$, if and only if $a^l_i = a^R_i = 0, b^l_i = b^R_i = 1$ or $a^l_i = a^R_i = 1, b^l_i = b^R_i = 0$;
(2) $E(A) = 1$, if and only if $[a^l_i, a^R_i] = [b^l_i, b^R_i]$, for any $x_i \in A$;
(3) $E(A) = E(A^C)$;
(4) $E(A) \leq E(B)$, $\forall x_i \in X$, if $a^l_i \leq a^R_i \leq b^l_i \leq b^R_i$, then $E(A^C) \leq E(A)$, $E(A^C) \leq E(B)$;
then $a^l_i \geq a^R_i \geq a^l_i \leq a^R_i \leq b^l_i \geq b^R_i \geq b^l_i \geq b^R_i$, for any $x_i \in A$.

4. A new fuzzy decision-making method based on interval intuitionistic Choquet integral

Multiple attribute group decision making problem is the process that a group of decision makes (DMs) to select the best alternative from the overall feasible alternatives. Interval intuitionistic attribute group decision making problem is a special case of MAGDM, in which the attribute value takes the form of interval intuitionistic fuzzy number. In GDM problem, how to aggregate the expert opinions and how to obtain the interactive among the attributes are two important problems which should be deeply considered. In the following, based on the interval intuitionistic fuzzy entropy and interval intuitionistic fuzzy measure we have mentioned above, a new approach is proposed to solve interval intuitionistic attribute group decision making problem, we construct two models to determine the fuzzy measure which are used to calculate the interactive importance measure of attributes and correlation measure of the experts (e.g. absolute measure, relative measure). First, we describe the interval intuitionistic fuzzy multiple attribute group decision making problems in this paper. For an interval intuitionistic fuzzy multiple attribute group decision making problems, let $M = \{A_1, A_2, \cdots, A_m\}$ be the set of alternatives, $N = \{C_1, C_2, \cdots, C_n\}$ be the set of attributes, and $D = \{D_1, D_2, \cdots, D_p\}$ be the set of DMs. $A_j^{(i)} = ([a^l_{ij}, a^R_{ij}], [b^l_{ij}, b^R_{ij}])$ is the attribute value which is provided by decision maker $D_i$ for the alternative $A_i$ with respect to the attribute $C_j$. Here, $[a^l_{ij}, a^R_{ij}]$ indicates the degree of the alternative $A_i$ satisfies the attribute $C_j$, $[b^l_{ij}, b^R_{ij}]$ indicates the degree of the alternative $A_i$ does not satisfies the attribute $C_j$ with the condition $0 \leq a^R_{ij} + b^l_{ij} \leq 1$, the decision making matrix $DM_{(i)}$ which is provided by $D_i$. The decision-making steps are shown as follows:

Step 1. Construct the decision-making matrix.

$$DM_{(i)} = \begin{bmatrix}
([a^l_{11}, a^R_{11}], [b^l_{11}, b^R_{11}]) & ([a^l_{12}, a^R_{12}], [b^l_{12}, b^R_{12}]) & \cdots & ([a^l_{1n}, a^R_{1n}], [b^l_{1n}, b^R_{1n}]) \\
([a^l_{21}, a^R_{21}], [b^l_{21}, b^R_{21}]) & ([a^l_{22}, a^R_{22}], [b^l_{22}, b^R_{22}]) & \cdots & ([a^l_{2n}, a^R_{2n}], [b^l_{2n}, b^R_{2n}]) \\
\vdots & \vdots & \ddots & \vdots \\
([a^l_{m1}, a^R_{m1}], [b^l_{m1}, b^R_{m1}]) & ([a^l_{m2}, a^R_{m2}], [b^l_{m2}, b^R_{m2}]) & \cdots & ([a^l_{mn}, a^R_{mn}], [b^l_{mn}, b^R_{mn}]) 
\end{bmatrix}_{m \times n}$$

(13)

Step 2. Calculate the entropy of the decision-making matrix.

$$E_i = \begin{bmatrix}
E([a^l_{11}, a^R_{11}], [b^l_{11}, b^R_{11}]) & E([a^l_{12}, a^R_{12}], [b^l_{12}, b^R_{12}]) & \cdots & E([a^l_{1n}, a^R_{1n}], [b^l_{1n}, b^R_{1n}]) \\
E([a^l_{21}, a^R_{21}], [b^l_{21}, b^R_{21}]) & E([a^l_{22}, a^R_{22}], [b^l_{22}, b^R_{22}]) & \cdots & E([a^l_{2n}, a^R_{2n}], [b^l_{2n}, b^R_{2n}]) \\
\vdots & \vdots & \ddots & \vdots \\
E([a^l_{m1}, a^R_{m1}], [b^l_{m1}, b^R_{m1}]) & E([a^l_{m2}, a^R_{m2}], [b^l_{m2}, b^R_{m2}]) & \cdots & E([a^l_{mn}, a^R_{mn}], [b^l_{mn}, b^R_{mn}]) 
\end{bmatrix}_{m \times n}$$

(14)
Step 3. Establish importance measure based on the entropy of expert decision-making.

1) Establish the absolute measure

Based on Definition 6, we can establish the absolute measure as follows:

$$
\mu(D_i) = 1 - E_i
$$

(15)

$$
\mu(D_i, D_j) = 1 - E(E_i \cap E_j) = 1 - E \left( \min(E_{i11}, E_{j11}) \quad \min(E_{i12}, E_{j12}) \quad \ldots \quad \min(E_{i1n}, E_{j1n}) \right)
$$

(16)

$$
\mu(D_{i1}, D_{i2}, \ldots D_{in}) = 1 - E(E_{i11} \cap E_{i12} \cap \ldots \cap E_{i1n})
$$

(17)

2) Establish the relative measure

Due to the importance measure should satisfy the normalization condition, so we should normalize the absolute measure before calculation, the concrete conversion formula is defined as follows:

$$
\overline{\mu(D_{i1}, D_{i2}, \ldots D_{in})} = \frac{\mu(D_{i1}, D_{i2}, \ldots D_{in})}{\mu(D_1, D_2, \ldots D_n)}
$$

(18)

Step 4. Calculate the weight of experts.

Based on Definition 10 and Eq.(5), we can obtain the experts weights based on Shapely value.

Step 5. Calculate the group decision making matrix.

Utilize the interval intuitionistic average operator (see Eq.(3)), and then collect the decision making information into the group decision making matrix:

$$
G = \left[ \begin{array}{cccc}
([a_{11}^L, a_{11}^R], [b_{11}^L, b_{11}^R])_G & ([a_{12}^L, a_{12}^R], [b_{12}^L, b_{12}^R])_G & \ldots & ([a_{1n}^L, a_{1n}^R], [b_{1n}^L, b_{1n}^R])_G \\
([a_{21}^L, a_{21}^R], [b_{21}^L, b_{21}^R])_G & ([a_{22}^L, a_{22}^R], [b_{22}^L, b_{22}^R])_G & \ldots & ([a_{2n}^L, a_{2n}^R], [b_{2n}^L, b_{2n}^R])_G \\
\vdots & \vdots & \ddots & \vdots \\
([a_{mn}^L, a_{mn}^R], [b_{mn}^L, b_{mn}^R])_G & ([a_{m2}^L, a_{m2}^R], [b_{m2}^L, b_{m2}^R])_G & \ldots & ([a_{mn}^L, a_{mn}^R], [b_{mn}^L, b_{mn}^R])_G \\
\end{array} \right]_{m \times n}
$$

(19)

Step 6. Calculate the group decision making score matrix and the entropy matrix.

Based on Definition 3 and Eq. (9), we can get the score matrix $S_G$ and entropy matrix $E_G$, which are defined as follows:

$$
S_G = \left[ \begin{array}{cccc}
s_{11} & s_{12} & \cdots & s_{1n} \\
s_{21} & s_{22} & \cdots & s_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
s_{m1} & s_{m2} & \cdots & s_{mn} \\
\end{array} \right],
E_G = \left[ \begin{array}{cccc}
e_{11} & e_{12} & \cdots & e_{1n} \\
e_{21} & e_{22} & \cdots & e_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
e_{m1} & e_{m2} & \cdots & e_{mn} \\
\end{array} \right]
$$

(20)

Step 7. Construct the programming model for solving attribute weights based on relative entropy.

In order to determine the optimal weight vector, the basic idea is to maximize the score function of each alternative, and minimize the entropy function of each alternative. According to the definition of relative entropy, we can establish the following linear programming model corresponding to the i-th alternative:

$$
\min W_i = \sum_{j=1}^{n} w_{ij} \ln \frac{e_j}{s_j}, \quad i = 1, 2, \ldots m
$$

s.t. \quad \sum_{j=1}^{n} w_{ij} = 1 \quad w_{ij} \geq 0

(21)

Solving this linear programming model, we can get the solution of the weight vector $W_i(i = 1, 2, \ldots m)$. 
Step 8. Solve the optimal attribute weight vector.

Based on the definition of Shannon entropy, we can determine the attribute weight vector as follows:

$$H_j = -\frac{1}{m \ln 2} \sum_{i=1}^{m} w_{ij}^{(i)} \ln w_{ij}^{(i)}, \quad j = 1, 2, \ldots, n$$  (22)

$$w_j = \frac{1 - H_j}{n - \sum_{k=1}^{n} H_k}, \quad j = 1, 2, \ldots, n$$  (23)

Step 9. Calculate the interaction coefficient $\lambda$.

We can calculate the value of $M_j$ with the following formula:

$$M_j = (1 + \frac{1}{m \ln 2} \sum_{i=1}^{m} w_{ij}^{(i)} \ln w_{ij}^{(i)})^2, \quad j = 1, 2, \ldots, n$$  (24)

Based on Eq. (5), we can get the interaction correlation coefficient based on the following equation:

$$\lambda = 1 + \prod_{j=1}^{n} (1 + M_j)$$  (25)

Step 10. Establish importance measure of attribute weights.

$$\mu(x_i) = \left[ (w_i^{(1,-)} - \lambda) \tan((1-H_i)\frac{\pi}{4}) \right], \left[ (1 - w_i^{(1,-)} \sin((1-H_i)\frac{\pi}{4}))^2, 1 - w_i^{(1,-)} \right]$$  (26)

$$\mu(x_{i_1}, x_{i_2}, \ldots, x_{i_n}) = \left[ (\sum_{i=i_1}^{i_2} w_i^{(1,-)} - \lambda) \tan\left( \prod_{i=i_1}^{i_2} (1-H_i) \frac{\pi}{4} \right) \right], \left[ (1 - (\sum_{i=i_1}^{i_2} w_i^{(1,-)} \sin(\prod_{i=i_1}^{i_2} (1-H_i) \frac{\pi}{4}))^2, 1 - (\sum_{i=i_1}^{i_2} w_i^{(1,-)} \right]$$  (27)

Step 11. Utilize Choquet integral to integrate the interval intuitionistic information of decision matrix.

Step 12. Utilize the score function to get the score of the overall alternatives.

Step 13. Utilize the score value to rank the alternatives.

5. Conclusions

In this paper, we have proposed a new method to determine the fuzzy measures of Choquet integral from decision making information directly. Based on the theory of interval intuitionistic fuzzy entropy, we construct a model with interval intuitionistic fuzzy measure to derive the experts weights and attribute weights, and through the strict mathematical reasoning to verify the method satisfies the axiomatic system of fuzzy measures. Using the Shapely value to aggregate the expert weights in the sense of fuzzy measure. The advantage of this fuzzy measure is the most to minimize the loss of the weight information and improve the accuracy of the decision making process. On the basis of our theoretical results, we propose a new Choquet integral method to handle interval intuitionistic fuzzy multiple attribute decision making problems. By establishing relative entropy model, an approach to determine the attribute weights of the problem is proposed. With these attribute weights, the fuzzy measures of Choquet integral is generated. This method is simple in the principle of mathematics, and can be easily fused with other multi-attribute group decision-making methods, therefore, it can be viewed as an effective expansion method under interval intuitionistic fuzzy decision making environment. However, due to the complexity of the problem in the real world, our method has inevitably exist some problems under complex decision-making environment, such as information fusion, some computational problems for large dimensional problems etc. Furthermore, how to expand this method to other fields such as computing with words, hesitate fuzzy sets and type-2 fuzzy sets etc. Hence, there are still many vital problems should be considered in the further research.
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